Measuring scrambling and topological invariants via randomized measurements

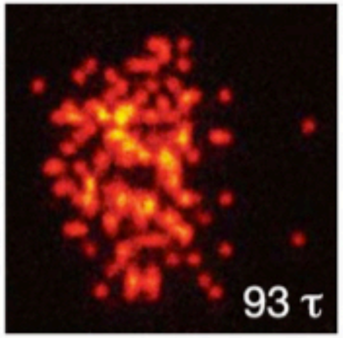
B. Vermersch (University of Innsbruck)

with A. Elben, L. Sieberer, J. Yu, G. Zhu, N. Yao, M. Hafezi, and P. Zoller

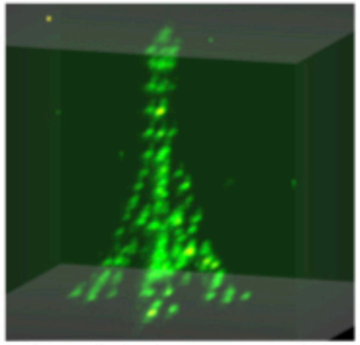
UNIVERSITY OF INNSBRUCK ÖAW UQUAM ERC Synergy Grant PASQuanS

Quantum simulators

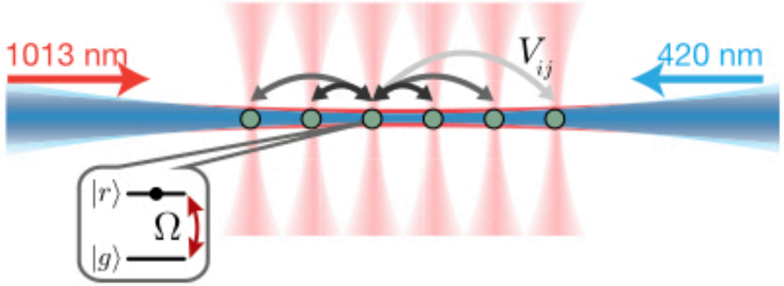
Ultracold atoms — Rydberg atoms



Choi et al., Science (2016)

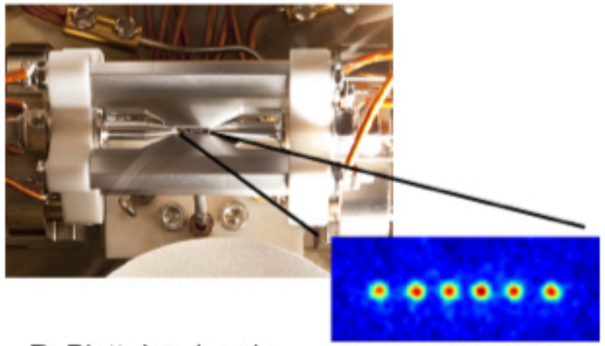


Barredo et al., Science (2016)



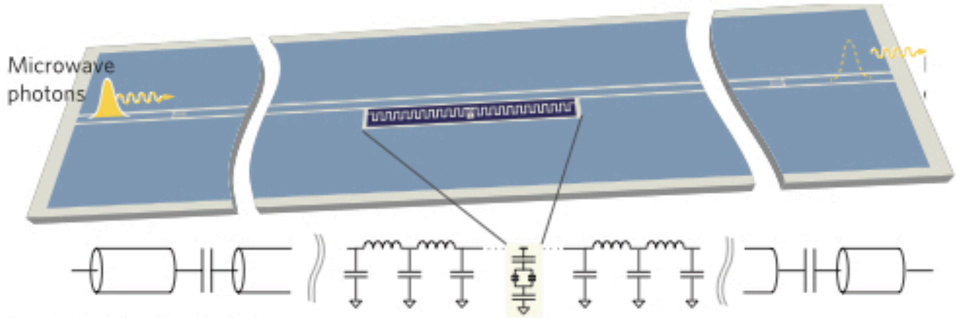
Bernien et al Nature 551, 579 (2017).

Trapped Ions



R. Blatt, Innsbruck

Superconducting circuits



A. Houck, Princeton

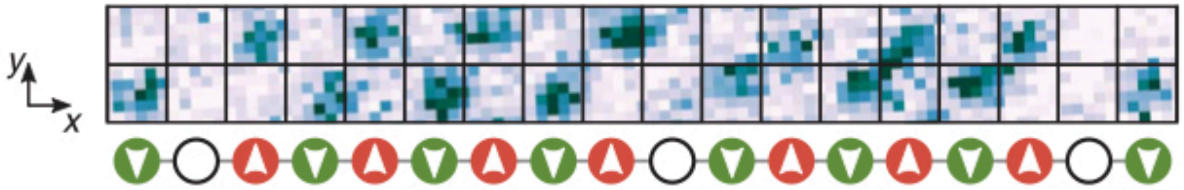
and Quantum dots, NV centers, cavity QED,..

Unique ways to create, probe, and understand quantum matter

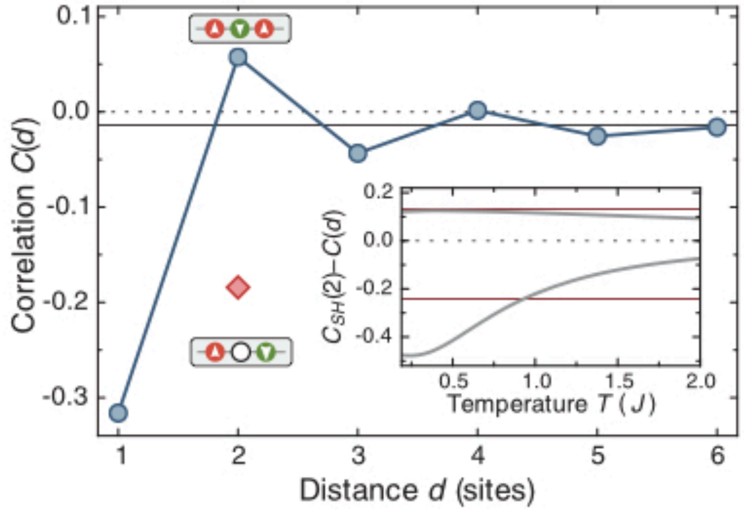
Characterizing quantum matter via correlation functions

Fermi-Hubbard model - Quantum Gas microscopes

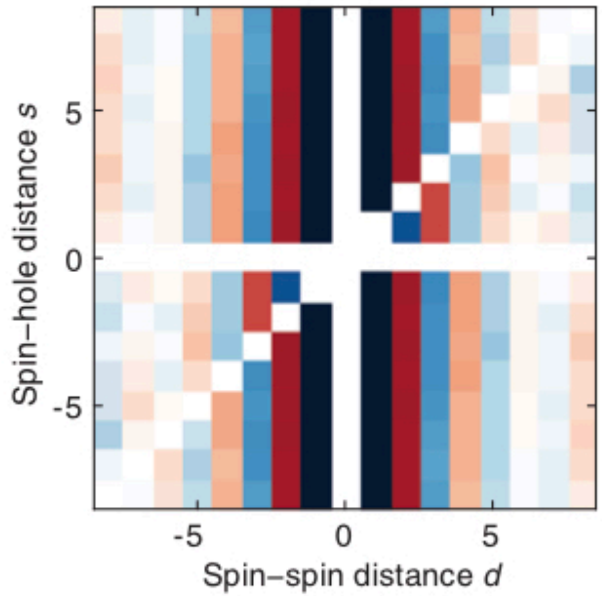
Hilker et al, Science 2016



2-point Correlation functions



String orders



Correlations functions are ``observable”

$$C = \mathrm{Tr}(\rho \hat{C})$$

→ Most common tool in AMO quantum simulation experiments.

Beyond ``standard" correlations: Entanglement

Definitions

$$\rho_A = \mathrm{Tr}_B(\rho)$$

Von Neumann entropy

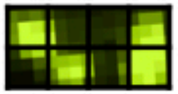
$$S = -\mathrm{Tr}(\rho_A \log(\rho_A))$$

Rényi entropies

$$S_n = \frac{1}{1-n} \log \mathrm{Tr}(\rho_A^n)$$

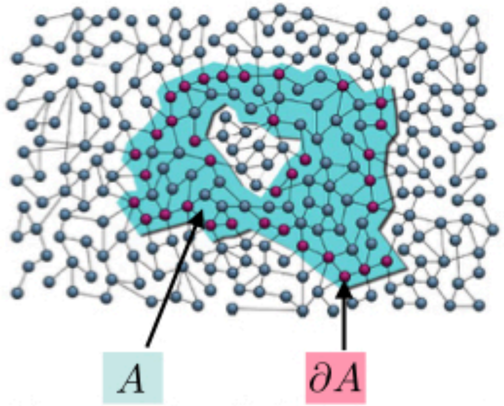
Measurement techniques

Copies

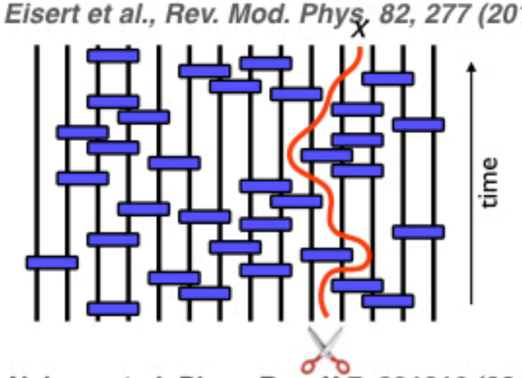


Daley et al . PRL, 109(2), 20505 (2012)
Islam et al., Nature 528, 77–83 (2015)

Equilibrium: Area laws, Quantum phase transitions, Thermalization…



Eisert et al., Rev. Mod. Phys. 82, 277 (2010)



Nahum et al, Phys. Rev. X 7, 031016 (2017)

Verification of quantum simulators

Coherence, entanglement

Random measuremenents (single copies)

van Enk et al, PRL (2012)
Elben, A.,BV, et al PRL 120(5), 50406 (2018)
BV .et al, . PRA, 97(2), 23604.(2018)
Brydges et al 2018 arXiv:1806.05747

Measuring entanglement via random measurements

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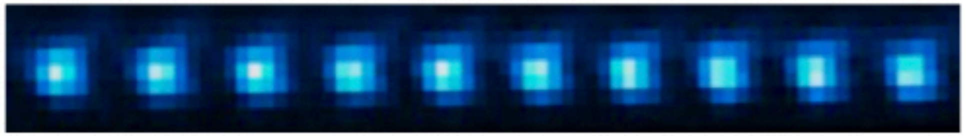
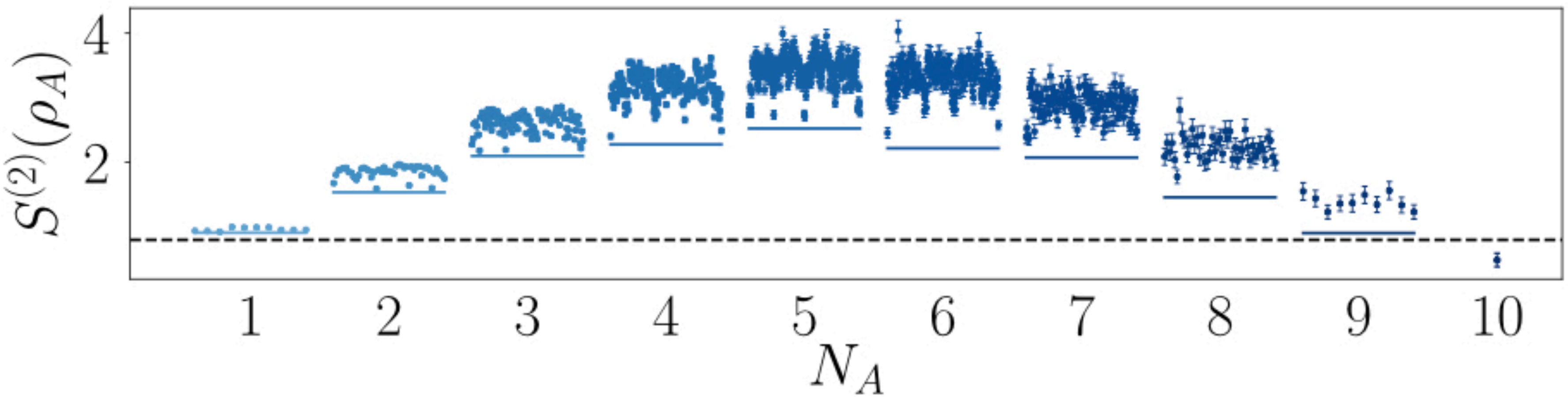
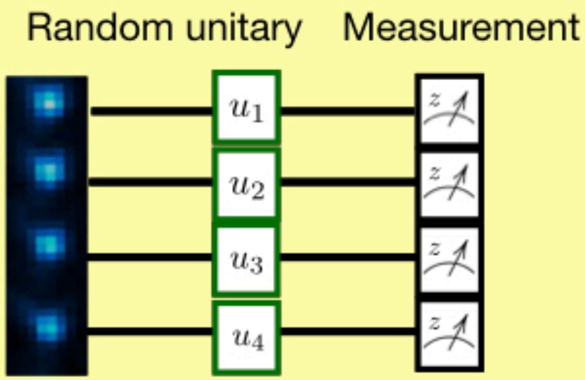
Brydges et al 2018 arXiv:1806.05747

(Collaboration with C. Roos-R. Blatt group)





The tool: random measurements



- Single copies
- Local Measurements
- Robust
- Simple

This talk: How to use this new tool to characterize and classify quantum matter

Take-home message Part 1

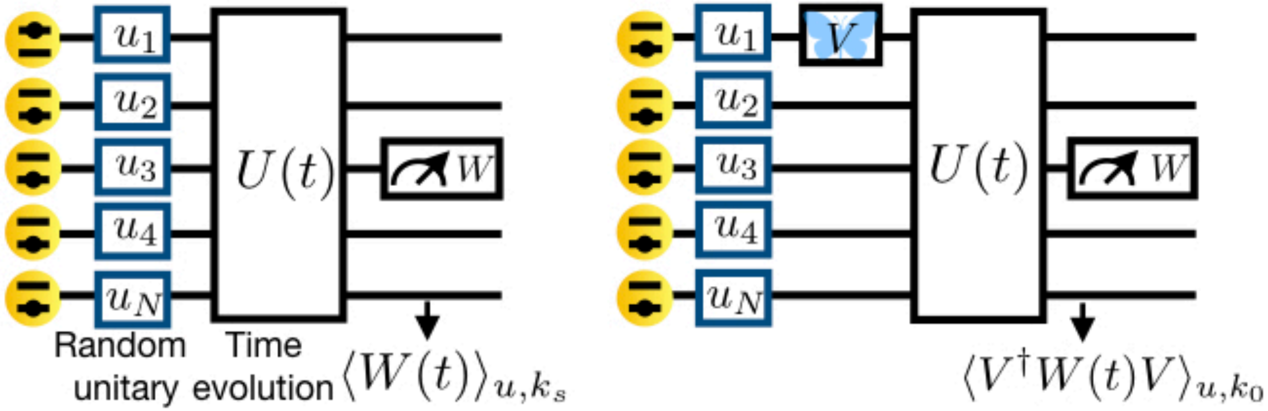
Out-of-time-ordered correlation functions via statistical correlations

Quantum Gravity

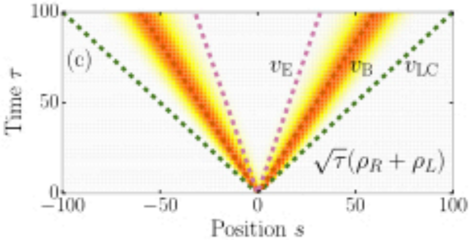
"Scrambling"

$$O = \mathrm{Tr}(\rho W(t) V W(t) V)$$

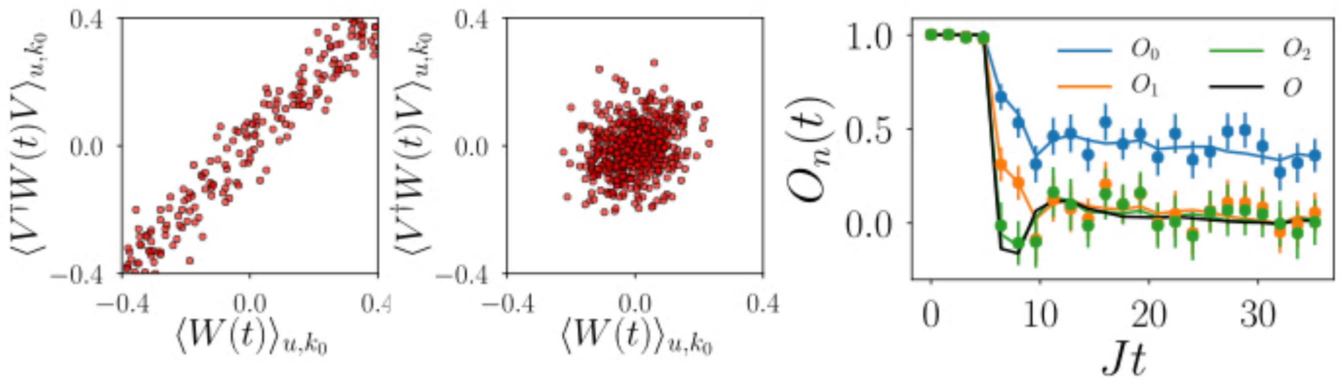
Protocol



Condensed-Matter



Phys. Rev. X 8, 031058 (2018)



→ No time-reversal → No overlap measurements → Applicable to many platforms

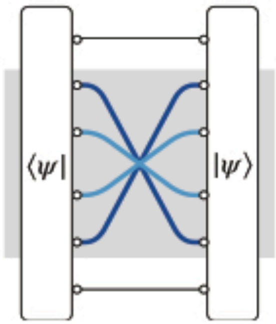
Take-home message Part 2

Statistics from correlated random unitaries = Topological invariants of SPT

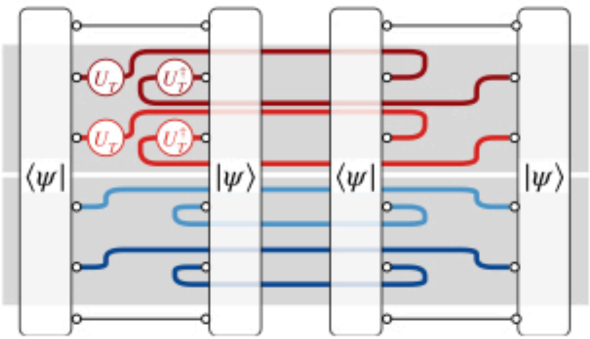
A toolbox for the classification of topological phases

Topological invariants

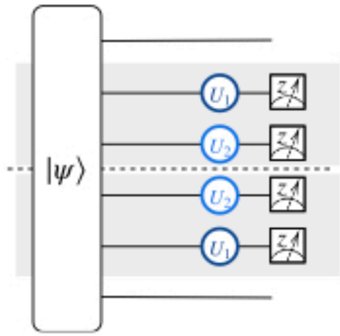
Inversion

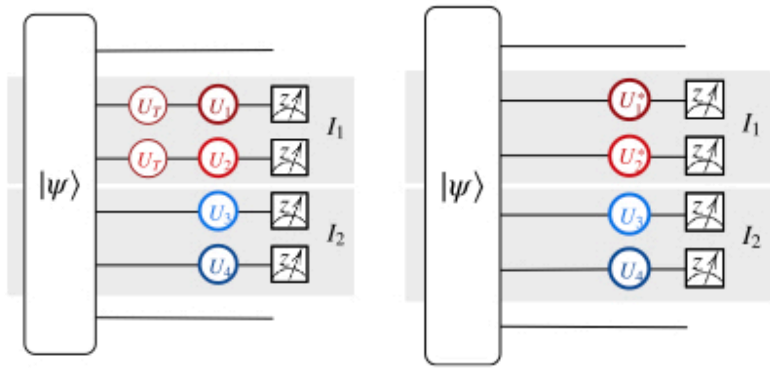


Time-Reversal



Protocols





→ First protocols → Applicable to any platform → New meaning to these quantities?

Measuring scrambling with random measurements

B. Vermersch, A. Elben, L. Sieberer, N. Yao, and P. Zoller

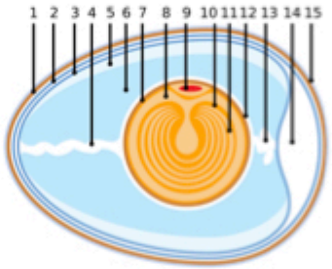
arxiv: 1807.09087

Classification of interacting topological phases (SPT)

A. Elben, B. Vermersch, J. Yu, G. Zhu, M. Hafezi and P. Zoller

in preparation

Information scrambling:
Loss of accessible information



→

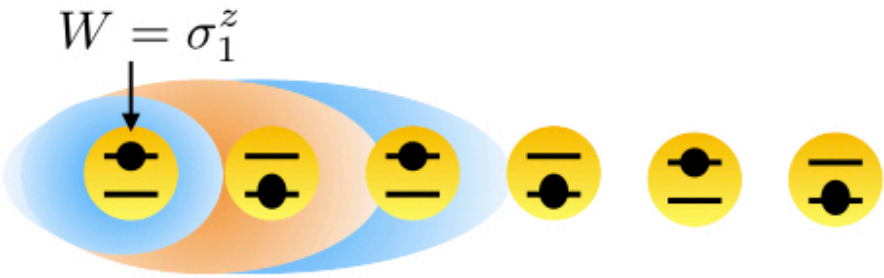
Black Hole Information Paradox:
How can information be trapped in a black hole while we receive Hawking radiation?

One conjecture: **Black holes are fast scramblers**: Hawking radiation Is only made of ``scrambled information", i.e cannot be decrypted

S. H. Shenker and D. Stanford, "Black Holes and the Butterfly Effect," J. High Energy Phys. **2014**, 67 (2014).

A. Kitaev, Talk at Fundamental Physics Prize Symposium Nov. 10, 2014.

J. Maldacena, S. H. Shenker, and D. Stanford, "A Bound on Chaos," J. High Energy Phys. **2016**, 106 (2016).



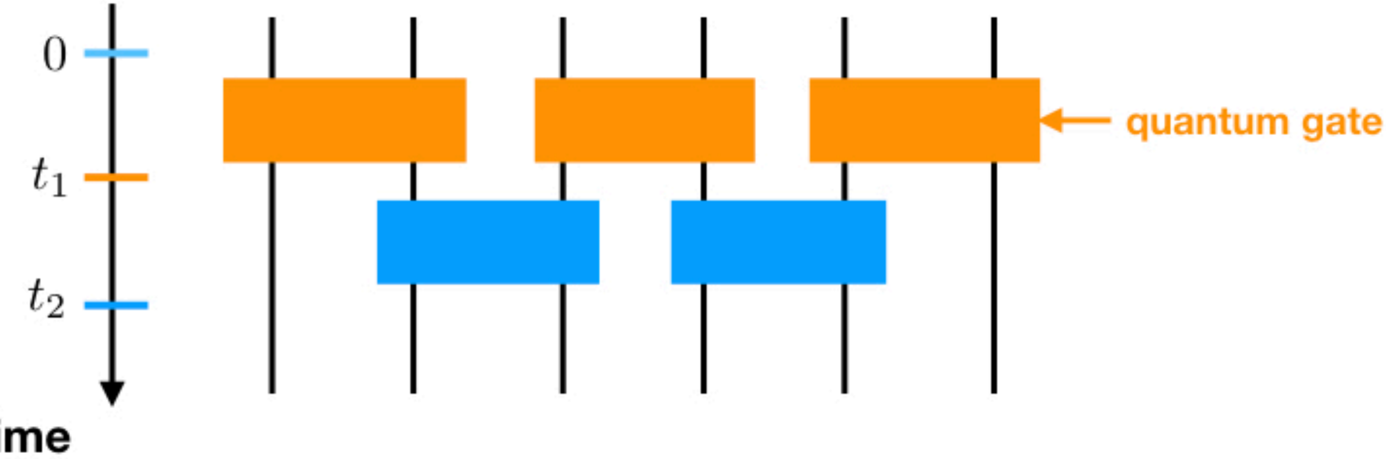
Operator spreading
(Heisenberg picture)

$$W(t) = U^\dagger(t) W U(t)$$

$U(t)$

Time evolution operator

ex: quantum gates



$$W(0) = \boxed{\sigma_1^z} \otimes I \otimes \cdots$$

$$W(t_1) = \sum_{\gamma_1, \gamma_2 = x,y,z,0} c_{\gamma_1,\gamma_2} \boxed{\sigma_1^{\gamma_1} \otimes \sigma_2^{\gamma_2}} \otimes I \cdots$$

⋮

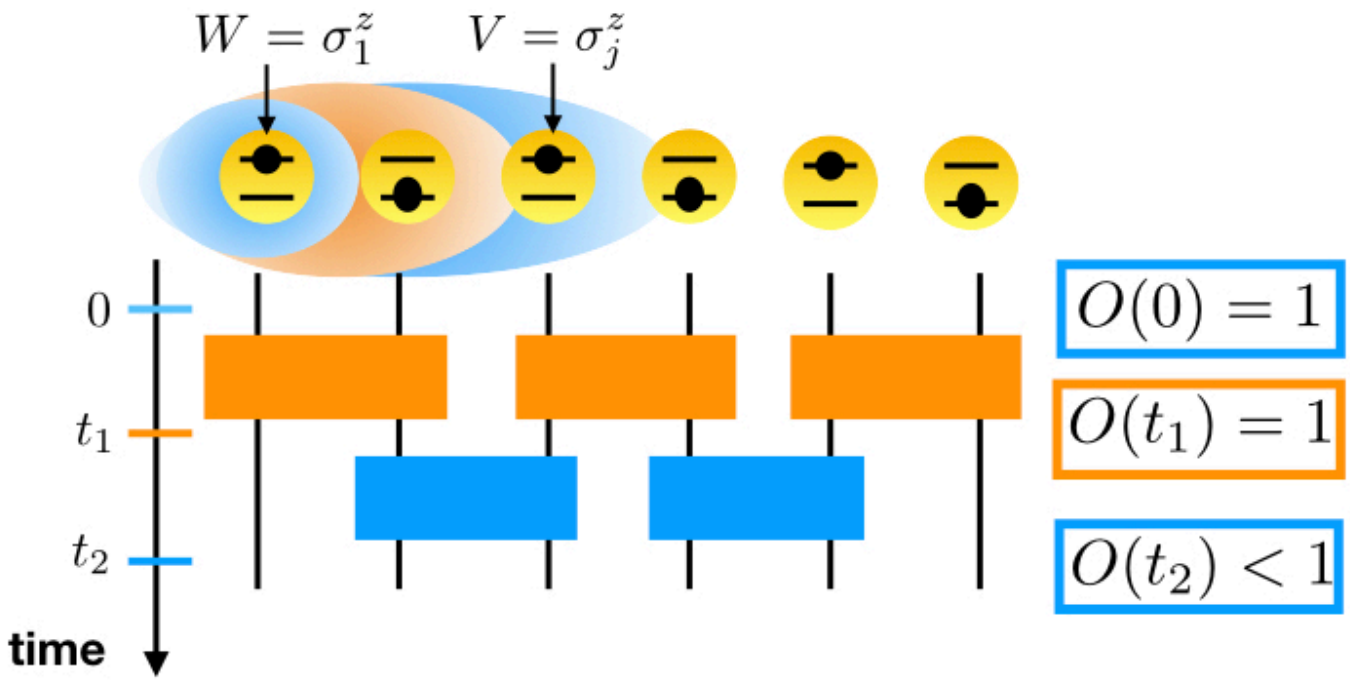
OTOC $O = \mathrm{Tr}(\rho W(t) V W(t) V)$

quantum state (here: V, W unitary and Hermitian operators)

$[W(t), V] = 0 \rightarrow O = 1$

$[W(t), V] \neq 0 \rightarrow O < 1$

Describe the spreading of an operator with respect to a `reference' V



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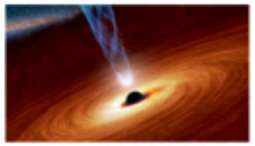
OTOCS: Information scrambling in many-body systems

Fast scramblers (analog black holes) → **Quantum Thermalization And chaos** → **Slow scramblers**

Sachdev-Ye-Kitaev (SYK) model



$$H_{SYK} = \frac{1}{4!}\sum_{jklm} J_{jklm}\chi_j\chi_k\chi_l\chi_m$$

$$O(t,r) \sim c_0 - c_1 e^{\lambda_L(t-r/v_B)}$$

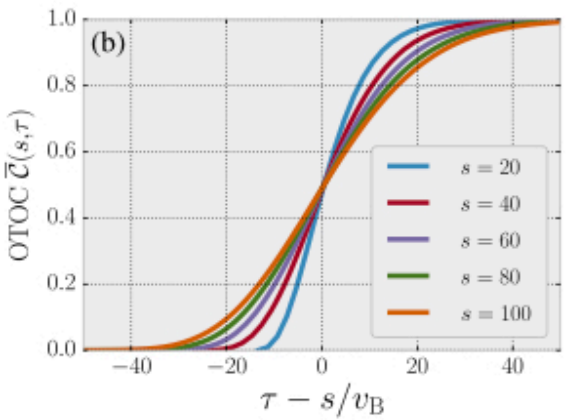
$$\lambda_L = 2\pi k_B T/\hbar$$

Sachdev, et al . PRL 1993 70(21), 3339–3342
Kitaev, A. KITP 2015
Banerjee et al 2017 PRB 95(13), 134302.

Random circuits

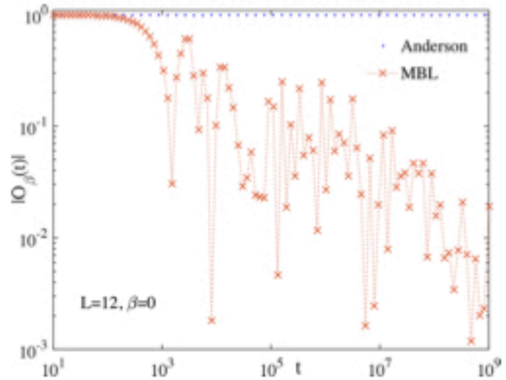
Quantum Ising models (inc. Long rang interactions)

Bose Hubbard



A. Bohrdt et al, New J. Phys. 19, 063001 (2017).
A. Nahum et al Phys. Rev. X 8, 021014 (2018).
C. W. von Keyserlingk et al Phys. Rev. X 8, 021013 (2018).
M. C. Tran, et al A. V. Gorshkov, arxiv:1808.05225 .

Many-body localization



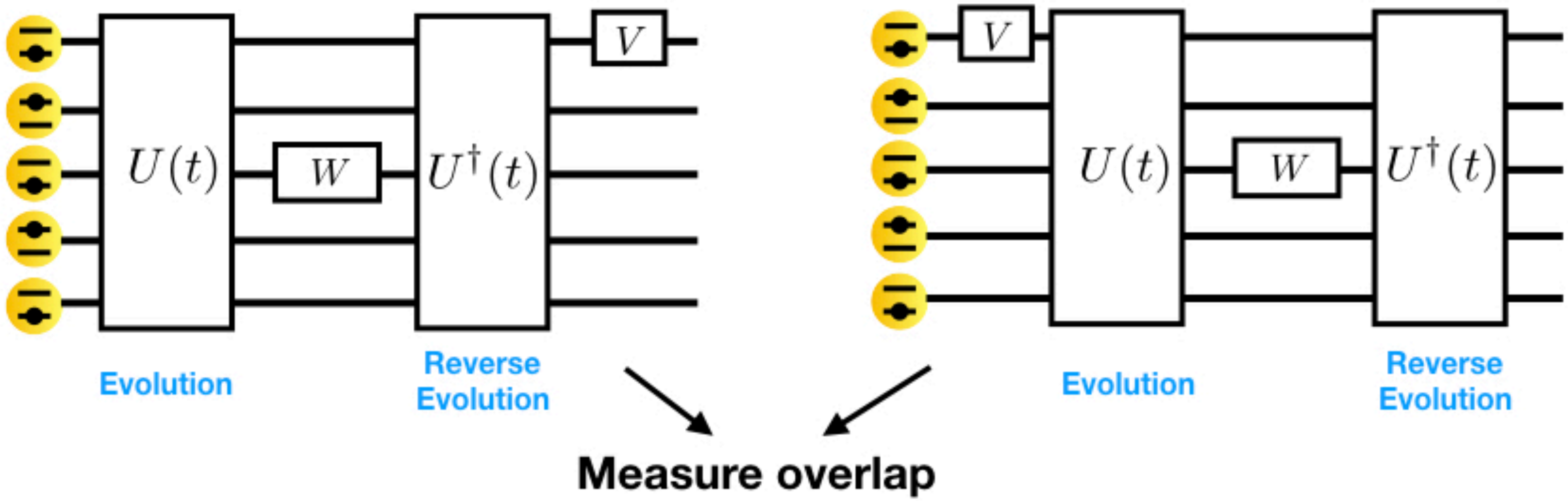
Fan, et al . Science Bulletin, 62(10), 707–711
Chen, X et al. Annalen Der Physik, 529(7)
...

Peculiar time-ordering in the definition implies `challenging' protocols

$$O = \mathrm{Tr}(\rho W(t) V W(t) V)$$

For a pure state $\rho = |\psi\rangle\langle\psi|$ $\quad O = \langle\psi_1|\psi_2\rangle$

$|\psi_1\rangle = V U^\dagger(t) W U(t) |\psi\rangle$ $\qquad |\psi_2\rangle = U^\dagger(t) W U(t) V |\psi\rangle$

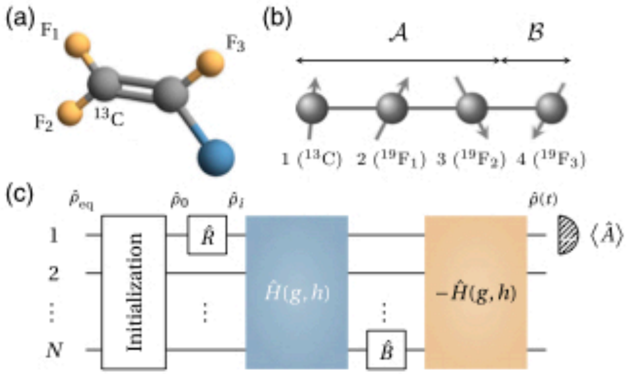


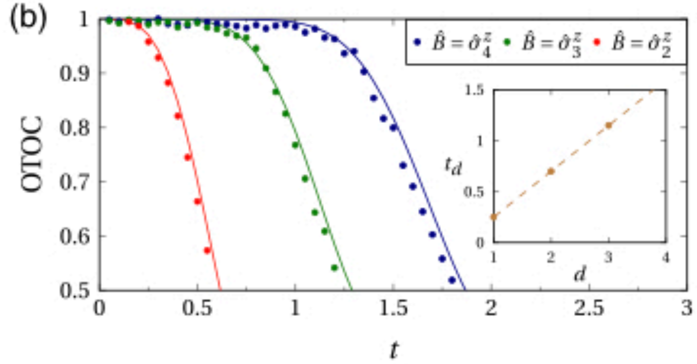
Requires time-reversal and/or copies and/or ancillas

Zhu et al . Phys. Rev. A 94 062329 (2016)
Swingle, B.et al Phys. Rev. A 94, 1–6 (2016)
Yao et al, arxiv:1607.01801
Garttner, M.,et al Nature Physics, 13(8), 781–786

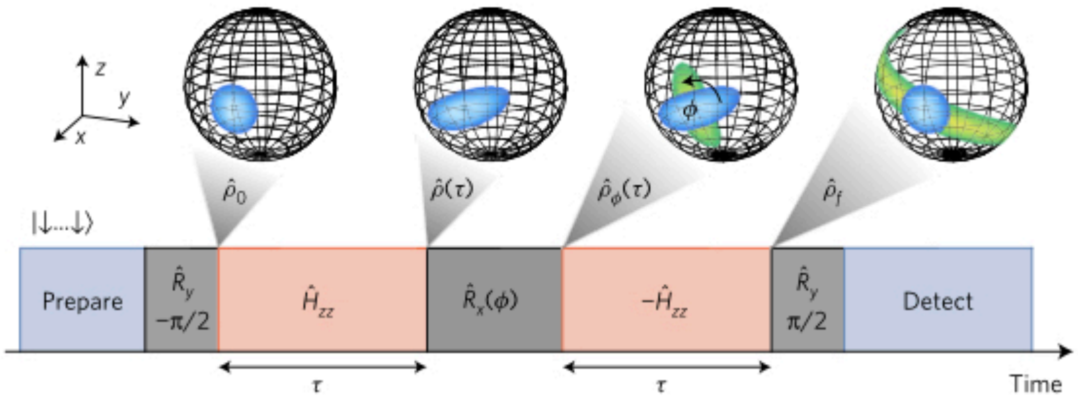
The two first measurements

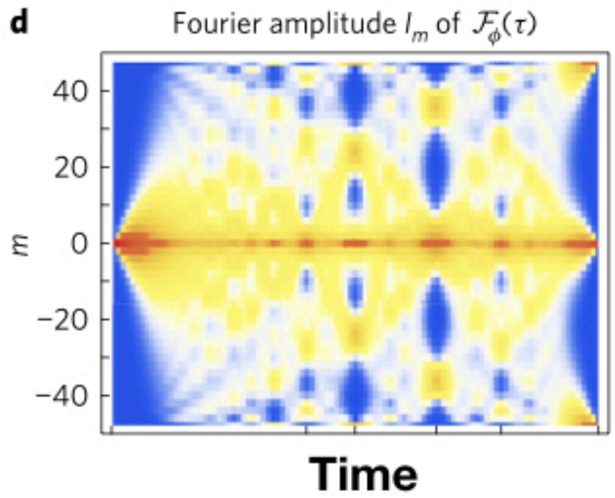
NMR Four spins Trotter evolution





Trapped ions (all-to-all interactions)

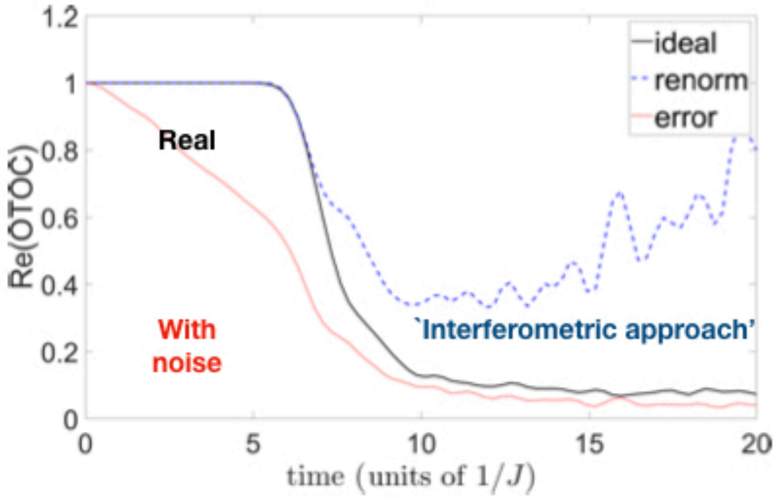


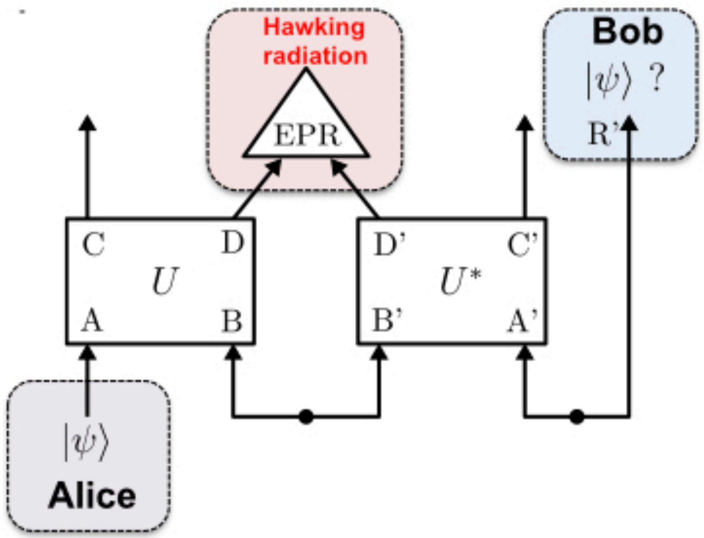


Key challenges: → **Implementing time-reversal** → **The role of decoherence**

J. Li, R. Fan, H. Wang, B. Ye, B. Zeng, H. Zhai, X. Peng, and J. Du, Phys. Rev. X 7, 031011 (2017).
M. Gärttner, J. G. Bohnet, A. Safavi-Naini, M. L. Wall, J. J. Bollinger, and A. M. Rey, Nat. Phys. 13, 781 (2017)
See also Viewpoint on Physics by Norm Yao and B. Swingle.

Decoherence versus scrambling





B. Swingle and N. Yunger Halpern, Phys. Rev. A 97, 062113 (2018).

B. Yoshida and N. Y. Yao, arXiv:1803.10772

K. A. Landsman et al, arxiv: 1806.02807

- Very important technological **challenges..**
- **Our approach:** Replace time-reversal by **statistical correlations**

Protocol: a simple butterfly experiment

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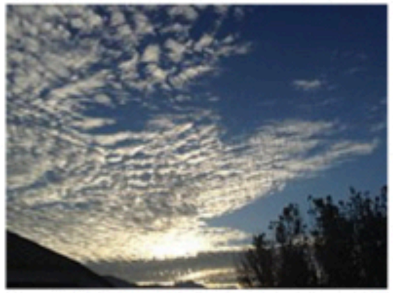
Inspiration: The Butterfly thought experiment

D. Diderot

E. Lorenz

Initial state → **Perturbation** → **Time-evolution** → **Observation**



Source: Wikipedia

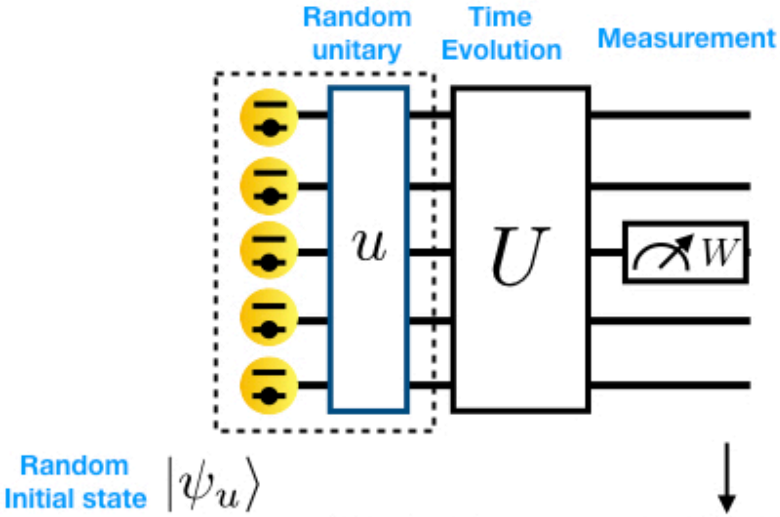
Perturbation → **Correlations?** ← Observation

Initial state	Perturbation	Time-evolution	Observation
random initial state	$V = \sigma_i^z$	$U(t)$	$W = \sigma_1^z$

Key idea: analyze **statistical correlations** over **random initial states** (instead of time reversal operations)

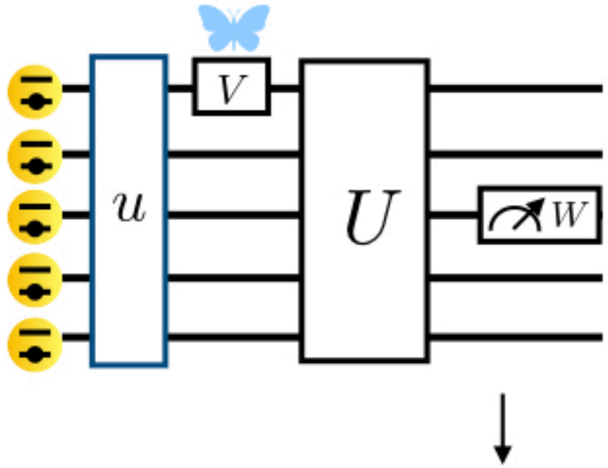
Protocol: a simple butterfly experiment

1st measurement (day 1)

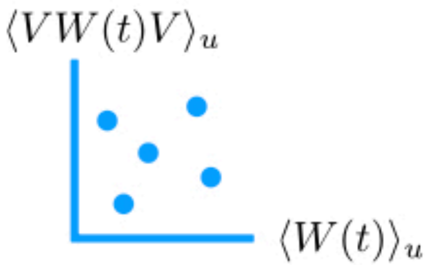


$$\langle W(t)\rangle_u = \langle \psi_u | W(t) | \psi_u \rangle$$

2nd measurement (day 2)



$$\langle V W(t) V\rangle_u = \langle \psi_u V | W(t) | V \psi_u \rangle$$



Statistical correlations = OTOCs $(T=\infty)$

$$O(t) = \frac{1}{\mathcal{D}^{(G)}} \overline{\langle W(t)\rangle_{u,k_0} \langle V^\dagger W(t) V\rangle_{u,k_0}}$$

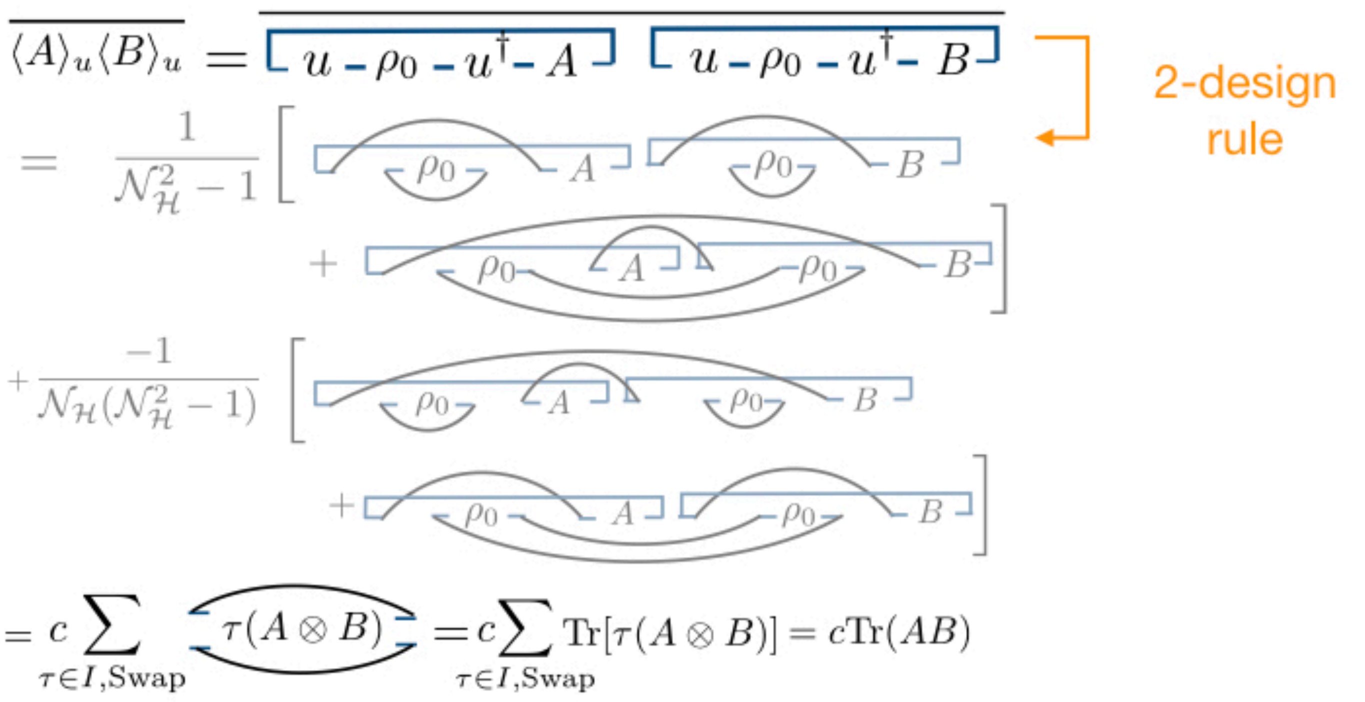
ensemble average over the circular unitary ensemble (CUE)

Elben, A.,BV, Dalmonte, M., Cirac, J. I., & Zoller, P. PRL 2018 120(5), 50406.
BV, Elben, A., Dalmonte, M., Cirac, J. I., & Zoller, P. PRA, 2018 97(2), 23604.

CUE (2-design):

$$
\begin{aligned}
&\overline{u_{m_1,n_1} u^*_{m'_1,n'_1} u_{m_2,n_2} u^*_{m'_2,n'_2}} \qquad (3)\\
&= \frac{\delta_{m_1,m'_1}\delta_{m_2,m'_2}\delta_{n_1,n'_1}\delta_{n_2,n'_2} + \delta_{m_1,m'_2}\delta_{m_2,m'_1}\delta_{n_1,n'_2}\delta_{n_2,n'_1}}{\mathcal{N}_{\mathcal{H}}^2 - 1}\\
&- \frac{\delta_{m_1,m'_1}\delta_{m_2,m'_2}\delta_{n_1,n'_2}\delta_{n_2,n'_1} + \delta_{m_1,m'_2}\delta_{m_2,m'_1}\delta_{n_1,n'_1}\delta_{n_2,n'_2}}{\mathcal{N}_{\mathcal{H}}(\mathcal{N}_{\mathcal{H}}^2 - 1)},
\end{aligned}
$$

(Collins 2006)

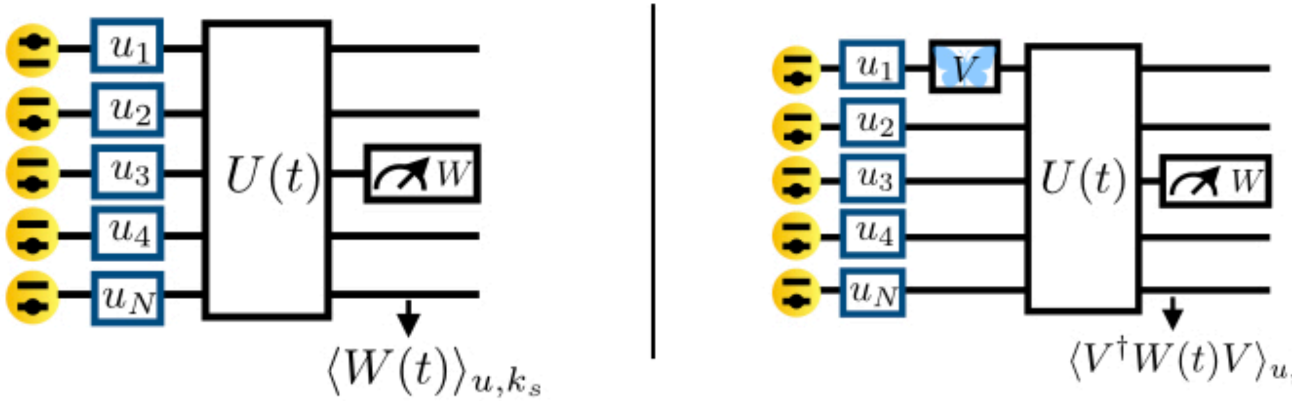


$$
O(t) = \frac{1}{\mathcal{D}^{(\mathrm{G})}} \overline{\langle W(t)\rangle_{u,k_0} \langle V^\dagger W(t) V\rangle_{u,k_0}}
$$

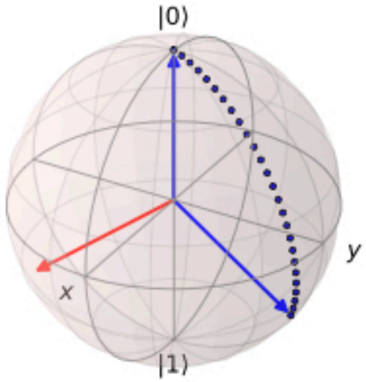
Protocol for spin systems with local unitaries

A much simpler protocol (for spins)

1st measurement (day 1) — 2nd measurement (day 2)



Single spin random rotation

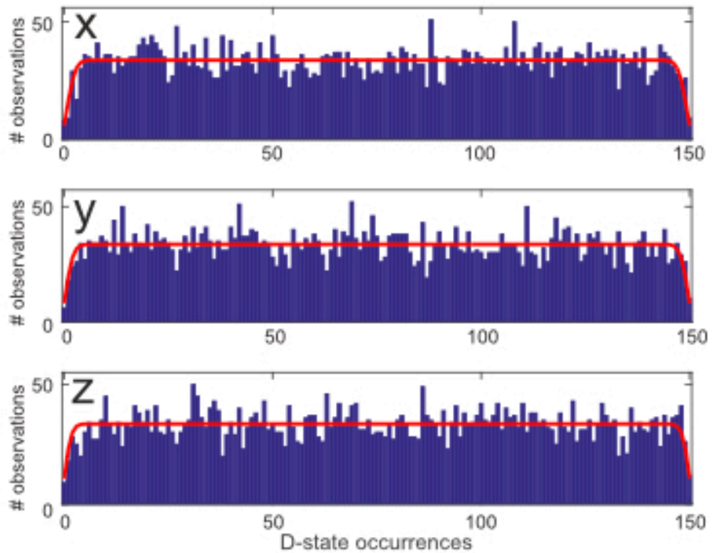


$$u_i \equiv u(\alpha_i, \beta_i, \gamma_i)$$

$$= Z(\alpha_i) Y(\pi/2) Z(\beta_i) Y(-\pi/2) Z(\gamma_i)$$

Available in state-of-the-art setups

Brydges et al 2018 arXiv:1806.05747



Protocol: a simple butterfly experiment

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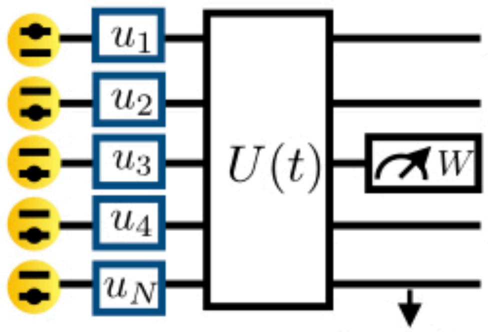
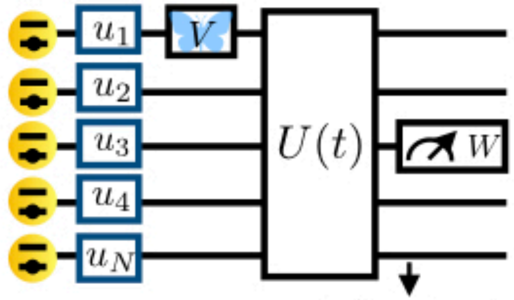
1st measurement (day 1)



$\langle W(t)\rangle_{u,k_s}$ ← Initial configuration

2nd measurement (day 2)



$\langle V^\dagger W(t) V\rangle_{u,k_0}$

Local statistical Correlations = Modified OTOCs

$\langle VW(t)V\rangle_u$

$\langle W(t)\rangle_u$

$$O_n(t) = \frac{1}{\mathcal{D}_n^{(\mathrm{L})}} \sum_{k_s \in E_n} c_{k_s} \overline{\langle W(t)\rangle_{u,k_s} \langle V^\dagger W(t) V\rangle_{u,k_0}},$$

$k_s \in E_n$ ← Set of 2^n initial states, n=0,..,N

$$O_n(t) = \frac{\sum_{A, B_n \subseteq A} \mathrm{Tr}_A \left(W(t)_A (VW(t)V)_A \right)}{\sum_{A, B_n \subseteq A} \mathrm{Tr}_A \left(W(t)_A W(t)_A \right)}$$

Modified OTOCs: → $O_N(t) = O(t)$

→ Fast converging series: n=0,1,2 is generically sufficient

Fast convergence of modified OTOCs: chaos

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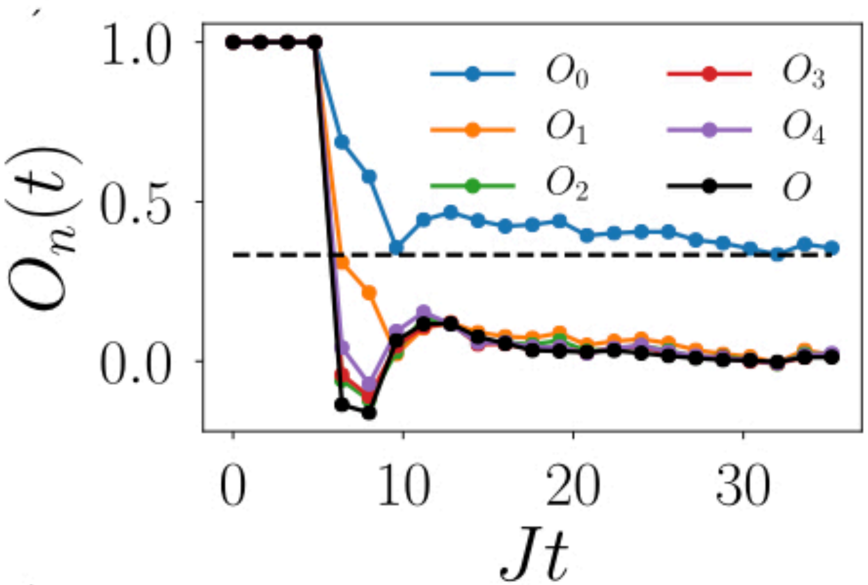
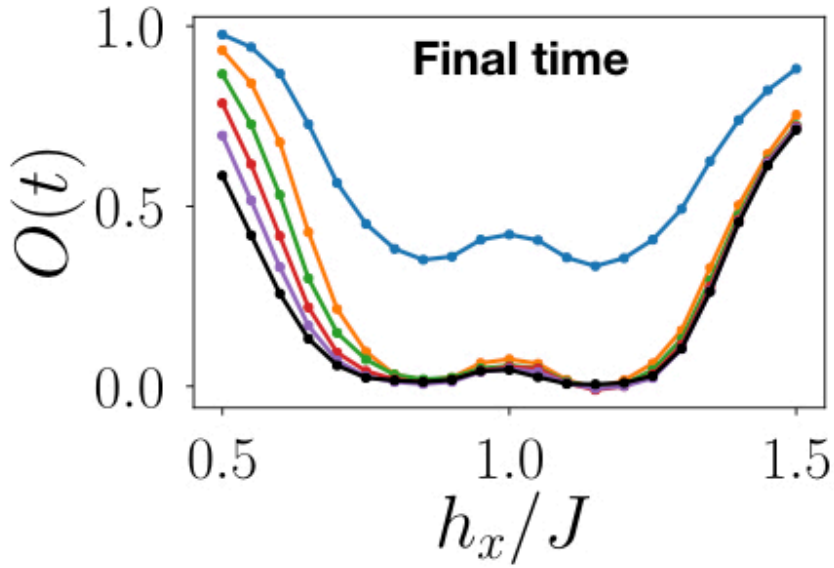
Example of Many-body Chaos: Kicked Ising with 8 sites





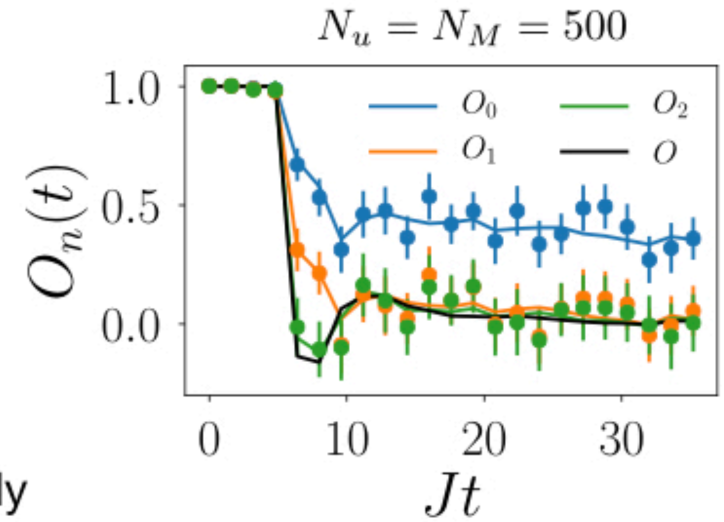
Statistical errors

$L(t) \approx v_B t$ "Scrambling length"

$N_M = 2^{L(t)} \longrightarrow$ error $1/\sqrt{N_u}$

projective measurements

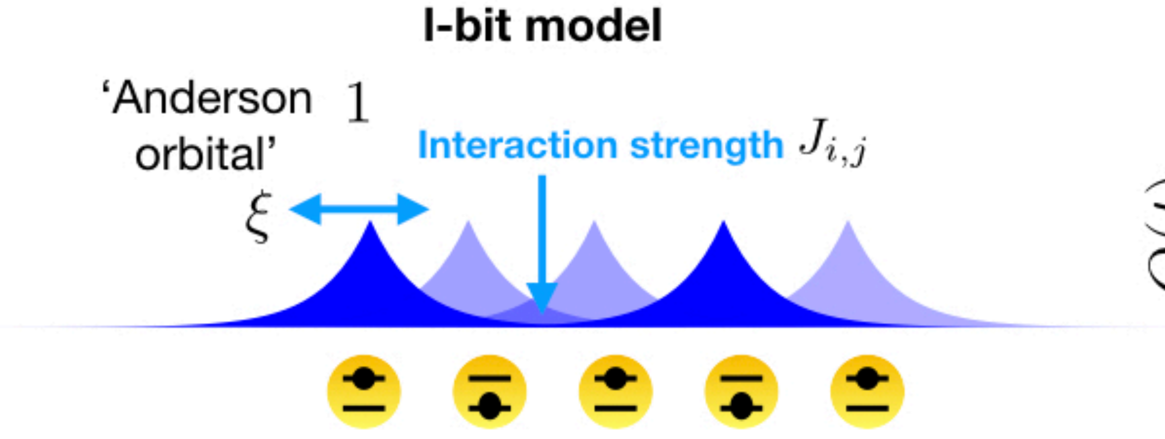
unitaries

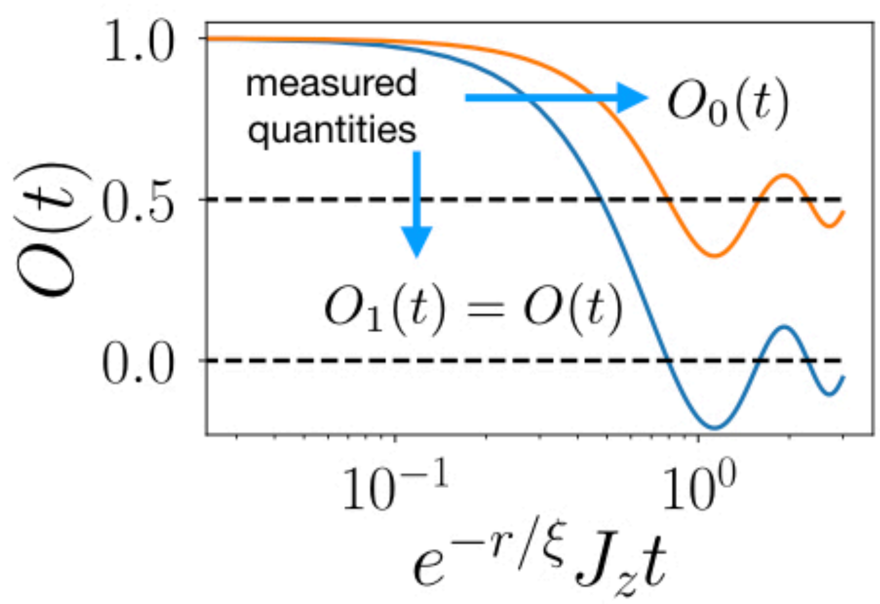


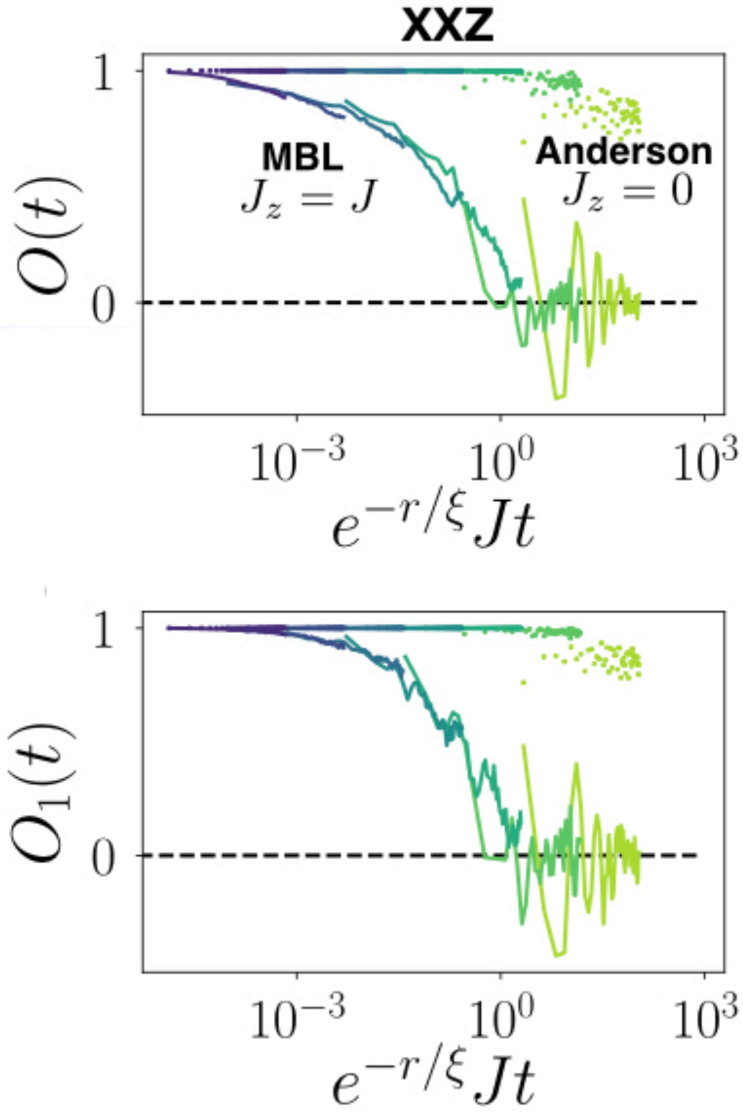
Independent of System Size

All N operators W are measured simultaneously

Example with Many-body Localization







OTOCs as diagnosis of scrambling

arxiv:1807.09087

can be measured in many-body systems with current technology

Random measurements are a generic tool

AMO implementations (no copies)

Statistical errors are not a fundamental issue

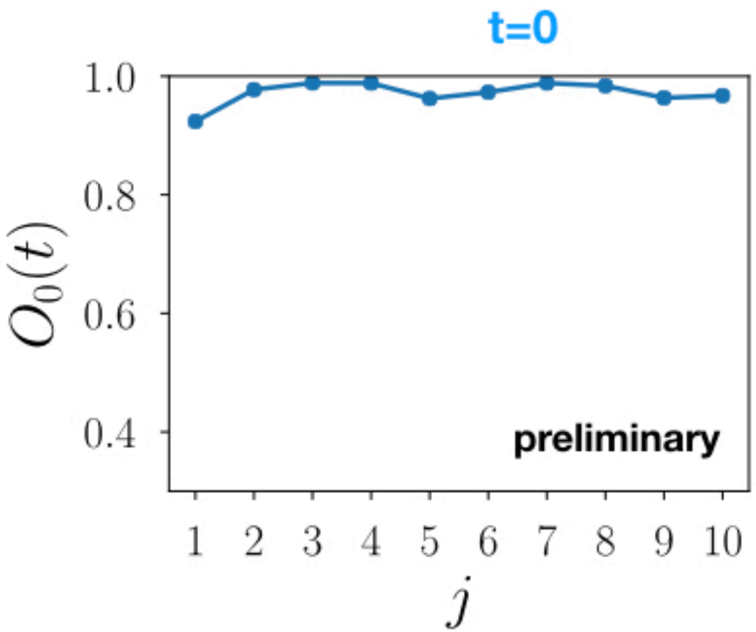
Natural robustness against errors and imperfections (ex: depolarization)

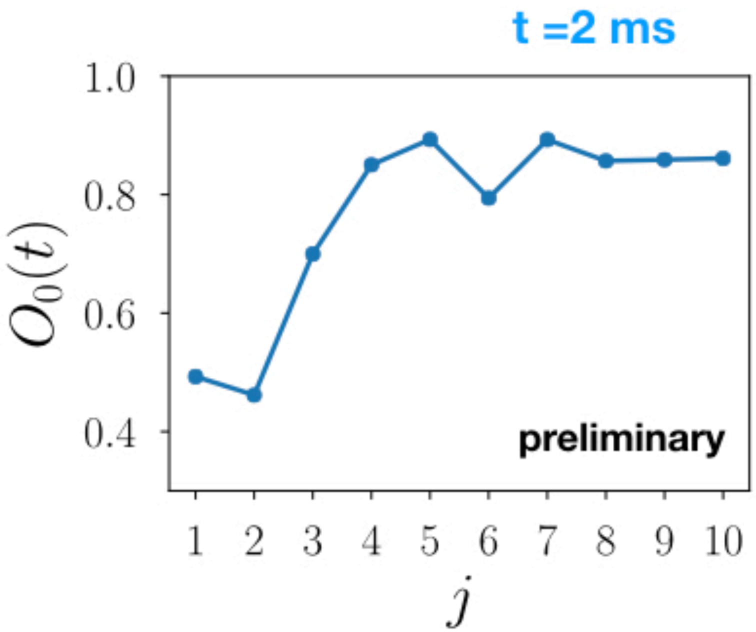
First experimental pictures

Collaboration with **M. Joshi, T. Brydges, C. Maier, C. Roos, and R. Blatt**

10 ions, evolution with long-range XY model

OTOCs in the x-basis

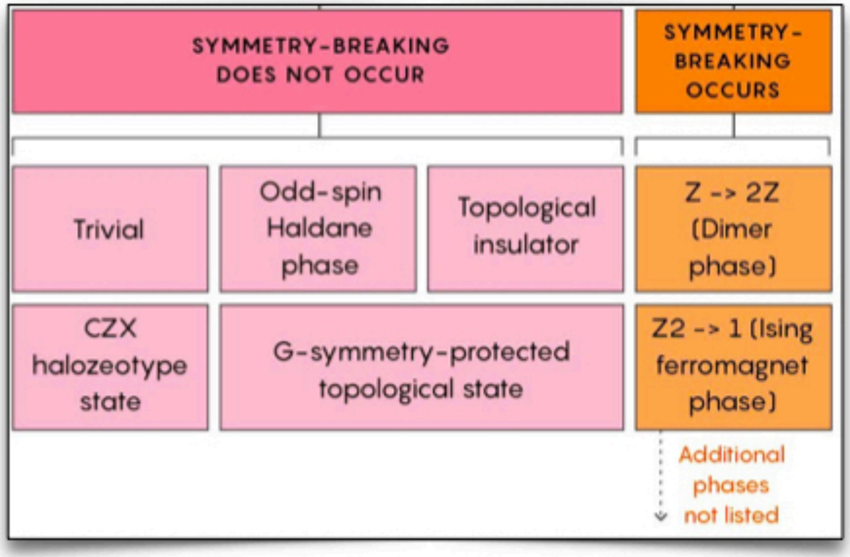




Measuring scrambling with random measurements

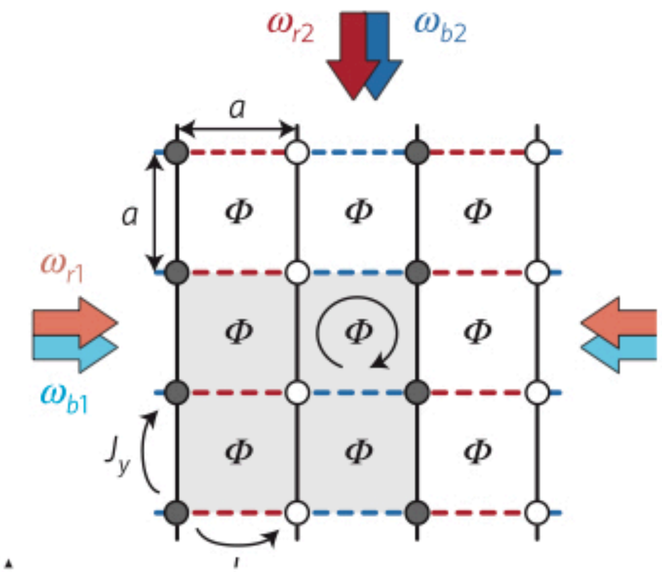
B. Vermersch, A. Elben, L. Sieberer, N. Yao, and P. Zoller

Classification of interacting topological phases (SPT)

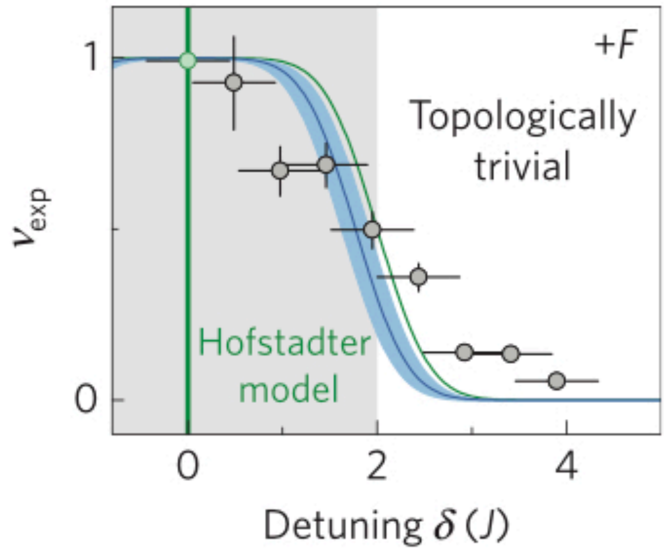


A. Elben, B. Vermersch, J. Yu, G. Zhu, M. Hafezi and P. Zoller

Measurements of Chern numbers in the lab



Aidelsburger et al Nature Physics 2015



The question of the Classification:

What are the equivalent of ``Chern Numbers" for **interacting topological phases?**

Topological invariants: Quantized non-local order parameters

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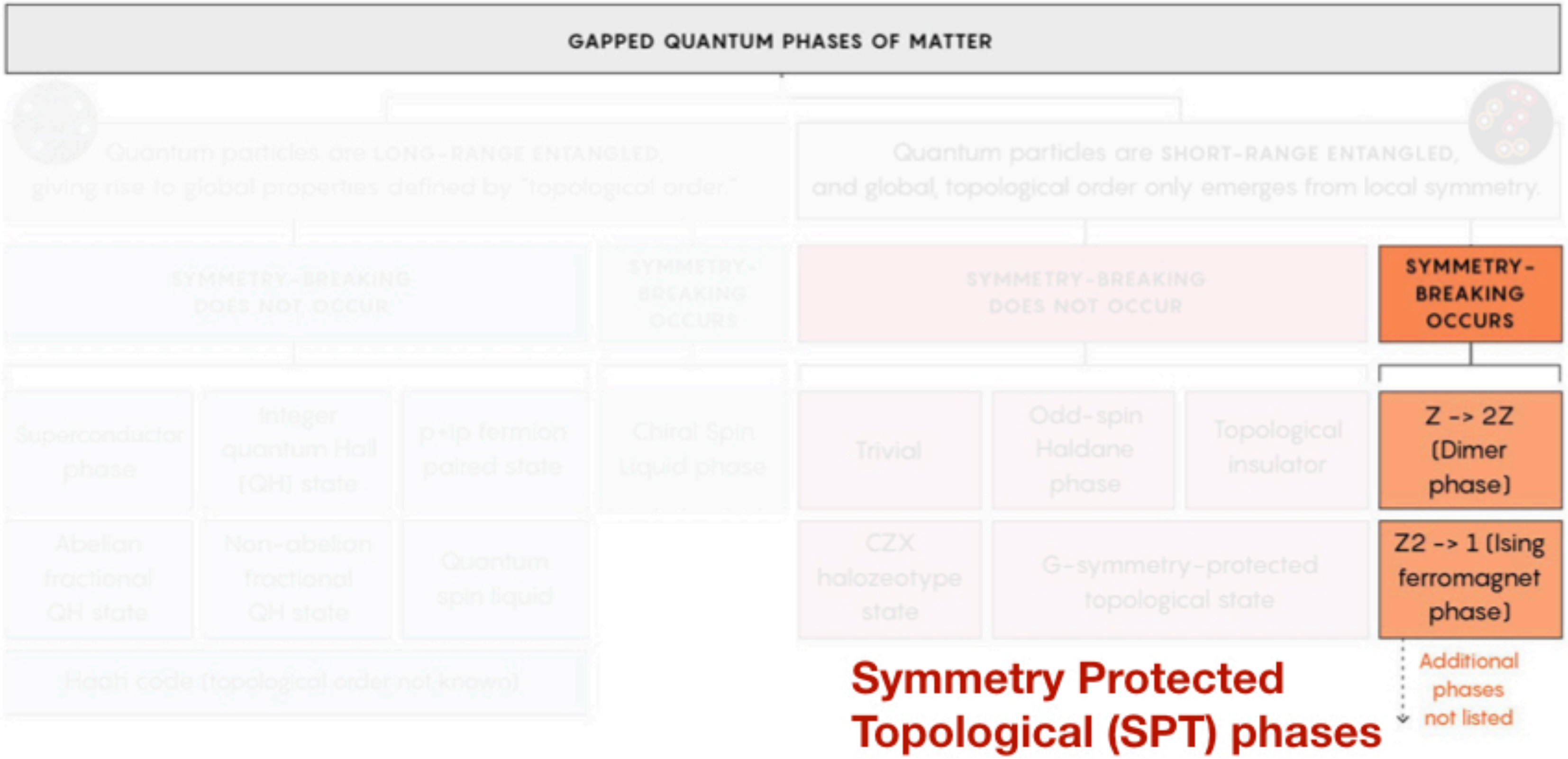
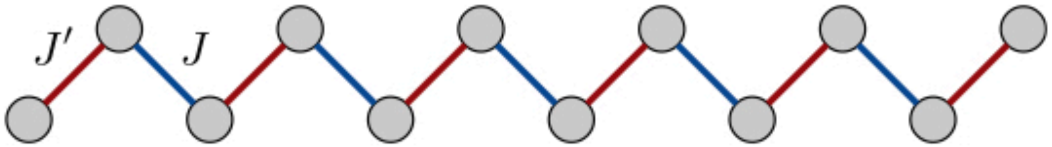


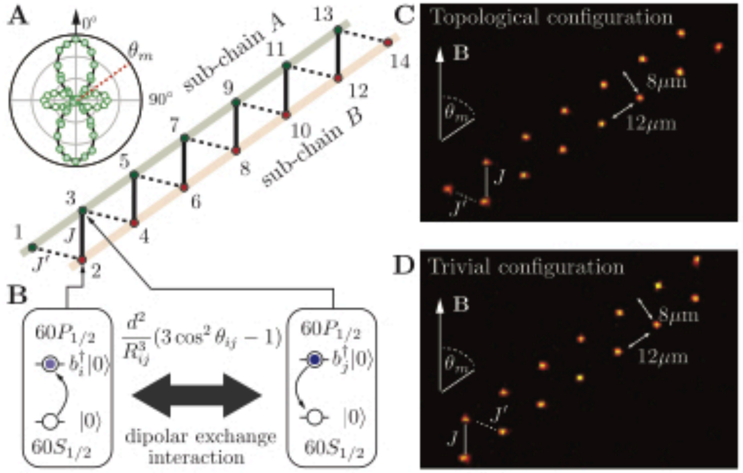
Pollmann et al. PRB 2010, Schuch et al. PRB 2011, Chen et al., Science 2012, PRB 2013, …

How to probe their classification in the lab?

Picture from Quanta Magazine, adapted from Xiao-Gang Wen

1D spin-1/2 model with alternating hoppings





Leseleuc et al., arXiv:1810.13286

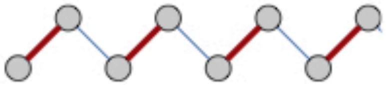
See talk by Antoine Browaeys

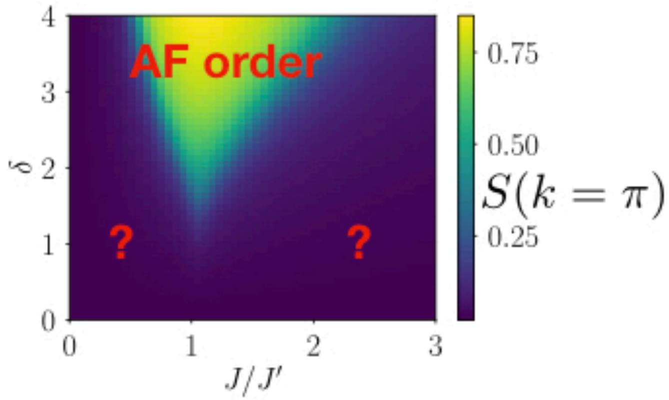
$$H = J' \sum_{i=1}^{N} \left(\sigma_{2i-1}^{-} \sigma_{2i}^{+} + \text{h.c.} + \frac{\delta}{2} \sigma_{2i-1}^{z} \sigma_{2i}^{z} \right.$$
$$\left. + J \sum_{i=1}^{N-1} \left(\sigma_{2i}^{-} \sigma_{2i+1}^{+} + \text{h.c.} + \frac{\delta}{2} \sigma_{2i}^{z} \sigma_{2i+1}^{z} \right) \right)$$

SB phase $|J'| \approx |J|, \delta \gg 1$

Trivial phase

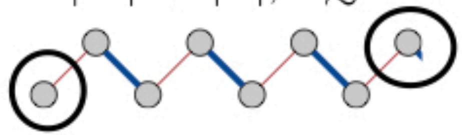
$|J'| \gg |J|, \delta \lesssim 1$





Topological phase

$|J'| \ll |J|, \delta \lesssim 1$

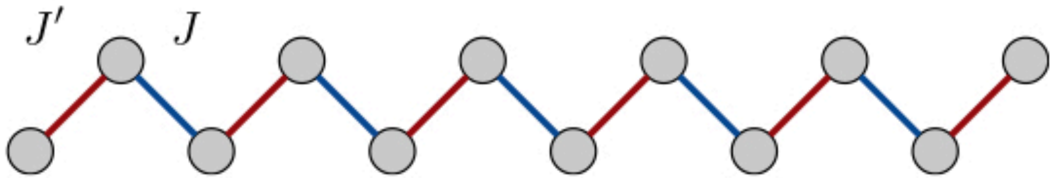


No local order parameter: **topological order**

Case study: the SSH model

Classification of SPT phases ⟷ Classification of the action of symmetry groups

Pollmann et al. PRB 2010, Schuch et al. PRB 2011, Chen et al., Science 2012, PRB 2013, …



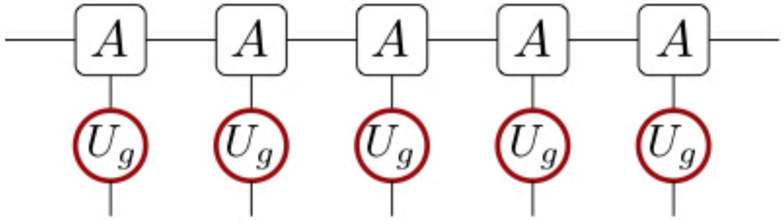
Type of symmetries

- Internal symmetries (rotations)
- Bond-centered inversion
- Time-reversal

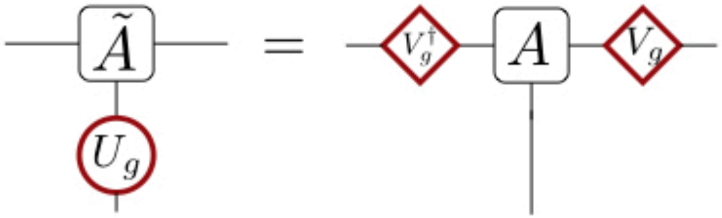
How to access and measure the action of symmetry groups?

Classification of 1D SPT phases via Matrix-Product-States

Haegeman et al., PRL, 2012
Pollmann et al. PRB 2012

IMPS: $\bigotimes_i U_g^i |\psi_{GS}\rangle =$  $= |\psi_{GS}\rangle$

Transformation under symmetry action



with

Projective representations of G

$$V_g \tilde{V}_h = e^{i\phi(g,h)} V_{gh}$$

On-site unitary: $\tilde{A} = A$
Inversion: $\tilde{A} = A^T$
Time reversal: $\tilde{A} = A^*$

Classification of SPT phases

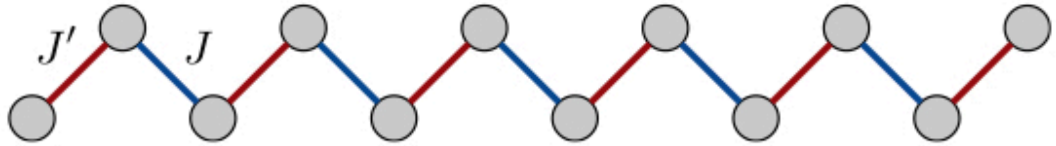
$$\left[e^{i\phi(g,h)}\right] \in H^2(G, U(1)_\phi)$$

Chen et al., Science 2012

← 2nd cohomology group of G

How to access and measure $\left[e^{i\phi(g,h)}\right]$ for a given G?

Case study: the SSH model



How to access and measure symmetry representations?

String-order

M. den Nijs and K. Rommelse, Phys. Rev. B 1989.

$$C^z_{\text{string}} = -\left\langle Z_2\, e^{i\frac{\pi}{2}\sum_{k=3}^{N-2} Z_k}\, Z_{N-1}\right\rangle$$

Can **detect** the effect of **internal symmetries**

Share the same values for different phases

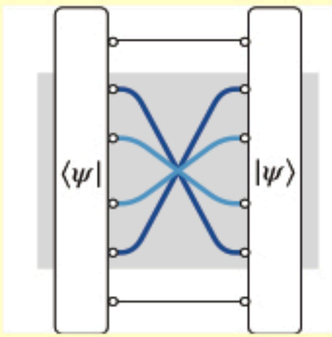
Cannot detect **other symmetries**

Pollmann, PRB, 2010

Not quantized

Topological invariants

Haegeman et al., PRL, 2012
Pollmann, Turner, PRB, 2012



Classification by direct identification of each symmetry representation $\left[e^{i\phi(g,h)}\right]$

Quantized

Strong Connections with topological quantum field theory, tensor-network theory

Ryu, PRL 2017

Key quantities for the classification

No protocols so far

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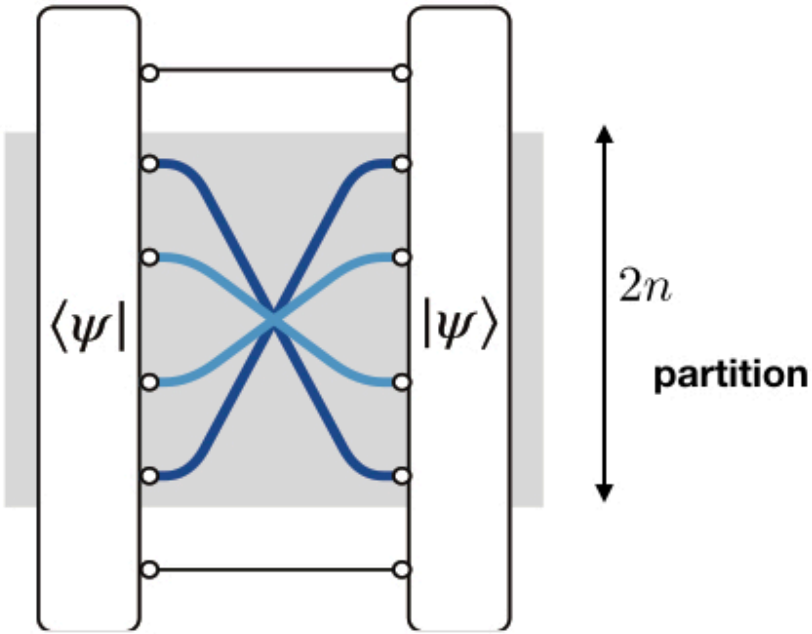
Partial inversion invariant

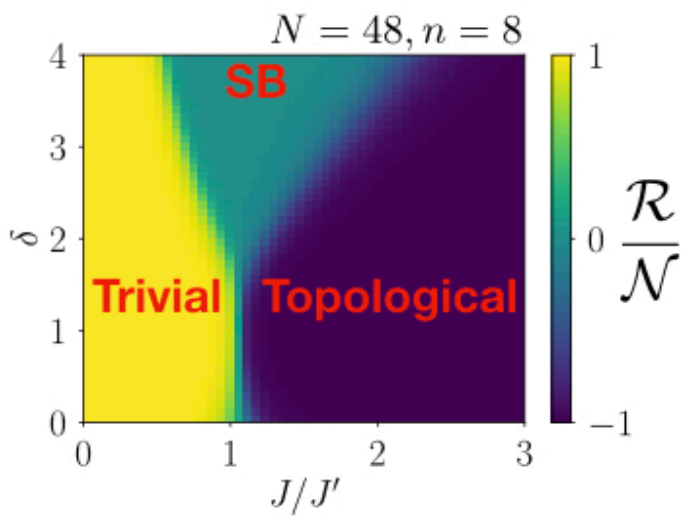
Pollmann, Turner, PRB 2012

$$\mathcal{R}(n) = \mathrm{Tr}\left[\mathbb{S}_{I_1,I_2} \left|\Psi\right\rangle \left\langle\Psi\right|\right]$$

$$\xrightarrow[n\to\infty]{\text{MPS theory}} \pm\mathrm{Tr}\left[\rho_{HP}^2\right] \leftarrow \text{purity}$$



Application to the SSH model

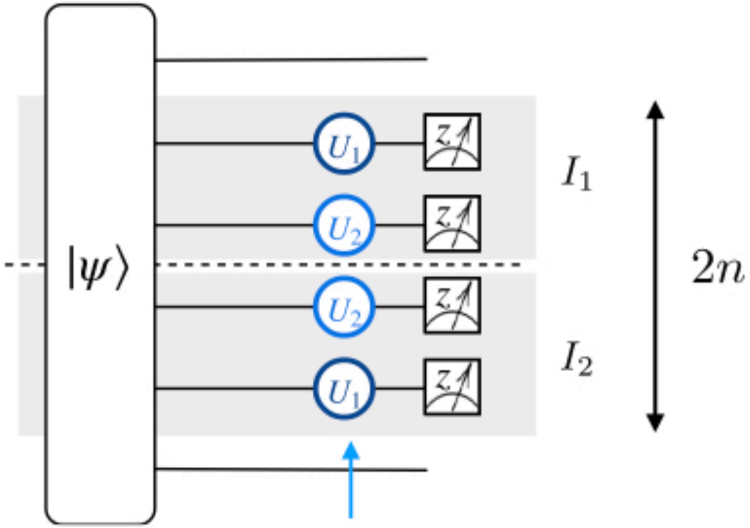


The partial inversion invariant classifies the whole SSH model

How to measure such *non-local* correlations in an experiment?

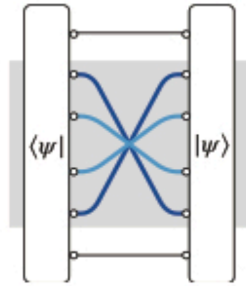
Measuring topological invariants

Idea: Correlate random unitaries *in space*



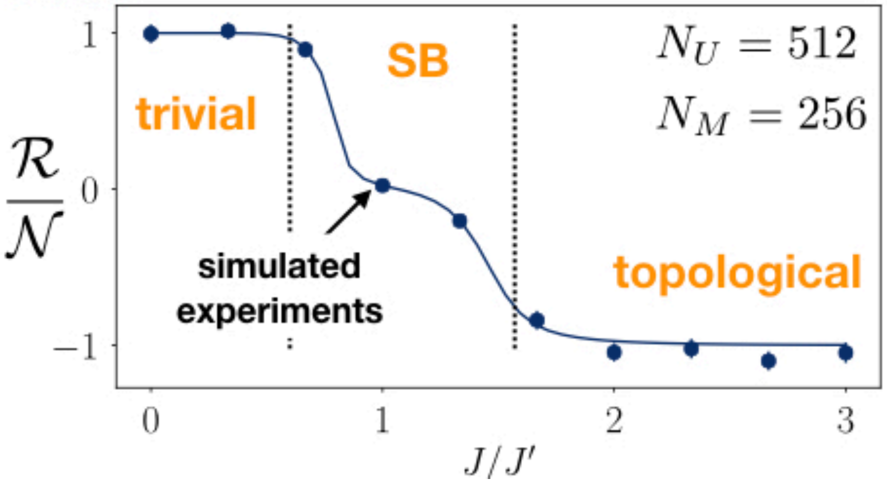
single qubit
random unitaries:

$$d^n \sum_{\mathbf{s}_{I_1}, \mathbf{s}'_{I_2}} (-d)^{-D[\mathbf{s}_{I_1}, \mathbf{s}'_{I_2}]} \overline{P_{U \otimes U}(\mathbf{s}_{I_1}, \mathbf{s}'_{I_2})}$$
$$= \mathrm{Tr}\left[\$_{I_1, I_2} \left|\Psi\right\rangle \left\langle\Psi\right|\right]$$
$$\xrightarrow[n \to \infty]{} \pm \mathrm{Tr}\left[\rho^2_{HP}\right]$$



Classification via random measurements:
apply a distribution with encodes the symmetry to characterize

SSH - model

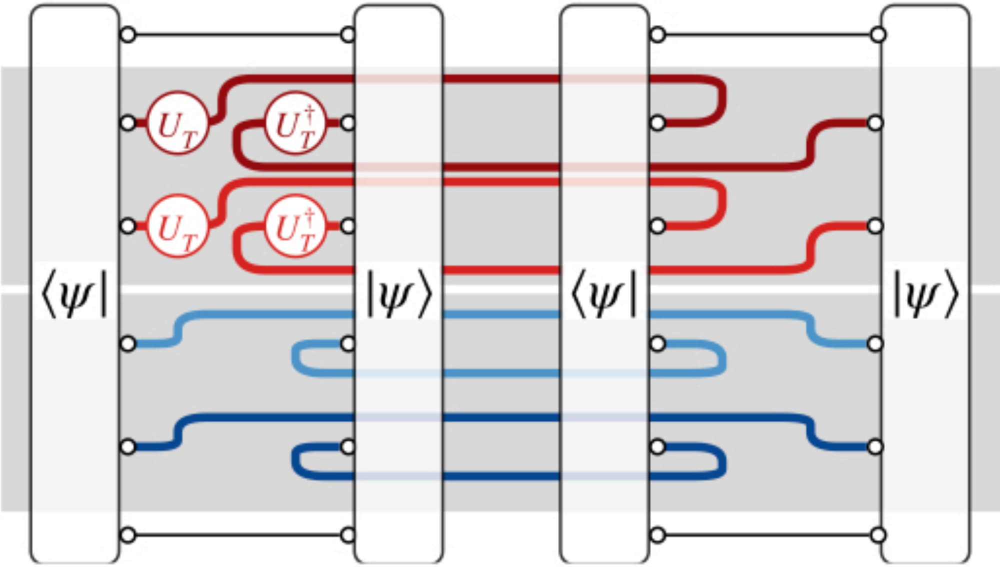


Error bars and bias correction with Jackknife resampling

$$N = 24, n = 6, \delta = 2.5$$

Time-reversal symmetry

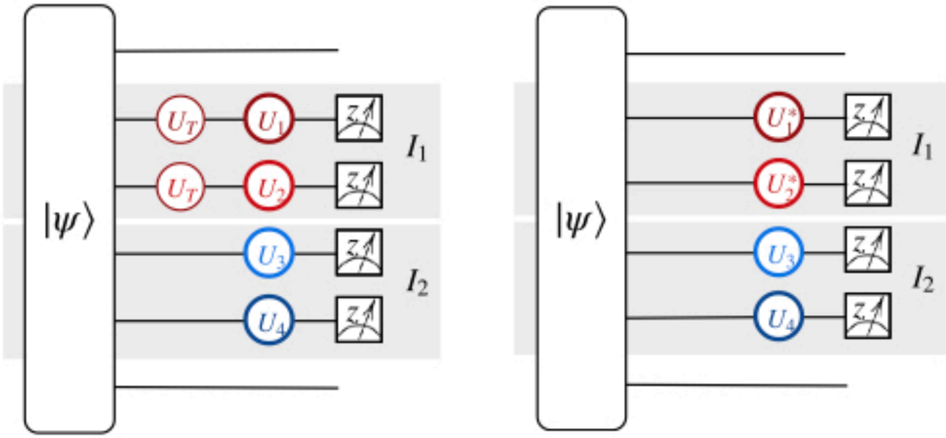
Partial transpose invariant



$$\mathcal{T}(n) = \mathrm{Tr}\left[R_{I_1} S_{I_2} \left|\Psi \otimes \Psi\right\rangle \left\langle \Psi \otimes \Psi\right|\right]$$

$$\xrightarrow[n \to \infty]{\text{MPS theory}} \pm \mathrm{Tr}\left[\rho_{HP}^2\right]^3$$

Protocol: Correlate *two* experiments



$\longrightarrow P_U^{(1)}(\mathbf{s})$ $\longrightarrow P_U^{(2)}(\mathbf{s}')$

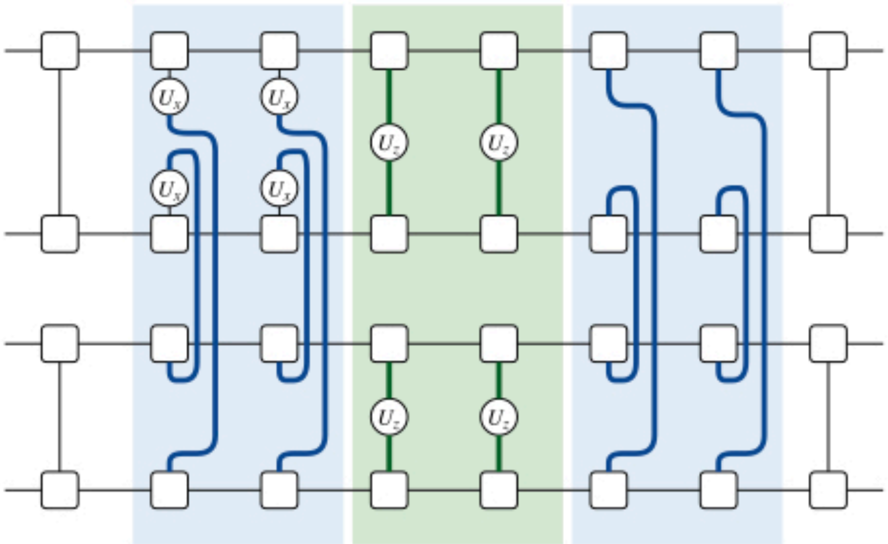
$$d^{2n} \sum_{\mathbf{s},\mathbf{s}'} (-d)^{-D[\mathbf{s},\mathbf{s}']} \overline{P_U^{(1)}(\mathbf{s}) P_U^{(2)}(\mathbf{s}')}$$

$$= \mathrm{Tr}\left[R_{I_1} S_{I_2} \left|\Psi \otimes \Psi\right\rangle \left\langle \Psi \otimes \Psi\right|\right]$$

Other Symmetries

Onsite unitary symmetry

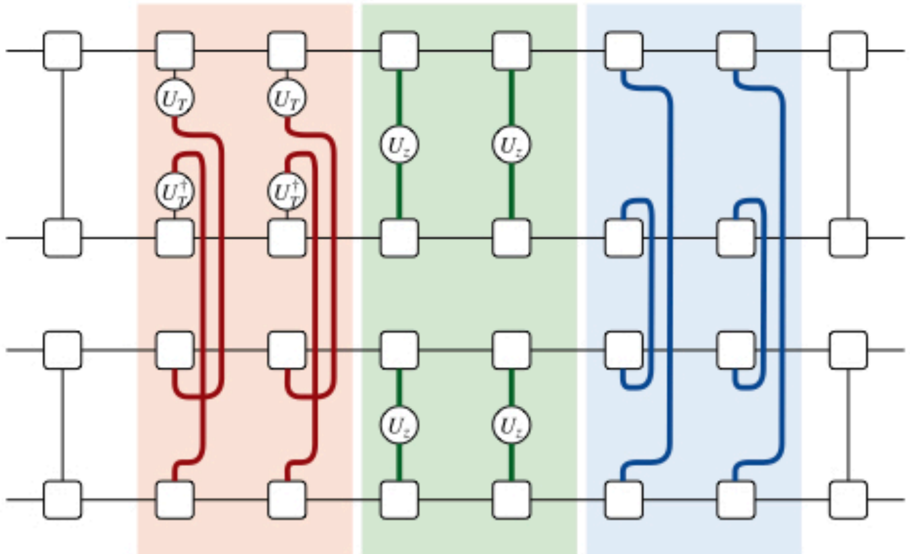
SSH: $\mathbb{Z}_2 \times \mathbb{Z}_2$ $\quad e^{i\pi/2\sigma_x}, e^{i\pi/2\sigma_z}$



Haegemann et al., PRL 2012

Time reversal + Onsite symmetry

SSH: Time reversal + $U(1)$



Shiozaki, Ryu, JHEP 2017

are accessible with a specific distribution of *local* random unitaires

Direct applications:

Bosonic SPT phases can be classified and tested in the lab now

Direct verification of topological order

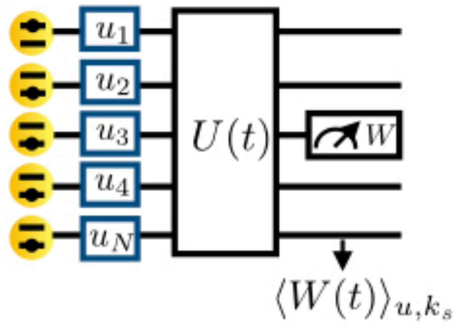
Quantum criticality

Non-equilibrium classification (see talk by N. Cooper)

Conclusion-Outlook

Statistical correlations of randomized measurements

- a tool to probe quantum states beyond standard observables
- applicable in any state-of-the art quantum simulation platform with high repetition rate
- **A tool to verify the quantum features of quantum simulators**



Rényi entropies

Elben, Vermersch et al. PRL, PRA 2018

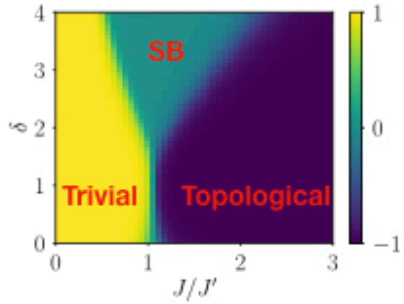
Brydges, Elben et al., arXiv:1806.05747

Out-of-time ordered correlation functions

Vermersch et al., arXiv:1807.09087

Topological invariants

with A.Elben, J. Yu, G. Zhu, M. Hafezi and P. Zoller

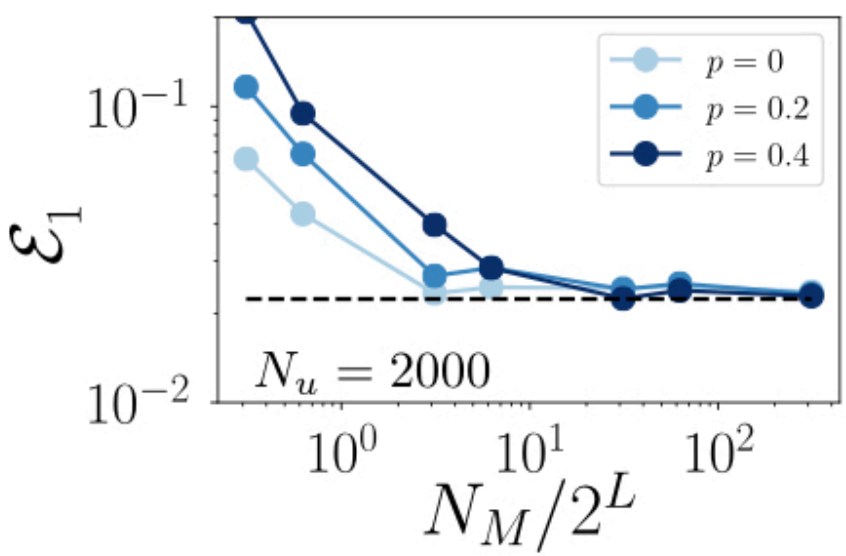


Prospects

- Detection/Classification of true topological order
- Protocols for Hubbard models (MBL as ressource?)
- Theory of random measurements
- Verification

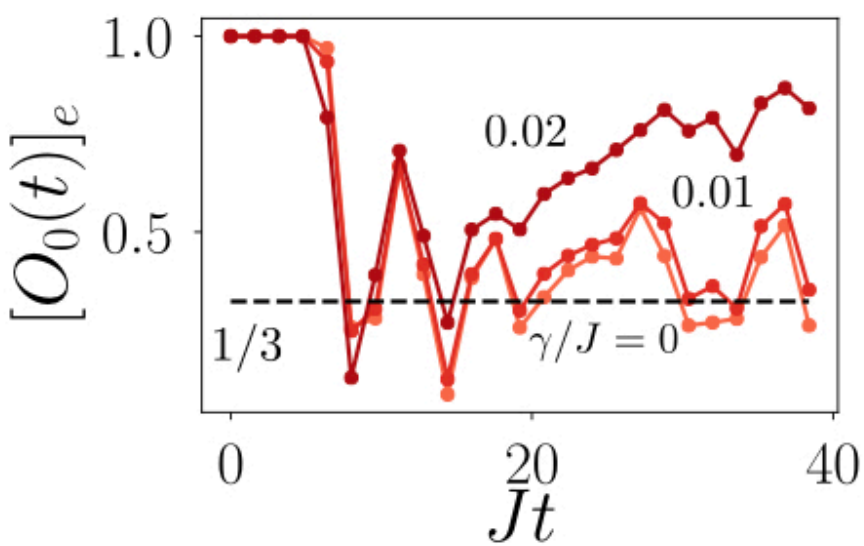
UNIVERSITY OF INNSBRUCK ÖAW UQUAM ERC Synergy Grant PASQuanS

Robust against local unitary errors depolarization



Spontaneous emission

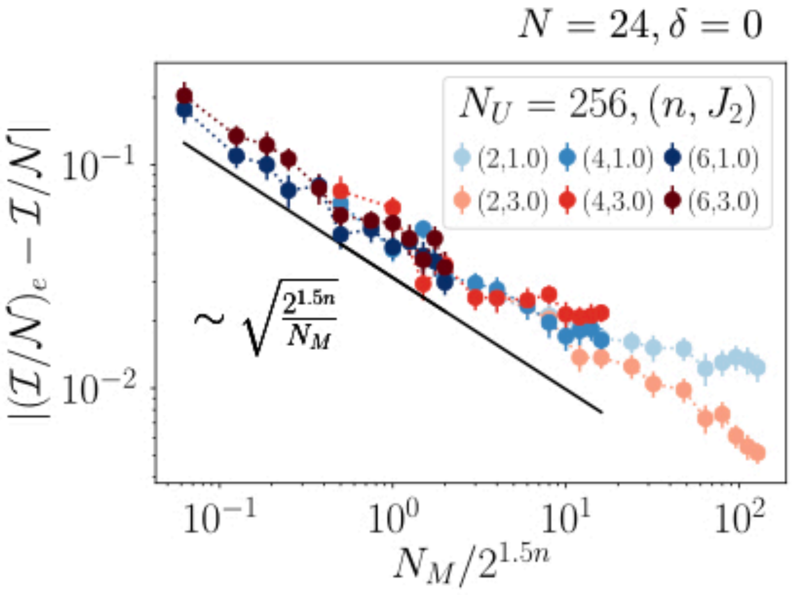
In contrast to time-reversal methods, decoherence and scrambling have opposite signatures



Average statistical errors

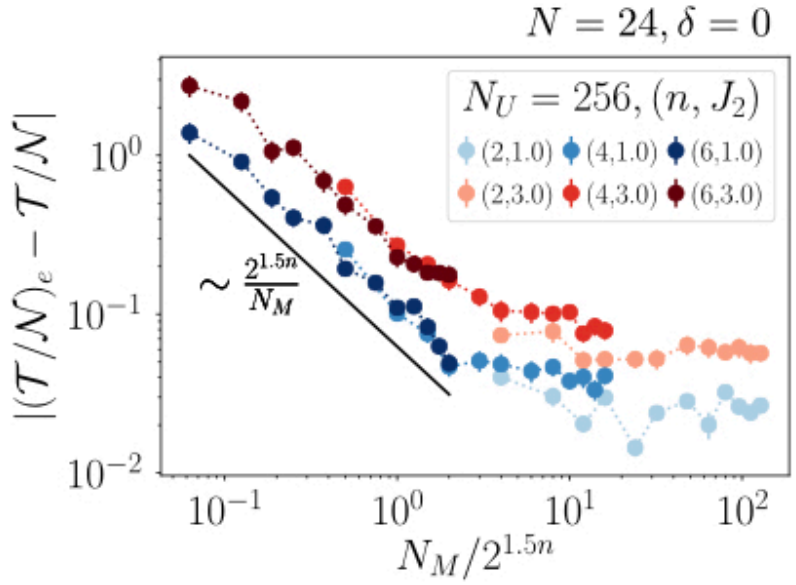
How does the required number of measurements scale with swapped sites n ?

Inversion invariant



$$\Delta_{\mathcal{I}_n} \sim \frac{1}{\sqrt{N_U}} \left(C_I(n) + \sqrt{\frac{2^{1.5n}}{N_M}} \right)$$

Time reversal invariant

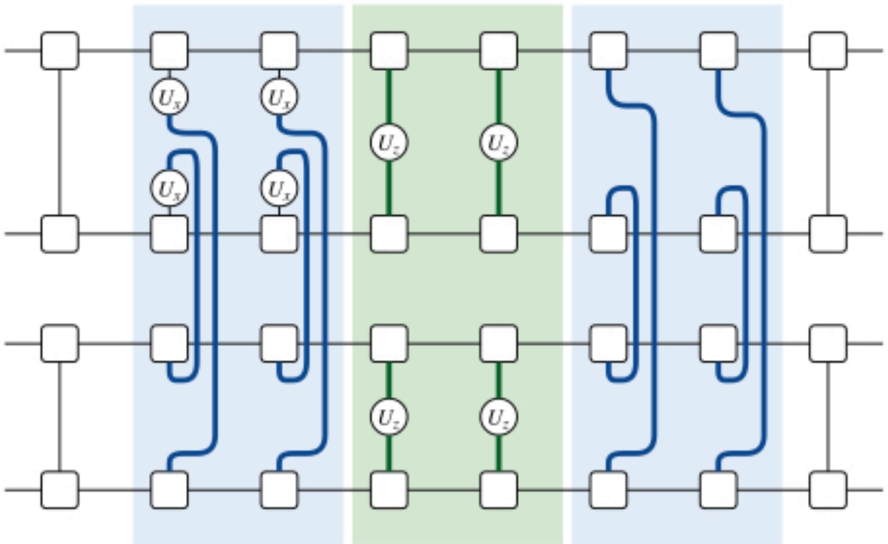


$$\Delta_{\mathcal{T}_n} \sim \frac{1}{\sqrt{N_U}} \left(C_T(n) + \frac{2^{1.5n}}{N_M} \right)$$

Exponential scaling with swapped sites - for relevant sizes within range of experimental possibilities (comparable to Renyi experiments)!

Onsite symmetry

Order parameter



$$\mathcal{C}_n = \langle \Psi \otimes \Psi | \bigotimes_{i \in I_1} (U_i^g)^\dagger \bigotimes_{i \in I_2} (U_i^h)^\dagger$$
$$S_{I_1} S_{I_3} \bigotimes_{i \in I_1} U_i^g \bigotimes_{j \in I_2} U_j^h | \Psi \otimes \Psi \rangle$$

Haegemann et al., PRL 2012

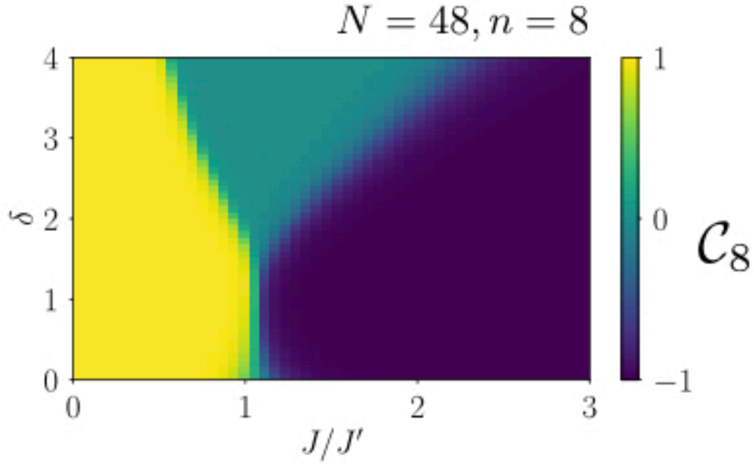
SSH model

Symmetry

$$D_2 = \mathbb{Z}_2 \times \mathbb{Z}_2$$

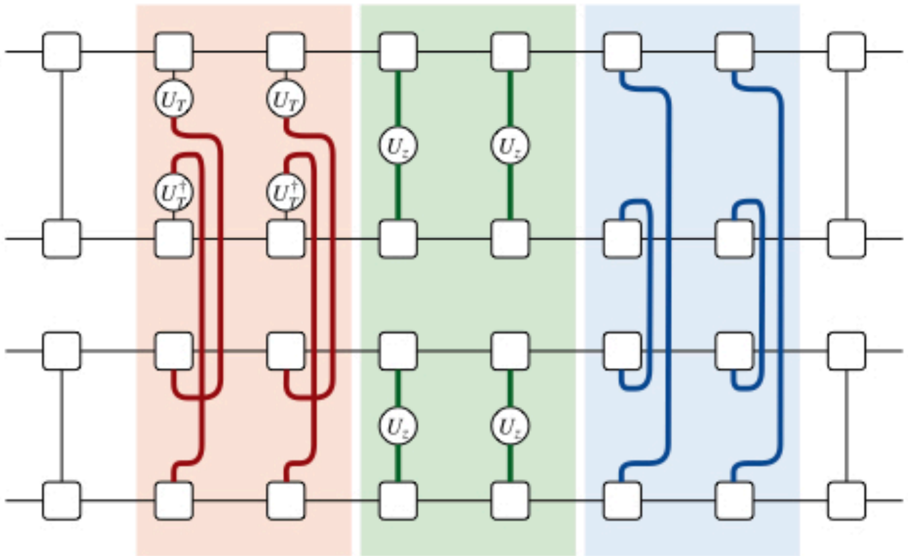
acting through

$$e^{i\pi S_x} \quad e^{i\pi S_z}$$



Klein-Bottle invariant

Time reversal + Onsite symmetry



Shiozaki, Ryu, JHEP 2017

$$\mathcal{K}_n = \langle \Psi \otimes \Psi | \bigotimes_{i \in I_1} (U_i^T)^\dagger \bigotimes_{i \in I_2} (U_i^g)^\dagger$$

$$R_{I_1} S_{I_3} \bigotimes_{i \in I_1} U_i^T \bigotimes_{j \in I_2} U_j^g | \Psi \otimes \Psi \rangle$$

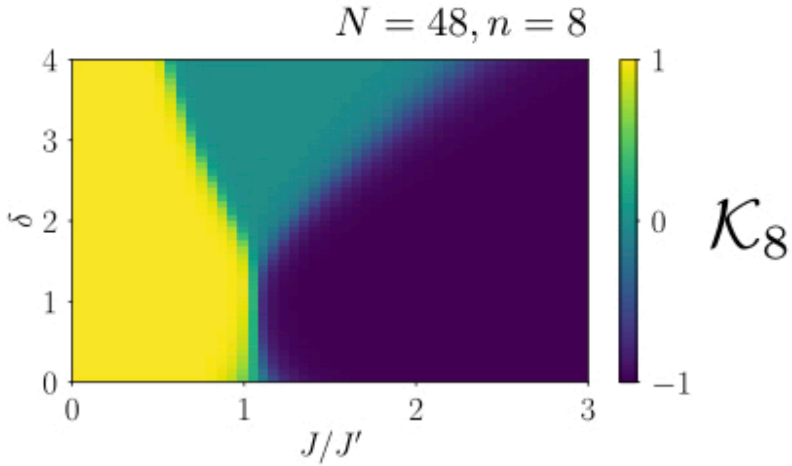
SSH model

Symmetry

$D_2 = \mathbb{Z}_2 \times \mathbb{Z}_2$ + Time reversal

acting through

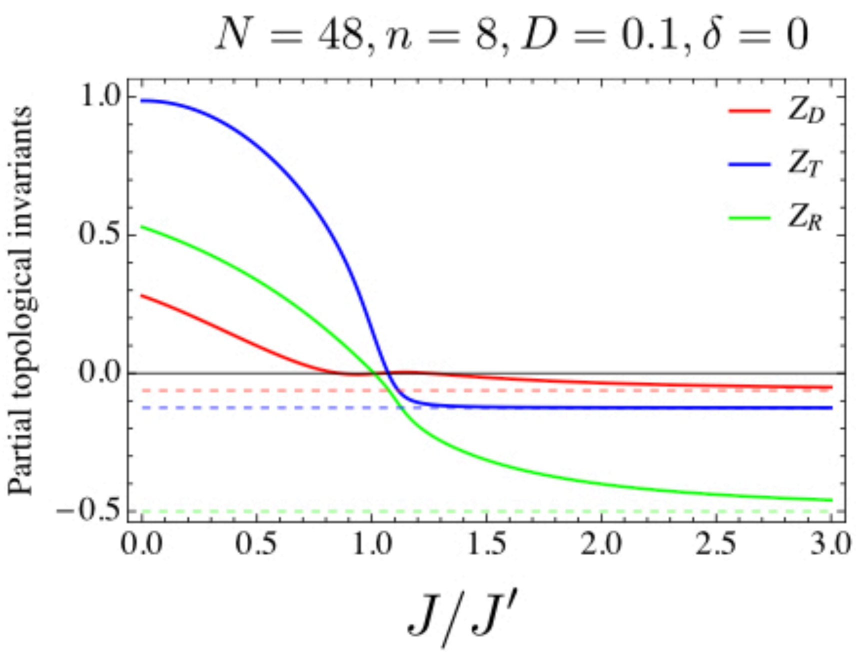
$e^{i\pi/2\sigma_y}\mathcal{K}, e^{i\pi/2\sigma_x}, e^{i\pi/2\sigma_z}$

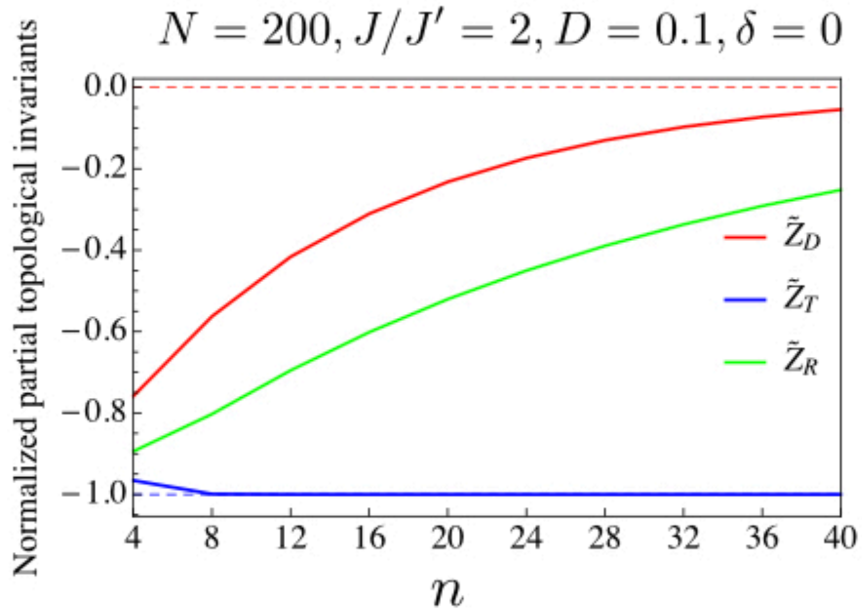


Explicit breaking of symmetries

$$H = H_{\mathrm{SSH}} + D \sum_j (\sigma_j^x \sigma_{j+1}^z - \sigma_j^z \sigma_{j+1}^x)$$

Inversion and D2 symmetry broken





Quench dynamics - Breaking time reversal symmetry

$$H = J' \sum_{i=1}^{N} \left(\sigma_{2i-1}^{-}\sigma_{2i}^{+} + \text{h.c.} + \delta\sigma_{2i-1}^{z}\sigma_{2i}^{z}\right) + J \sum_{i=1}^{N-1} \left(\sigma_{2i}^{-}\sigma_{2i+1}^{+} + \text{h.c.} + \delta\sigma_{2i}^{z}\sigma_{2i+1}^{z}\right)$$

Quench from topological to trivial phase: $(J', J) = (1, 2) \to (1, 1/2)$