Probing quantum matter via randomized measurements

Wikipedia

B. Vermersch (University of Innsbruck)

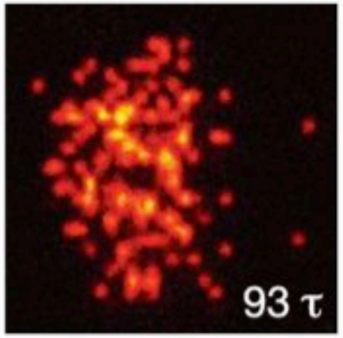
with A. Elben, M. Dalmonte, J. I Cirac, T. Brydges, C. Maier, P. Jurcevic, N. Lanyon, R. Blatt, L. Sieberer, J. Yu, G. Zhu, N. Yao, M. Hafezi, and P. Zoller

UNIVERSITY OF INNSBRUCK ÖAW UQUAM ERC Synergy Grant PASQuanS

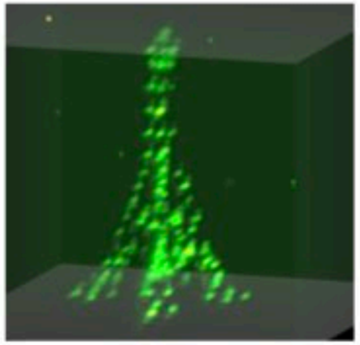
Research at Innsbruck…

Quantum simulators and computers

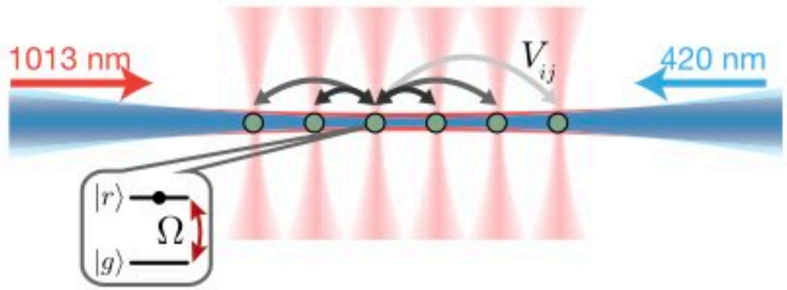
Ultracold atoms — Rydberg atoms



Choi et al., Science (2016)

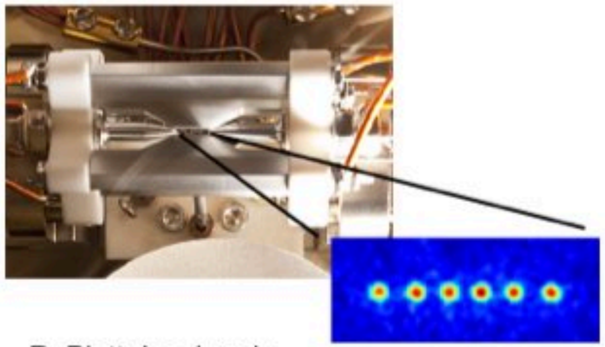


Barredo et al., Science (2016)



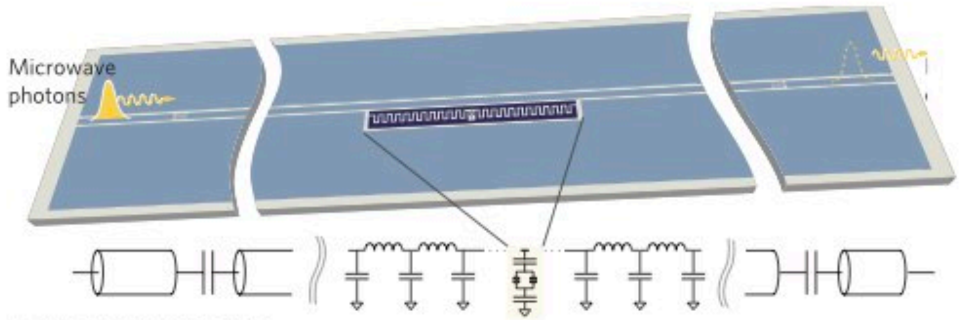
Bernien et al Nature 551, 579 (2017).

Trapped Ions



R. Blatt, Innsbruck

Superconducting circuits



A. Houck, Princeton

and quantum dots, NV centers, cavity QED,..

Unique ways to create, probe, and understand quantum matter

Introducing Random measurements: Measuring Entanglement

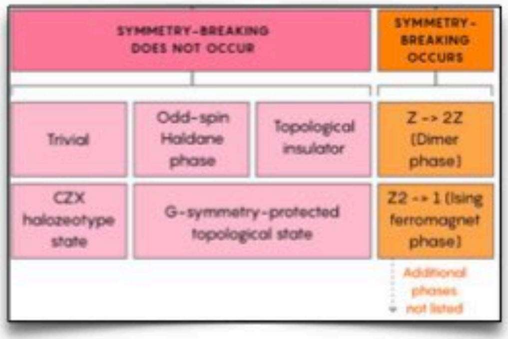
Elben, Vermersch et al. (PRL 2018)
Vermersch, Elben et al. (PRA 2018)
arXiv:1806.05747
arXiv:1812.02624

Measuring scrambling

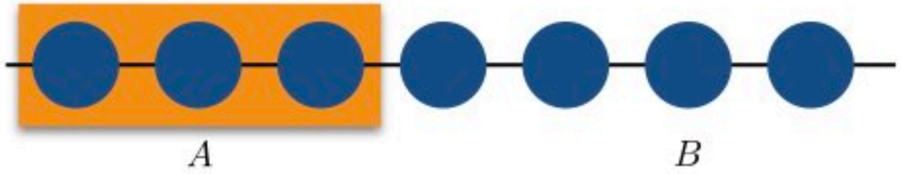
arxiv: 1807.09087

Classification of interacting topological phases

in preparation



Renyi entropies as an entanglement measure



Two subsystems A and B are **bipartite entangled** iff $|\Psi\rangle \neq |\Psi_A\rangle \otimes |\Psi_B\rangle$

Reduced density matrix

$$\boxed{\rho_A = \mathrm{Tr}_B \underbrace{|\Psi\rangle \langle\Psi|}_{\rho}}$$

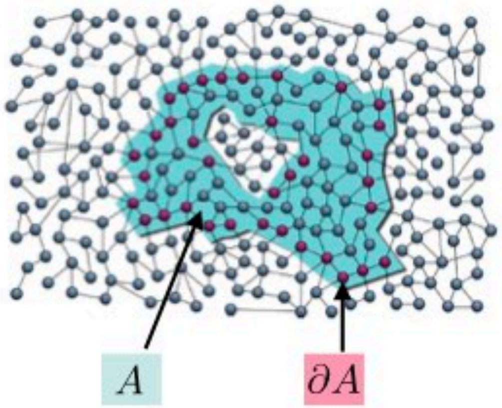
Purity of subsystems $\mathrm{Tr}\left[\rho_A^2\right], \mathrm{Tr}\left[\rho_B^2\right] < \mathrm{Tr}\left[\rho^2\right]$ Purity of full system

Entanglement entropies

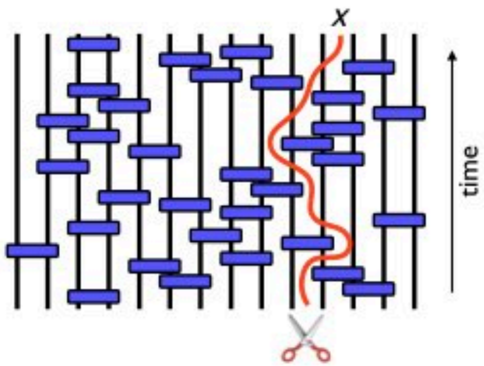
$$S_A = -\mathrm{Tr}_A\left[\rho_A \log \rho_A\right]$$ von-Neumann

$$\boxed{S_A^{(n)} = \frac{1}{1-n} \log \mathrm{Tr}_A\left[\rho_A^n\right]} \leq S_A$$ **Renyi**

Probe of Quantum Phases: Equilibrium, Quantum phase transitions, Thermalization…



Eisert et al., Rev. Mod. Phys. 82, 277 (2010)



Nahum et al, Phys. Rev. X 7, 031016 (2017)

Verification of quantum simulators and quantum computers

Coherence, entanglement

How to measure entanglement in (large)quantum systems?

How to measure Renyi entropies?

Initial state

Adiabatic preparation,
Quench dynamics,
...

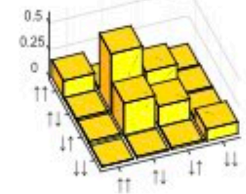
ρ_I

$\rho \to \rho_A$

Measurement of
Renyi entropy
of a (sub-)system
$\sim \mathrm{Tr}\left[\rho_A^n\right]$

Realization in a trapped ion quantum simulator existing today in the lab?!

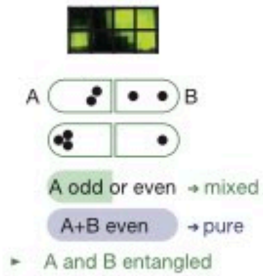
Quantum State Tomography



Gross et al. PRL 105, 150401 (2010)
B. P. Lanyon et al., Nat.Phys., (2017)

If available + feasible

Interference of copies

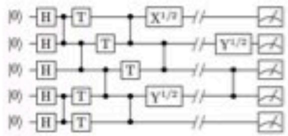


Islam et al., Nature 528, 77–83 (2015)

Daley et al . PRL, 109(2), 20505 (2012)

If available

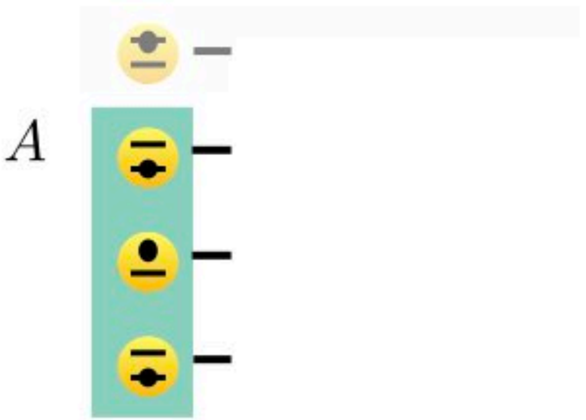
Random measurements on single copies in a QC



van Enk, Beenaker, PRL 108, 110503 (2012)

Random measurements in a ***quantum computer***

Random measurement



$$P_U(\mathbf{s}_A) =$$
$$\mathrm{Tr}\left[U_A \boldsymbol{\rho_A} U_A^\dagger \left|\mathbf{s}_A\right\rangle \left\langle\mathbf{s}_A\right|\right]$$
with $U_A \in \mathrm{CUE}$

Average over the circular unitary ensemble (CUE)

$$\overline{P_U(\mathbf{s_A})^2} = \frac{1 + \mathrm{Tr}\left[\rho_A^2\right]}{N_{\mathcal{H}_A}(N_{\mathcal{H}_A}+1)}$$

Hilbertspace
dimension of A

CUE (2-design): $$\overline{U_{ik}U_{il}^*U_{im}U_{in}^*} = \frac{\delta_{kl}\delta_{mn} + \delta_{kn}\delta_{ml}}{N_{\mathcal{H}_A}(N_{\mathcal{H}_A}+1)}$$

~ Gaussian

van Enk, Beenaker (PRL 2012)

Single qubit

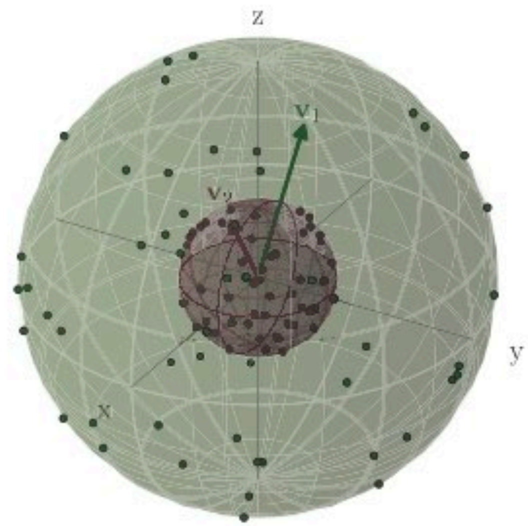
Bloch vector representation

$$\rho = \frac{1}{2}\left(\mathbb{1}_2 + \vec{a}\cdot\vec{\sigma}\right)$$

$$\longrightarrow \mathrm{Tr}\left[\rho^2\right] = \frac{1}{2}\left(1 + \boxed{|\vec{a}|^2}\right)$$

$: |\vec{a}| = 1$

$: |\vec{a}| = 0.33$



Random Measurement

Rotation: $U\rho U^\dagger = \frac{1}{2}\left(\mathbb{1}_2 + (Q_U\vec{a})\cdot\vec{\sigma}\right)$

z-Measurements: $Z_U = \langle\sigma_z\rangle_U = (Q_U\vec{a})_3$

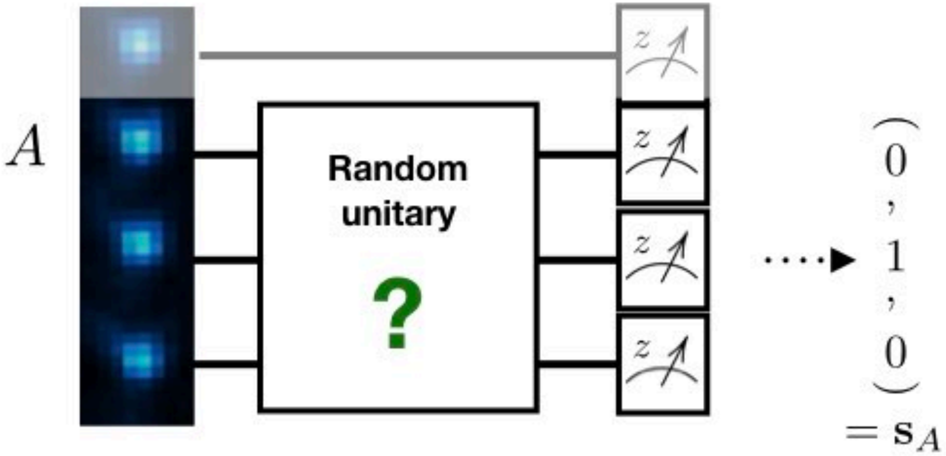
Z_U **uniformly distributed with variance** $\dfrac{|\vec{a}|^2}{3}$

$$\longrightarrow \mathrm{Tr}\left[\rho^2\right] = \frac{1}{2}\left(1 + 3\,\overline{\langle\sigma_z\rangle_U^2}\right)$$

The signal is in the noise

Random measurements in a *quantum simulator*

Random measurement in an experiment



$$P_U(\mathbf{s}_A) =$$

$$\mathrm{Tr}\left[U_A \rho_A U_A^\dagger \left|\mathbf{s}_A\right\rangle \left\langle \mathbf{s}_A\right|\right]$$

with $U_A \in \mathrm{CUE}$

Average over the circular unitary ensemble (CUE)

$$\overline{P_U(\mathbf{s}_A)^2} = \frac{1 + \mathrm{Tr}\left[\rho_A^2\right]}{N_{\mathcal{H}_A}(N_{\mathcal{H}_A} + 1)}$$

Hilbertspace dimension of A

How to generate **random unitaries from CUE?**

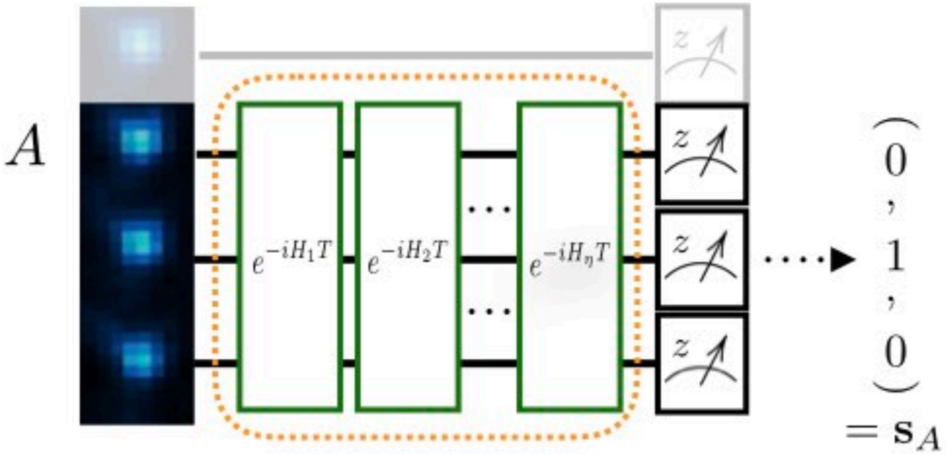
A **simpler** protocol?

How many measurements per unitary and **how many unitaries?**

Elben, Vermersch et al. (PRL 2018)

Random measurements via random quenches

Random unitaries via random quenches



$$P_U(\mathbf{s}_A) = \mathrm{Tr}\left[U_A \rho_A U_A^\dagger \left|\mathbf{s}_A\right\rangle \left\langle \mathbf{s}_A\right|\right]$$

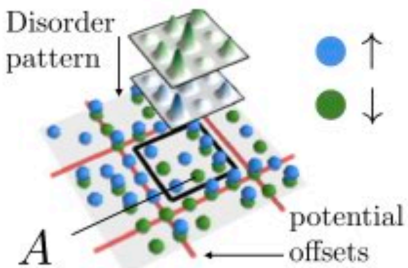
with

$$\boxed{U_A = e^{-iH_\eta T} \dots e^{-iH_1 T}}$$

Hamiltonian time evolution with

✓generic interactions

✓engineered, *timedependent* disorder

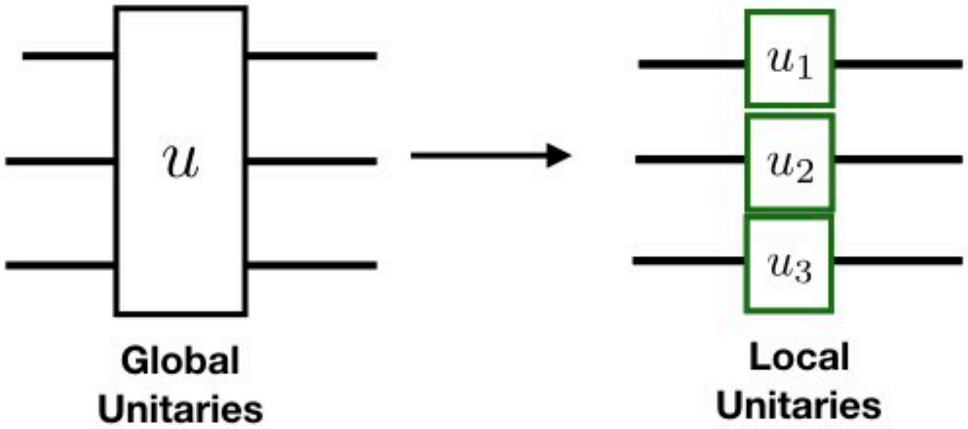


Applicable in Bose/Fermi Hubbard and Spin models

- Ultracold atoms
- Rydberg atoms
- Superconducting qubits
- Trapped ions
- ...

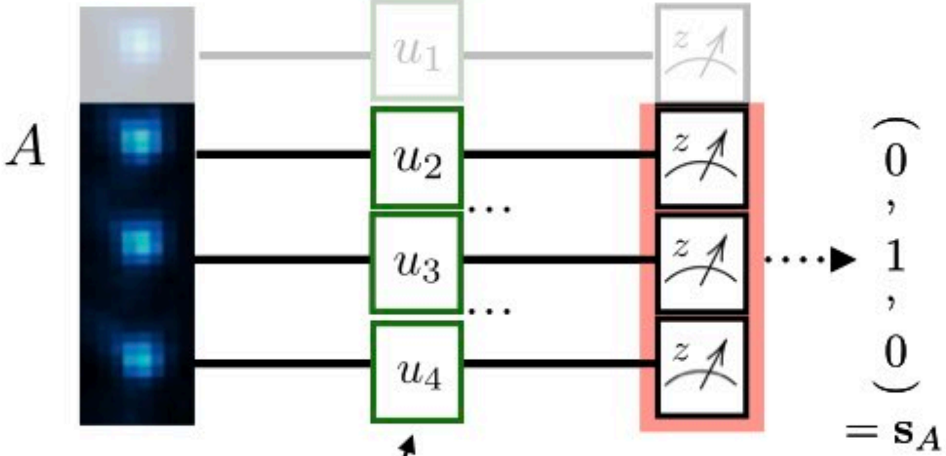
Elben, Vermersch et al. (PRL 2018)

A simpler protocol?



Elben, Vermersch et al. (PRL 2018)

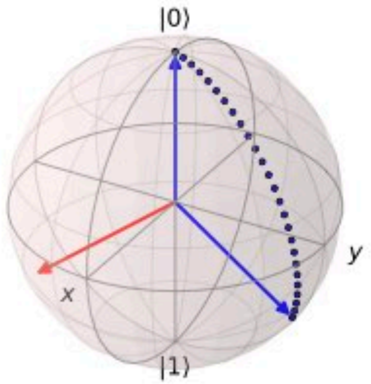
Random measurements with local unitaries



$$P_U(\mathbf{s}_A) = \mathrm{Tr}\left[U_A \rho_A U_A^\dagger \left|\mathbf{s}_A\right\rangle \left\langle \mathbf{s}_A\right|\right]$$

$$\boxed{U_A = \bigotimes u_i} \quad u_i \in \mathrm{CUE}(d)$$

Single spin random rotation



$$u_i \equiv u(\alpha_i, \beta_i, \gamma_i)$$

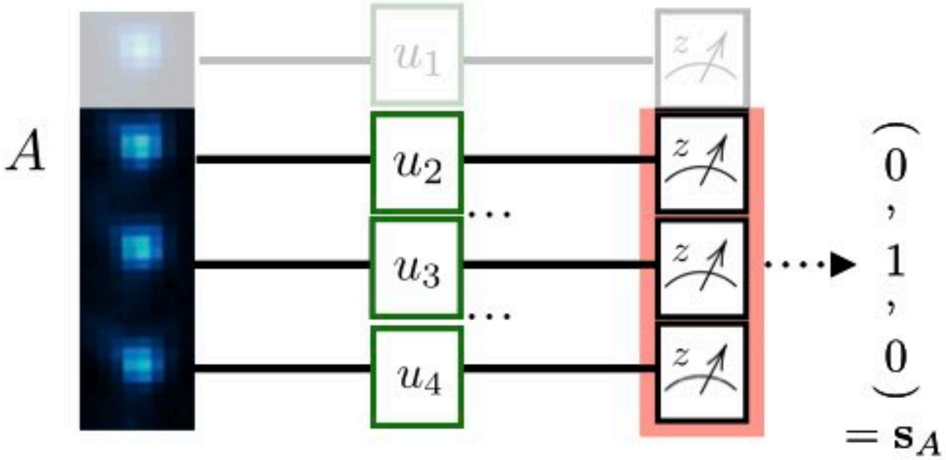
$$= Z(\alpha_i) Y(\pi/2) Z(\beta_i) Y(-\pi/2) Z(\gamma_i)$$

$\alpha_i, \beta_i, \gamma_i$ random

Elben, Vermersch et al. (PRL 2018)
Brydges, Elben et al (arXiv:1806.05747)

Random measurements with *local* unitaries

Random measurements with local unitaries



$$P_U(\mathbf{s}_A) = \mathrm{Tr}\left[U_A \rho_A U_A^\dagger \left|\mathbf{s}_A\right\rangle \left\langle \mathbf{s}_A\right|\right]$$

$$\boxed{U_A = \bigotimes u_i} \quad u_i \in \mathrm{CUE}(d)$$

Average over the random unitary ensemble

$$\overline{P_U(\mathbf{s}_A)} = \frac{1}{d^{N_A}} \qquad \overline{P_U(\mathbf{s}_A)^2} = \frac{\sum_{A' \subseteq A} \mathrm{Tr}(\rho_{A'}^2)}{d^{N_A}(d+1)^{N_A}}$$

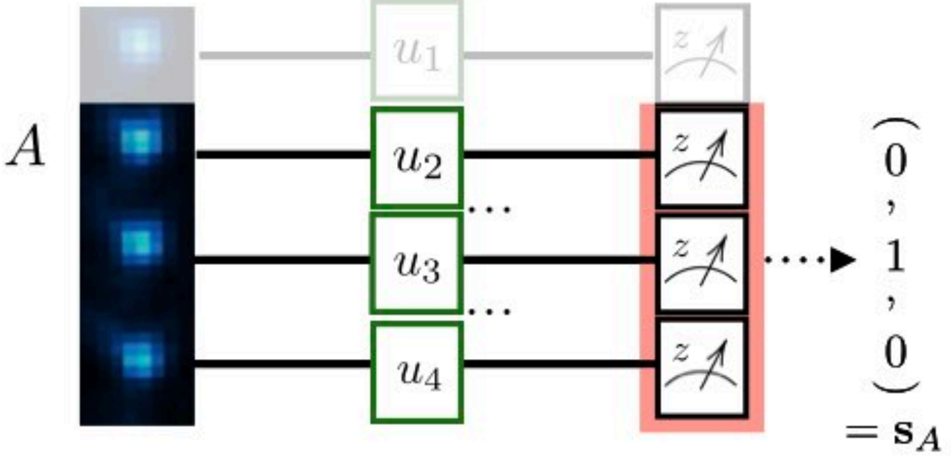
Exponential number of terms

How to invert, to obtain directly $\mathrm{Tr}\left[\rho_A^2\right]$ **?**

Elben, Vermersch et al. (PRL 2018)

Random measurements with *local* unitaries

Random measurement with local random unitaries



$$P_U(\mathbf{s}_A) = \mathrm{Tr}\left[U_A \rho_A U_A^\dagger \left|\mathbf{s}_A\right\rangle \left\langle \mathbf{s}_A\right|\right]$$

$$\boxed{U_A = \bigotimes u_i} \quad u_i \in \mathrm{CUE}(d)$$

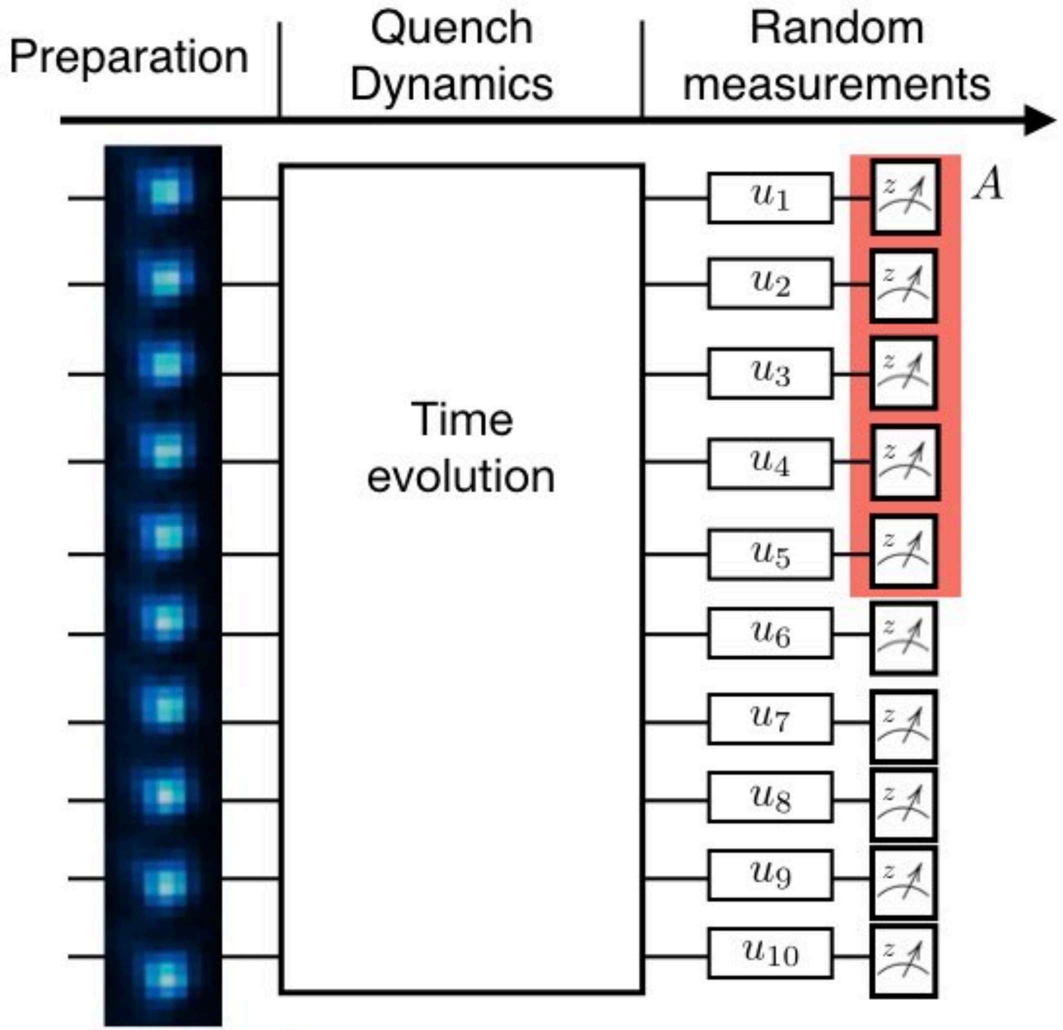
Average over the random unitary ensemble

Hamming distance

$$\boxed{\mathrm{Tr}\left[\rho_A^2\right] = \overline{X_U}} \quad \textbf{with} \quad X_U = 2^{N_A} \sum_{s_A, s'_A} (-2)^{-D[s_A, s'_A]} \underbrace{\boxed{P_U(s_A) P_U(s'_A)}}_{\text{Cross correlation}}$$

Explicit reconstruction

Trapped ion measurement scheme

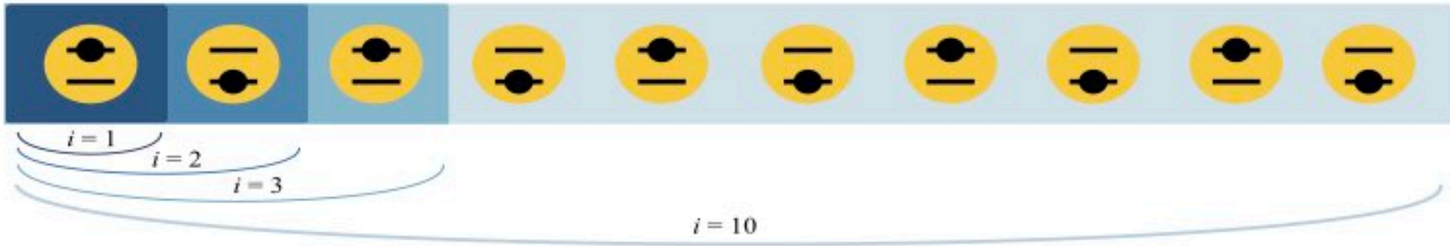


$$H_{XY} = \hbar \sum_{i<j} \boxed{J_{ij}} (\sigma_i^+ \sigma_j^- + \sigma_i^- \sigma_j^+) + \hbar \sum_j (B + \boxed{b_j}) \sigma_j^z$$

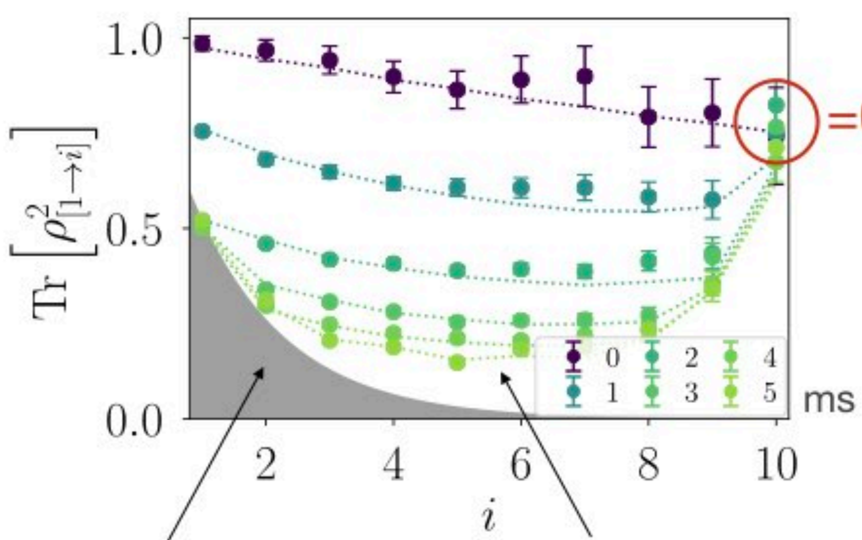
long range (algebraic) interaction

local disorder potentials

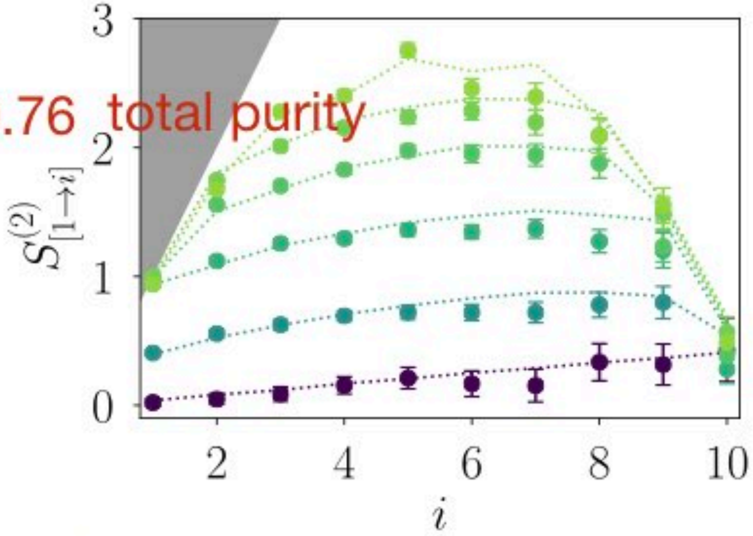
Experimental results - 10 Ions [no disorder]



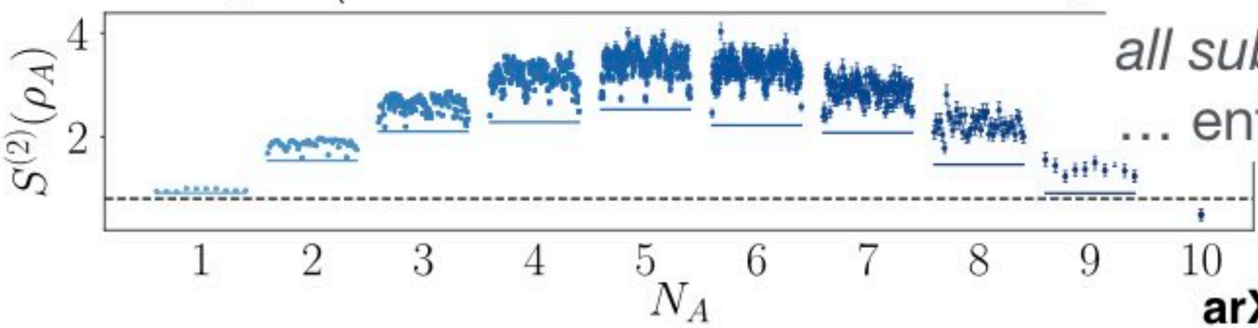
purity $\mathrm{Tr}[\rho_A^2]$



Renyi entropy



maximally



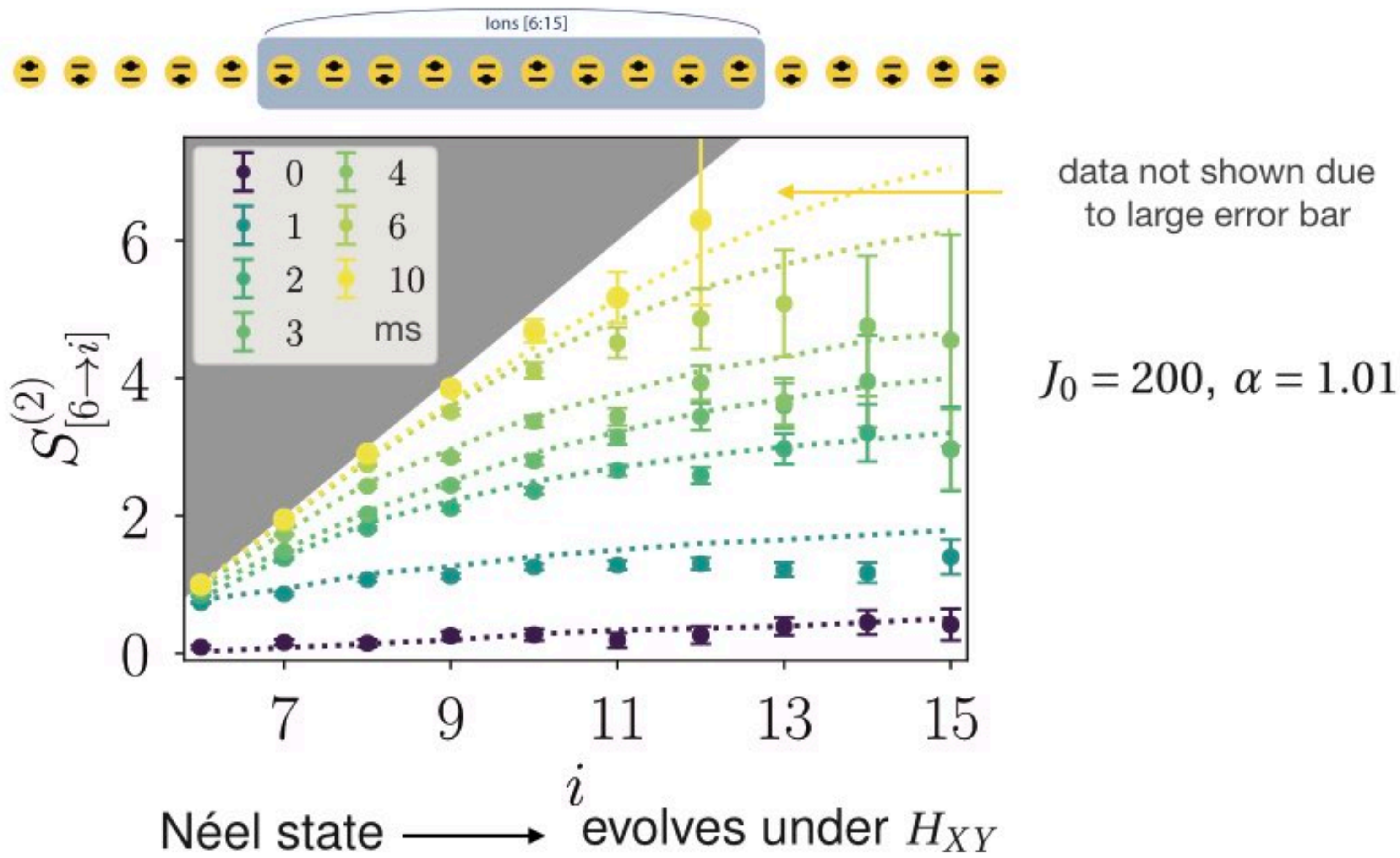
all sub partitions

... entangled

arXiv:1806.05747

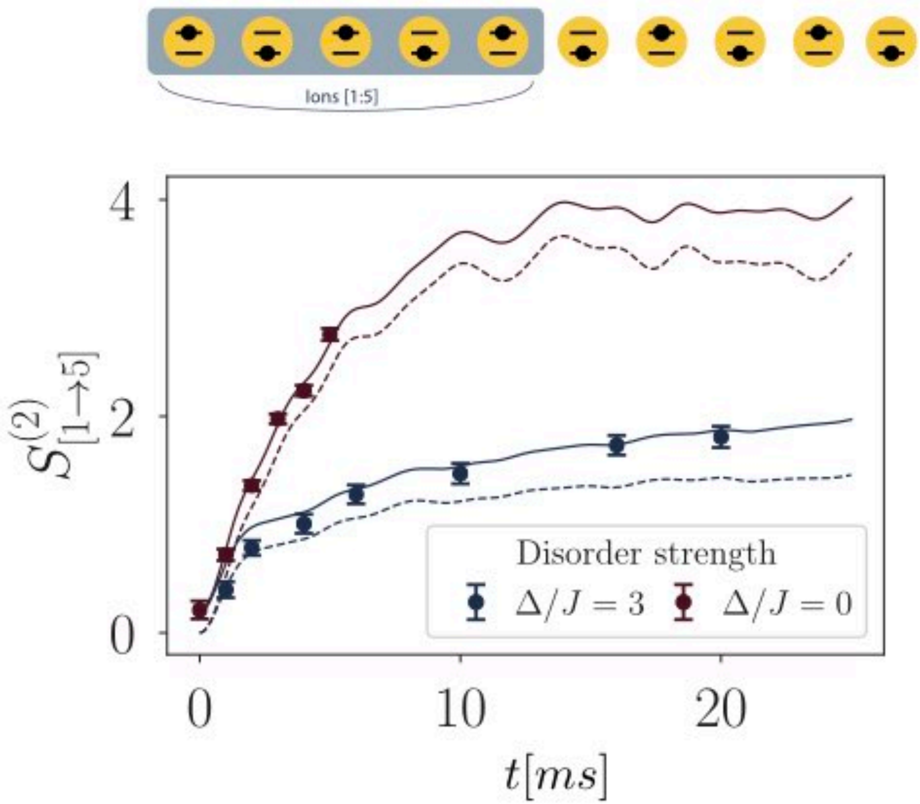
Experimental results - 20 Ions [no disorder]

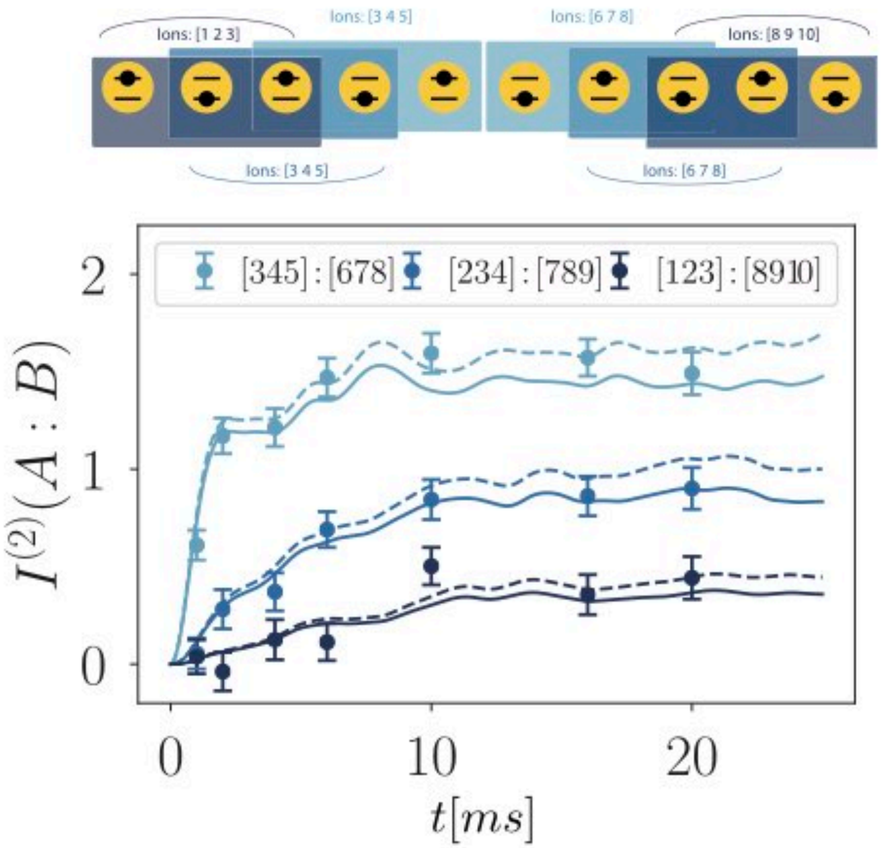
Renyi entropy 1-10-qubit partitions of a 20-qubit system.



With minor improvements of the apparatus Renyi entropies of 20 ions seem accessible

Experimental results - 10 Ions [disorder]





$$H_{XY} = \hbar \sum_{i<j} \boxed{J_{ij}}(\sigma_i^+\sigma_j^- + \sigma_i^-\sigma_j^+) + \hbar \sum_j (B + \boxed{b_j})\sigma_j^z$$

long range interaction

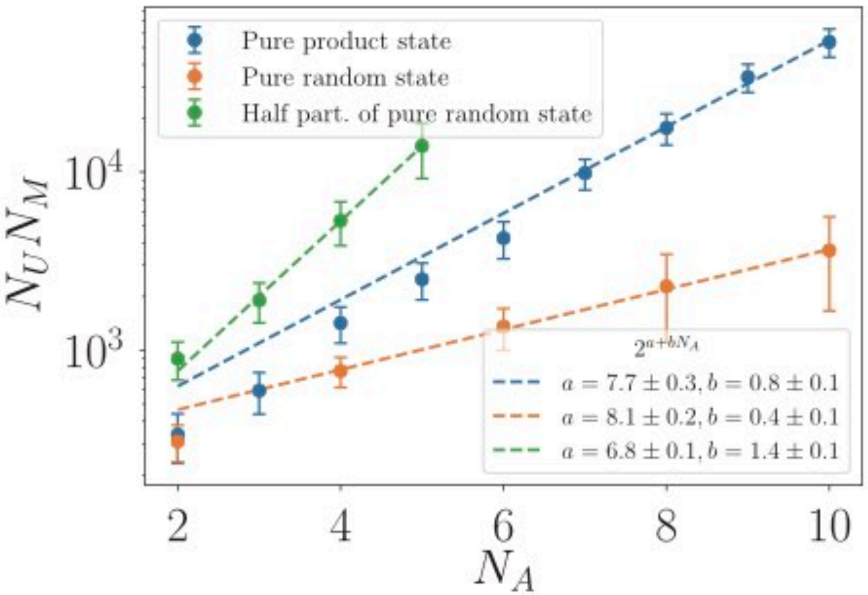
local disorder potentials

arXiv:1806.05747

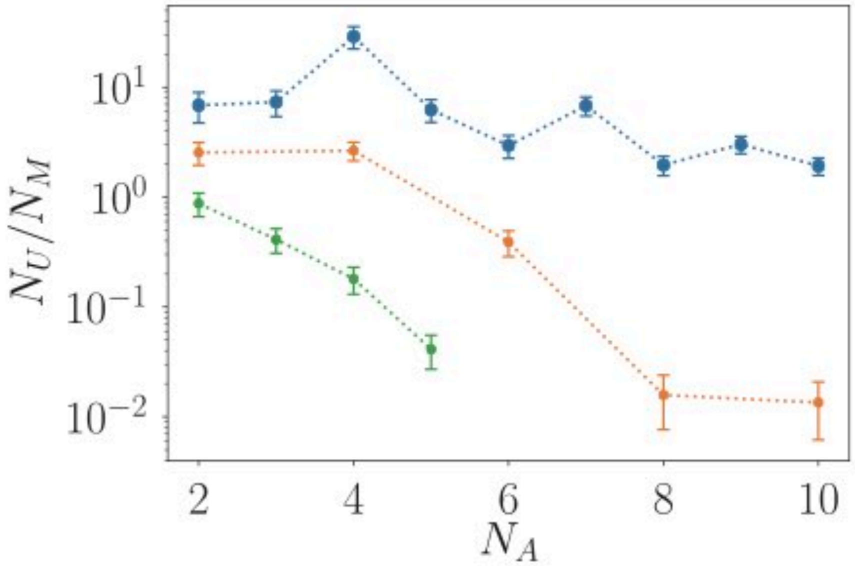
Scaling of the required number of measurements

Scaling of the number of measurements to obtain the Renyi entropy with an error smaller than 0.15

Total number of measurements



Optimal ratio

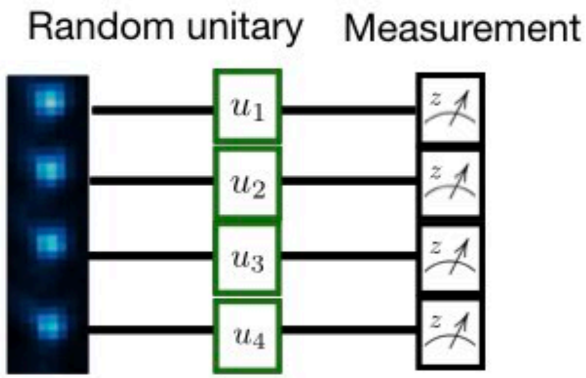


- Exponential scaling with exponents < 1.5
- Optimal ratio strongly dependent on the state of interest

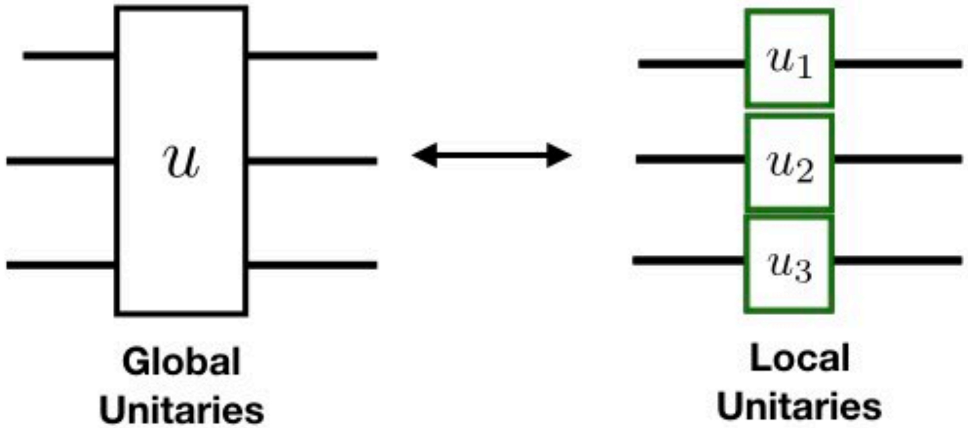
⟶ Adaptive sampling to find optimal parameters?

Our tool: random measurements

- Single copies
- Local Measurements
- Robust (ex: readout errors)
- Simple



- Two variants



A powerful tool to probe quantum matter?

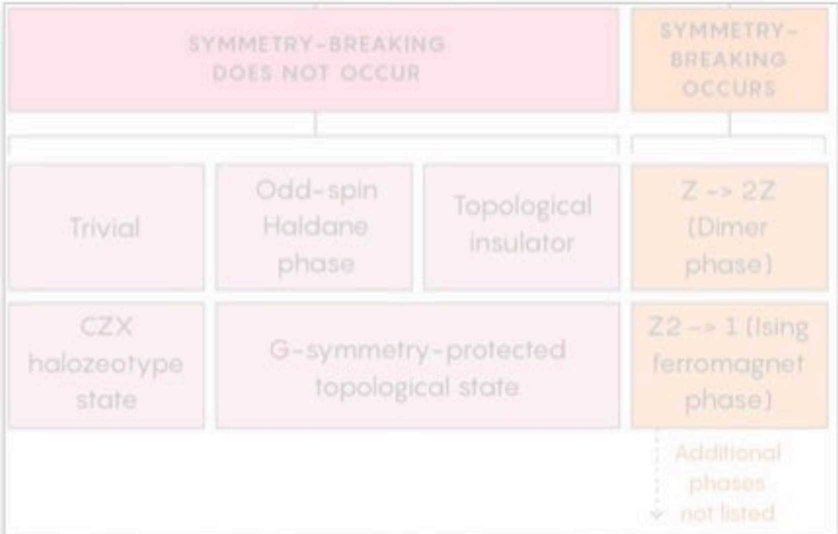
Measuring scrambling

with random measurements

B. Vermersch, A. Elben, L. Sieberer, N. Yao, and P. Zoller

arxiv: 1807.09087

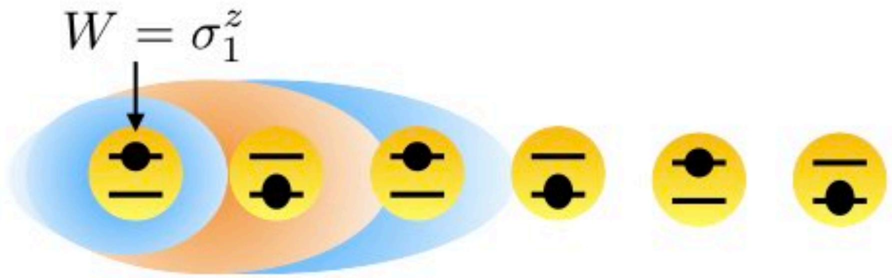
Classification of interacting topological phases (SPT)



A. Elben, B. Vermersch, J. Yu, G. Zhu, M. Hafezi and P. Zoller

in preparation

Introducing Scrambling and OTOCs



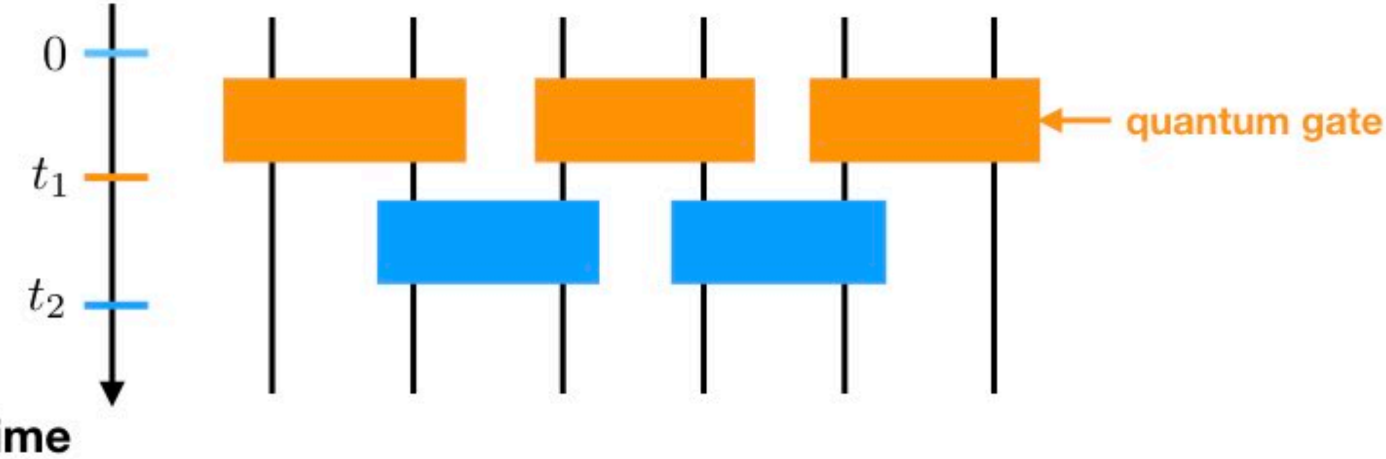
Operator spreading
(Heisenberg picture)

$$W(t) = U^\dagger(t) W U(t)$$

$U(t)$

Time evolution operator

ex: quantum gates



$$W(0) = \boxed{\sigma_1^z} \otimes I \otimes \cdots$$

$$W(t_1) = \sum_{\gamma_1, \gamma_2 = x,y,z,0} c_{\gamma_1,\gamma_2} \boxed{\sigma_1^{\gamma_1} \otimes \sigma_2^{\gamma_2}} \otimes I \cdots$$

$\vdots$

Out of time ordered correlation functions (OTOCs)

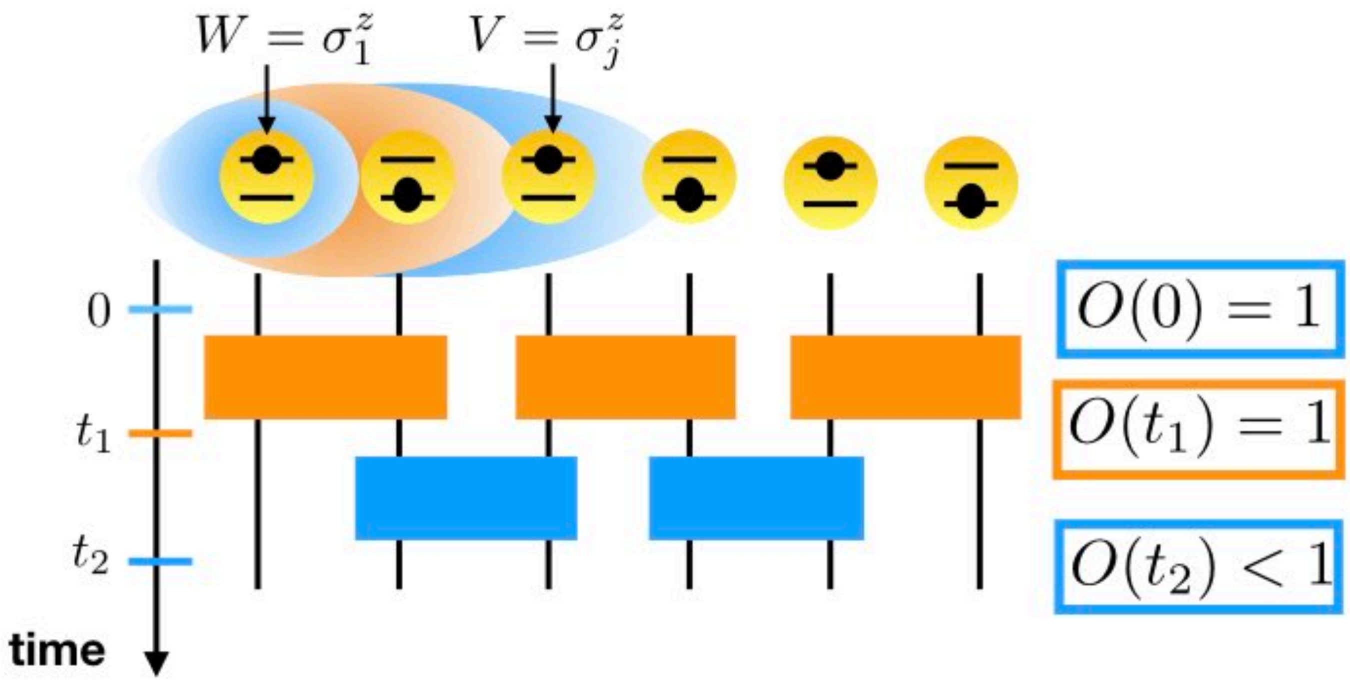
OTOC $$O = \mathrm{Tr}(\rho W(t) V W(t) V)$$

quantum state (here: V, W Pauli matrices)

$[W(t), V] = 0 \rightarrow O = 1$

$[W(t), V] \neq 0 \rightarrow O < 1$

Describe the spreading of an operator with respect to a `reference' V

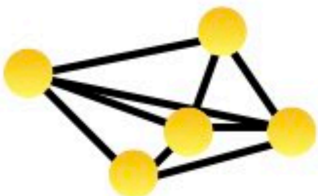


Introducing OTOCs

OTOCS: Information scrambling in many-body systems

Fast scramblers (analog black holes) → Quantum Thermalization And chaos → Slow scramblers

Sachdev-Ye-Kitaev (SYK) model



$$H_{SYK} = \frac{1}{4!}\sum_{jklm} J_{jklm}\chi_j\chi_k\chi_l\chi_m$$

$$O(t,r) \sim c_0 - c_1 e^{\lambda_L (t - r/v_B)}$$

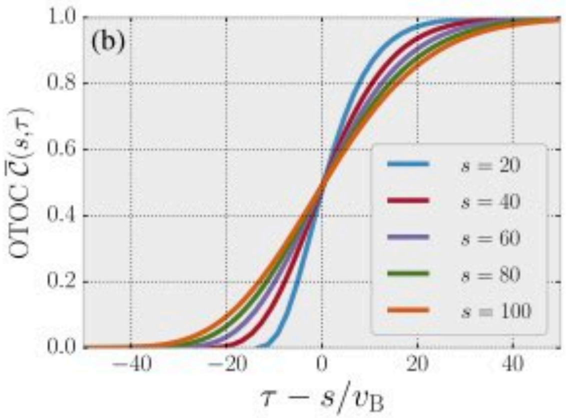
$$\lambda_{\mathrm{L}} = 2\pi k_{\mathrm{B}} T/\hbar$$

Sachdev, et al . PRL 1993 70(21), 3339–3342
Kitaev, A. KITP 2015
Banerjee et al 2017 PRB 95(13), 134302.

Random circuits

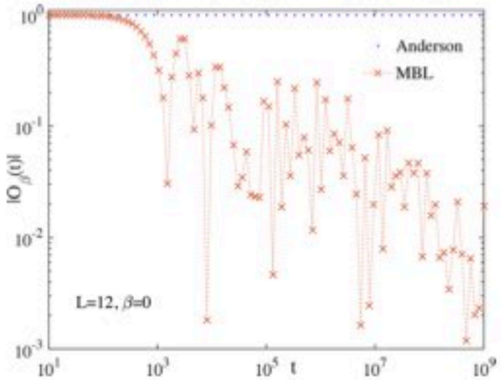
Quantum Ising models (inc. Long rang interactions)

Bose Hubbard



A. Bohrdt et al, New J. Phys. 19, 063001 (2017).
A. Nahum et al Phys. Rev. X 8, 021014 (2018).
C. W. von Keyserlingk et al Phys. Rev. X 8, 021013 (2018).
M. C. Tran, et al A. V. Gorshkov, arxiv:1808.05225 .

Many-body localization



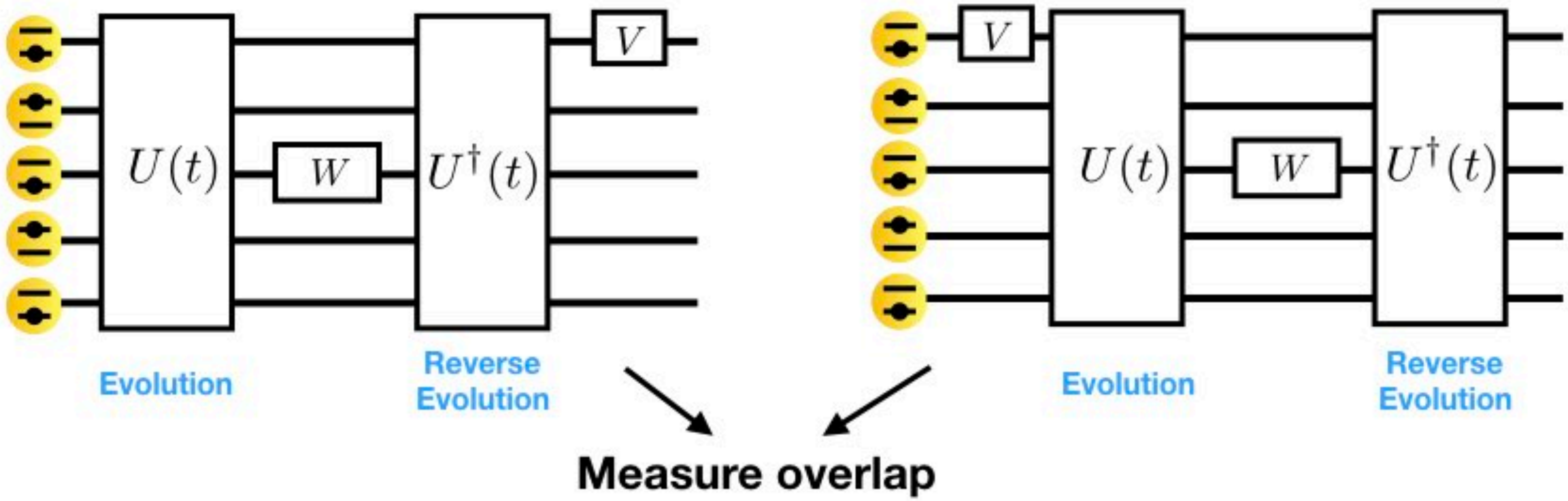
Fan, et al . Science Bulletin, 62(10), 707–711
Chen, X et al. Annalen Der Physik, 529(7)
...

Peculiar time-ordering in the definition implies `challenging' protocols

$$O = \mathrm{Tr}(\rho W(t) V W(t) V)$$

For a pure state $\rho = |\psi\rangle\langle\psi|$ $\quad O = \langle\psi_1|\psi_2\rangle$

$|\psi_1\rangle = VU^\dagger(t)WU(t)|\psi\rangle$ $\qquad |\psi_2\rangle = U^\dagger(t)WU(t)V|\psi\rangle$

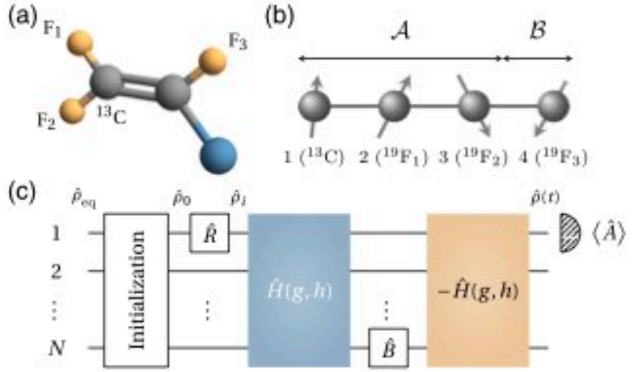


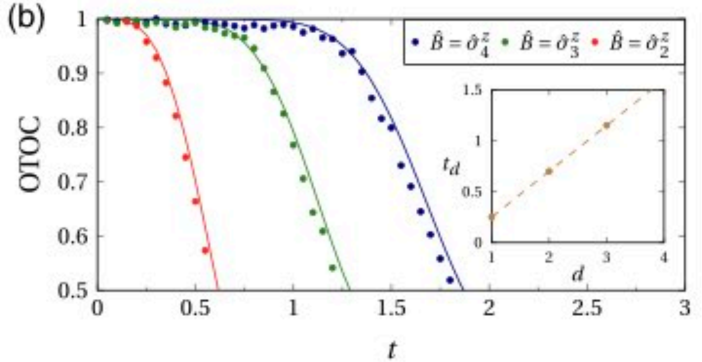
Requires time-reversal and/or copies and/or ancillas

Zhu et al . Phys. Rev. A 94 062329 (2016)
Swingle, B.et al Phys. Rev. A 94, 1–6 (2016)
Yao et al, arxiv:1607.01801
Garttner, M.,et al Nature Physics, 13(8), 781–786

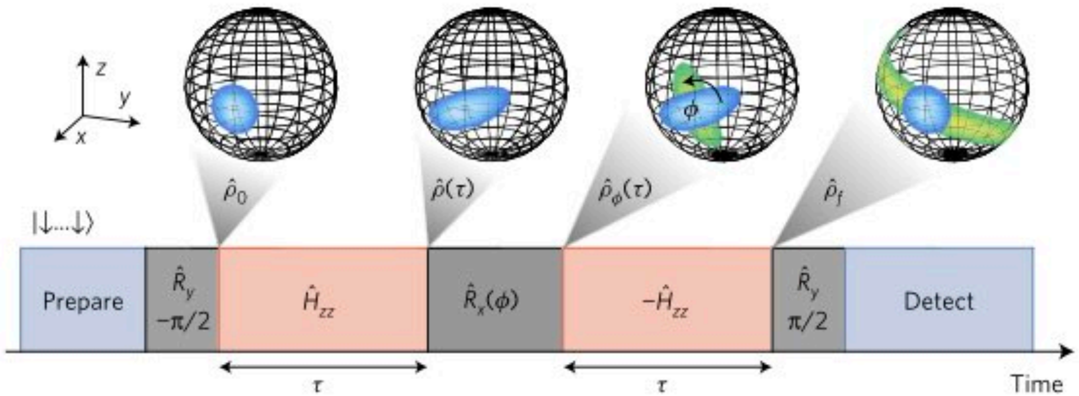
The two first measurements

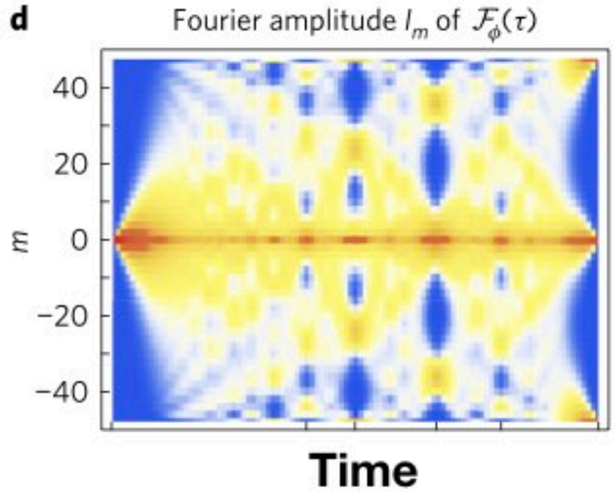
NMR Four spins Trotter evolution





Trapped ions (all-to-all interactions)



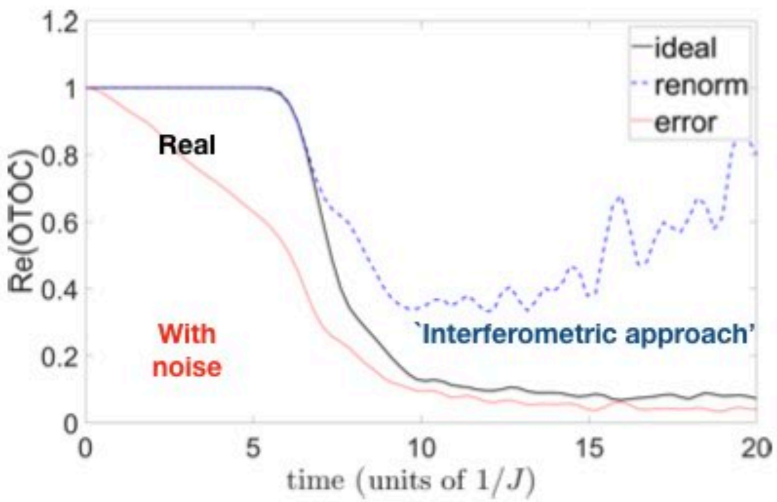


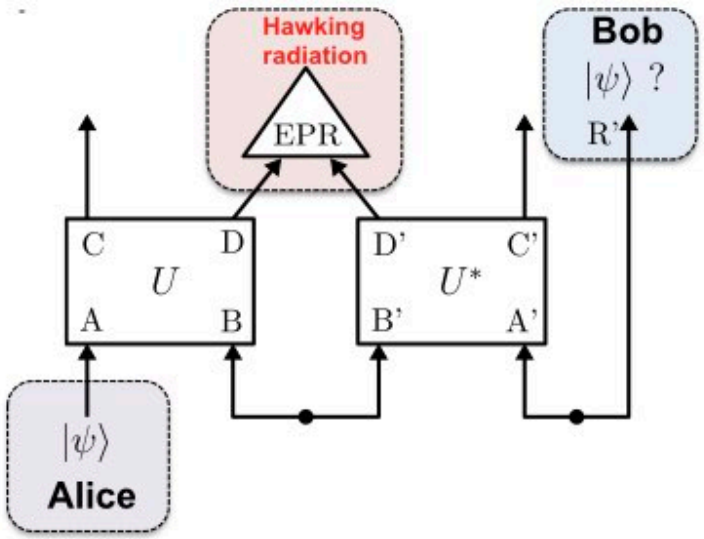
Key challenges: → **Implementing time-reversal** → **The role of decoherence**

J. Li, R. Fan, H. Wang, B. Ye, B. Zeng, H. Zhai, X. Peng, and J. Du, Phys. Rev. X 7, 031011 (2017).
M. Gärttner, J. G. Bohnet, A. Safavi-Naini, M. L. Wall, J. J. Bollinger, and A. M. Rey, Nat. Phys. 13, 781 (2017)
See also Viewpoint on Physics by Norm Yao and B. Swingle.

Decoherence with Time-reversal operations

Decoherence versus scrambling





B. Swingle and N. Yunger Halpern, Phys. Rev. A 97, 062113 (2018).

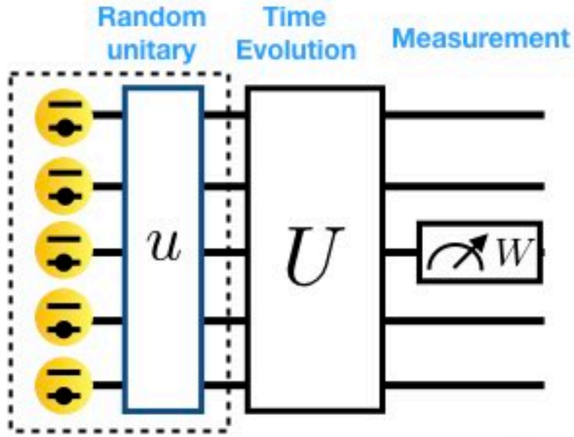
B. Yoshida and N. Y. Yao, arXiv:1803.10772

K. A. Landsman et al, arxiv: 1806.02807

- Very important technological **challenges..**
- **Our approach:** Replace time-reversal by **statistical correlations**

Protocol: a simple butterfly experiment

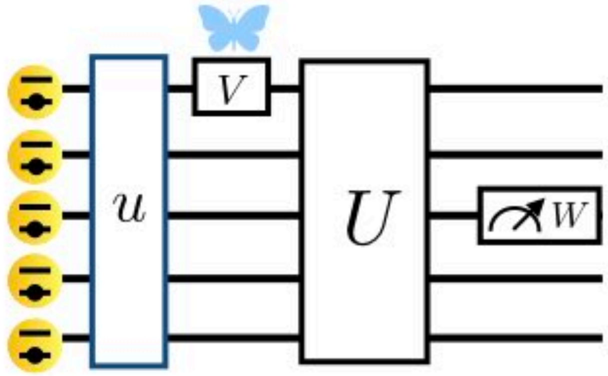
1st measurement (day 1)



Random Initial state $|\psi_u\rangle$

$$\langle W(t)\rangle_u = \langle\psi_u|W(t)|\psi_u\rangle$$

2nd measurement (day 2)



$$\langle VW(t)V\rangle_u = \langle\psi_u V|W(t)|V\psi_u\rangle$$

$\langle VW(t)V\rangle_u$

$\langle W(t)\rangle_u$

Statistical correlations = OTOCs $(T=\infty)$

$$O(t) = \frac{1}{\mathcal{D}^{(\mathrm{G})}}\overline{\langle W(t)\rangle_{u,k_0}\langle V^\dagger W(t)V\rangle_{u,k_0}}$$

ensemble average over the circular unitary ensemble (CUE)

Elben, A.,BV, Dalmonte, M., Cirac, J. I., & Zoller, P. PRL 2018 120(5), 50406.
BV, Elben, A., Dalmonte, M., Cirac, J. I., & Zoller, P. PRA, 2018 97(2), 23604.

Proof: The signal is in the noise

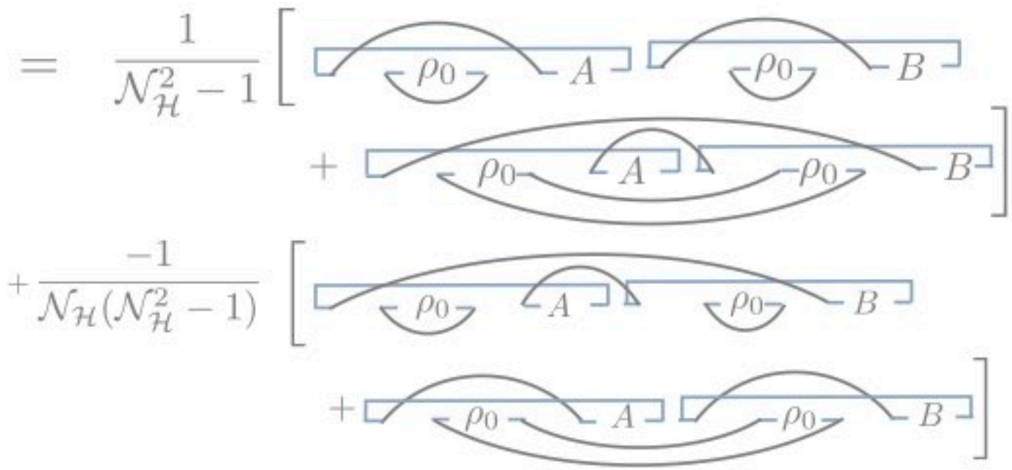
CUE (2-design):

$$\overline{u_{m_1,n_1}u^*_{m'_1,n'_1}u_{m_2,n_2}u^*_{m'_2,n'_2}} \tag{3}$$
$$= \frac{\delta_{m_1,m'_1}\delta_{m_2,m'_2}\delta_{n_1,n'_1}\delta_{n_2,n'_2} + \delta_{m_1,m'_2}\delta_{m_2,m'_1}\delta_{n_1,n'_2}\delta_{n_2,n'_1}}{\mathcal{N}_{\mathcal{H}}^2 - 1}$$
$$- \frac{\delta_{m_1,m'_1}\delta_{m_2,m'_2}\delta_{n_1,n'_2}\delta_{n_2,n'_1} + \delta_{m_1,m'_2}\delta_{m_2,m'_1}\delta_{n_1,n'_1}\delta_{n_2,n'_2}}{\mathcal{N}_{\mathcal{H}}(\mathcal{N}_{\mathcal{H}}^2 - 1)},$$

(Collins 2006)

$$\overline{\langle A\rangle_u \langle B\rangle_u} = \overline{\left[u - \rho_0 - u^\dagger - A\right]\left[u - \rho_0 - u^\dagger - B\right]}$$

2-design rule



$$= c \sum_{\tau \in I,\mathrm{Swap}} \tau(A \otimes B) = c \sum_{\tau \in I,\mathrm{Swap}} \mathrm{Tr}[\tau(A \otimes B)] = c\,\mathrm{Tr}(AB)$$

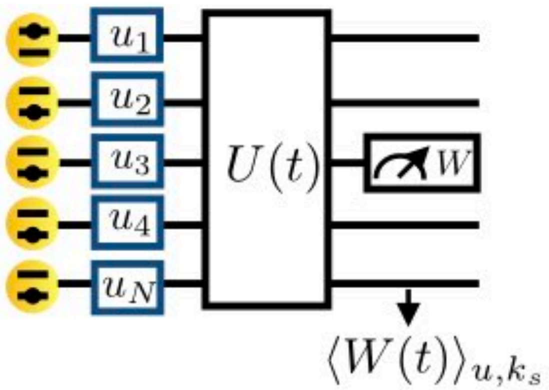
$$O(t) = \frac{1}{\mathcal{D}^{(\mathrm{G})}} \overline{\langle W(t)\rangle_{u,k_0} \langle V^\dagger W(t) V\rangle_{u,k_0}}$$

Protocol for spin systems with local unitaries

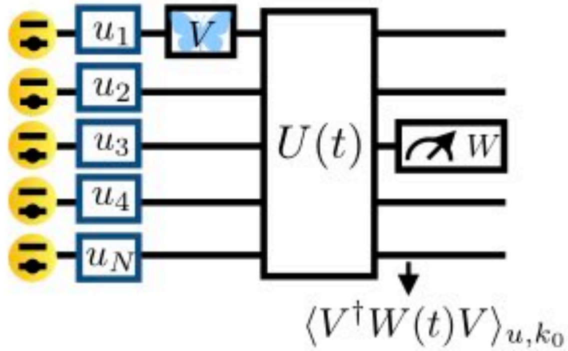
IQI uibk

A much simpler protocol (for spins)

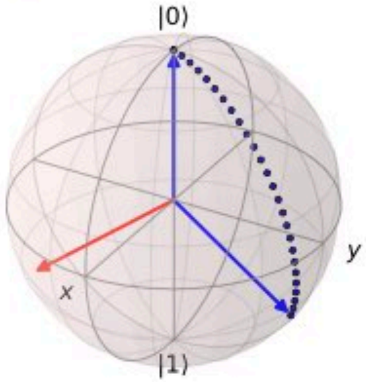
1st measurement (day 1)



2nd measurement (day 2)



Single spin random rotation

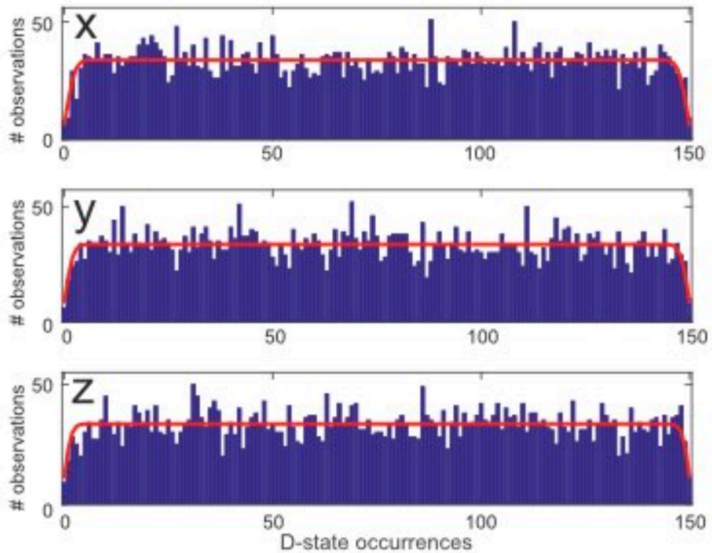


$$u_i \equiv u(\alpha_i, \beta_i, \gamma_i)$$

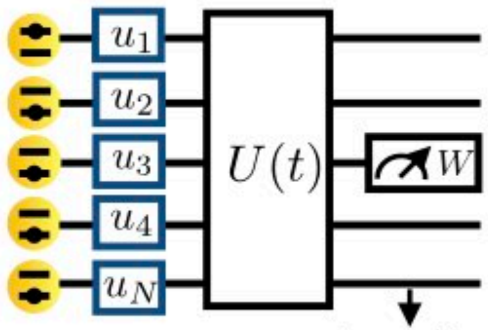
$$= Z(\alpha_i) Y(\pi/2) Z(\beta_i) Y(-\pi/2) Z(\gamma_i)$$

Available in state-of-the-art setups

Brydges et al 2018 arXiv:1806.05747

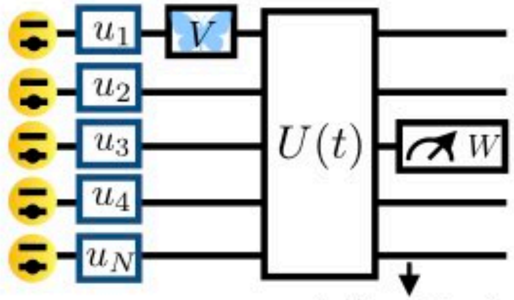


1st measurement (day 1)



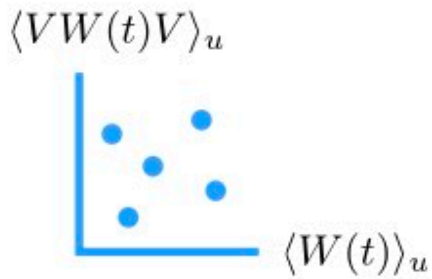
$\langle W(t)\rangle_{u,k_s}$ ← Initial configuration

2nd measurement (day 2)



$\langle V^\dagger W(t) V\rangle_{u,k_0}$

Local statistical Correlations = Modified OTOCs



$$O_n(t) = \frac{1}{\mathcal{D}_n^{(\mathrm{L})}} \sum_{k_s \in E_n} c_{k_s} \overline{\langle W(t)\rangle_{u,k_s} \langle V^\dagger W(t) V\rangle_{u,k_0}},$$

← Set of 2^n initial states, n=0,..,N

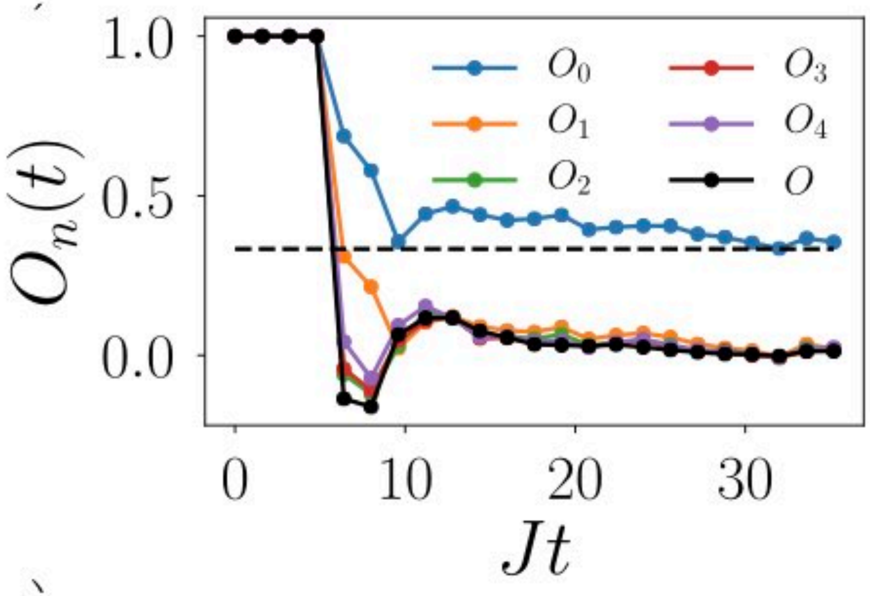
$$\boxed{O_n(t) = \frac{\sum_{A,B_n\subseteq A} \mathrm{Tr}_A\left(W(t)_A (VW(t)V)_A\right)}{\sum_{A,B_n\subseteq A} \mathrm{Tr}_A\left(W(t)_A W(t)_A\right)}}$$

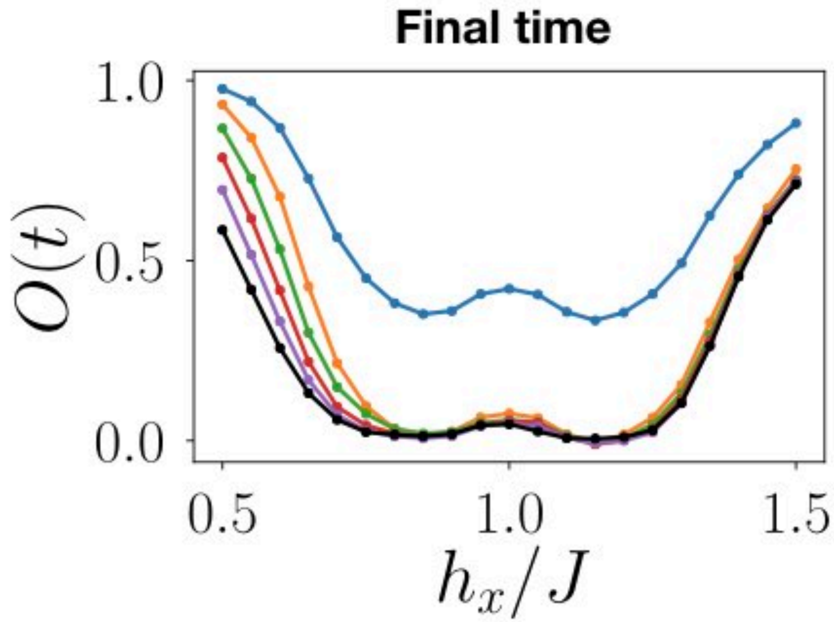
Modified OTOCs:

- → $O_N(t) = O(t)$
- → Fast converging series: n=0,1,2 is generically sufficient

Fast convergence of modified OTOCs: chaos

Example of Many-body Chaos: Kicked Ising with 8 sites





OTOCs as diagnosis of scrambling

arxiv:1807.09087

can be measured in many-body systems with current technology

AMO implementations (no copies)

Statistical errors are not a fundamental issue

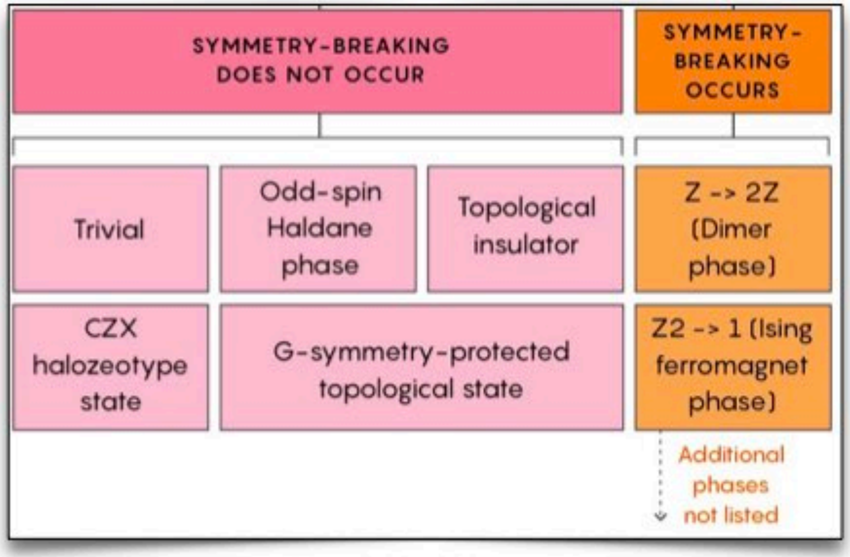
Natural robustness against errors and imperfections (ex: depolarization)

Experiment in progress (M. Joshi, T. Brydges, C. Maier, C. Roos, and R. Blatt)

Measuring scrambling with random measurements

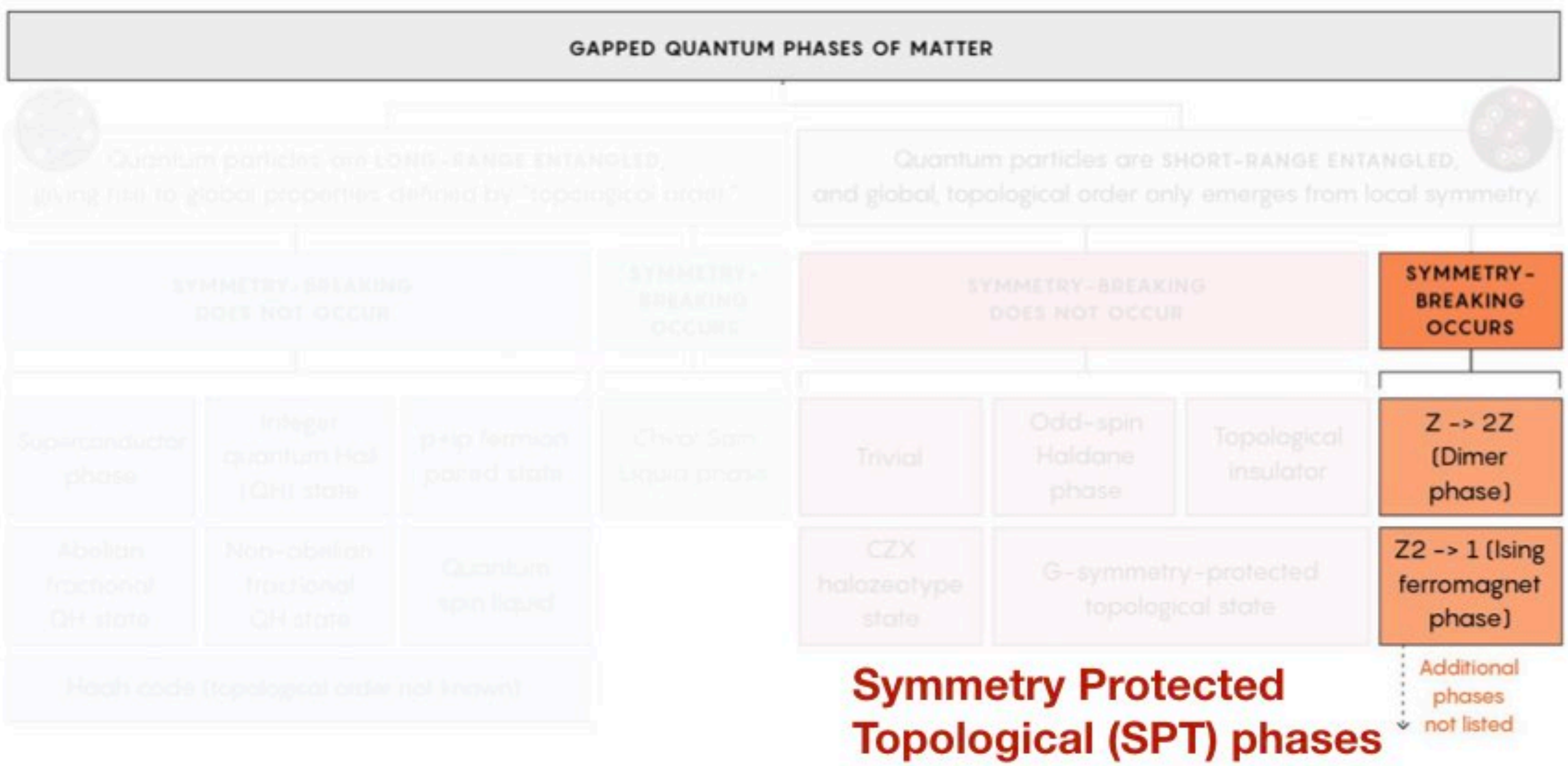
B. Vermersch, A. Elben, L. Sieberer, N. Yao, and P. Zoller

Classification of interacting topological phases (SPT)



A. Elben, B. Vermersch, J. Yu, G. Zhu, M. Hafezi and P. Zoller

Classification of interacting gapped quantum phases



Topological: Absence of Local order parameter

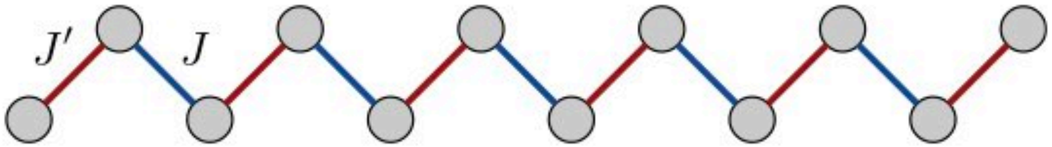
Local order parameter

Pollmann et al. PRB 2010, Schuch et al. PRB 2011, Chen et al., Science 2012, PRB 2013, …

How to probe their classification in the lab?

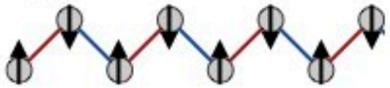
Picture from Quanta Magazine, adapted from Xiao-Gang Wen

1D spin-1/2 model with alternating hoppings



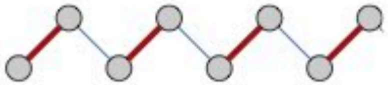
$$H = J' \sum_{i=1}^{N} \left(\sigma_{2i-1}^{-} \sigma_{2i}^{+} + \text{h.c.} + \frac{\delta}{2} \sigma_{2i-1}^{z} \sigma_{2i}^{z} \right. \\ \left. + J \sum_{i=1}^{N-1} \left(\sigma_{2i}^{-} \sigma_{2i+1}^{+} + \text{h.c.} + \frac{\delta}{2} \sigma_{2i}^{z} \sigma_{2i+1}^{z} \right) \right)$$

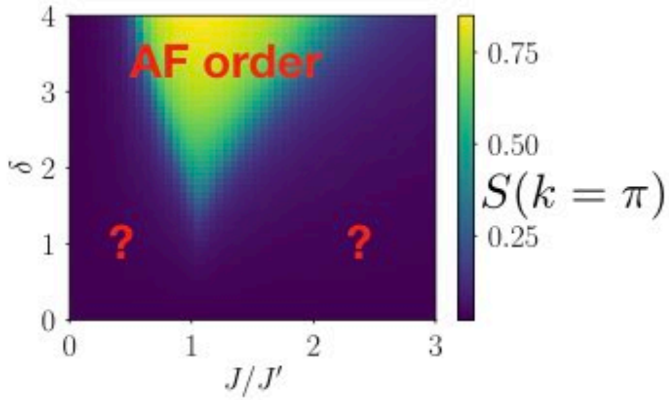
SB phase $|J'| \approx |J|, \delta \gg 1$



Trivial phase

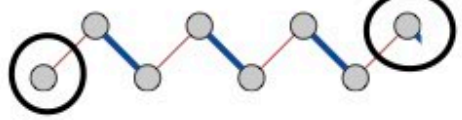
$|J'| \gg |J|, \delta \lesssim 1$





Topological phase

$|J'| \ll |J|, \delta \lesssim 1$



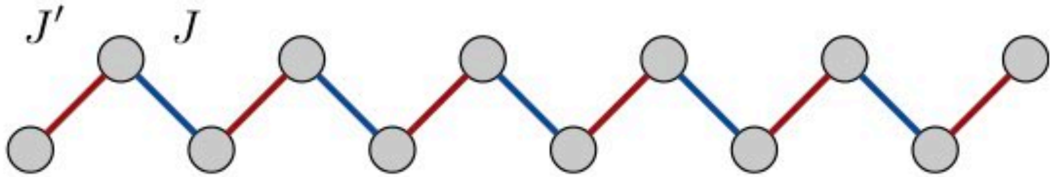
No local order parameter: **topological order**

Case study: the SSH model

IQI uibk

Classification of SPT phases ⟷ Classification of the action of symmetry groups

Pollmann et al. PRB 2010, Schuch et al. PRB 2011, Chen et al., Science 2012, PRB 2013, ...



Type of symmetries

- Internal symmetries (rotations)
- Bond-centered inversion
- Time-reversal

The action of symmetry groups is measured by SPT Topological invariants

Haegeman et al., PRL, 2012
Pollmann, Turner, PRB, 2012

Classification by direct identification of each symmetry representation $\left[e^{i\phi(g,h)}\right]$

Quantized

Strong Connections with topological quantum field theory, tensor-network theory

Ryu, PRL 2017

Key quantities for the classification

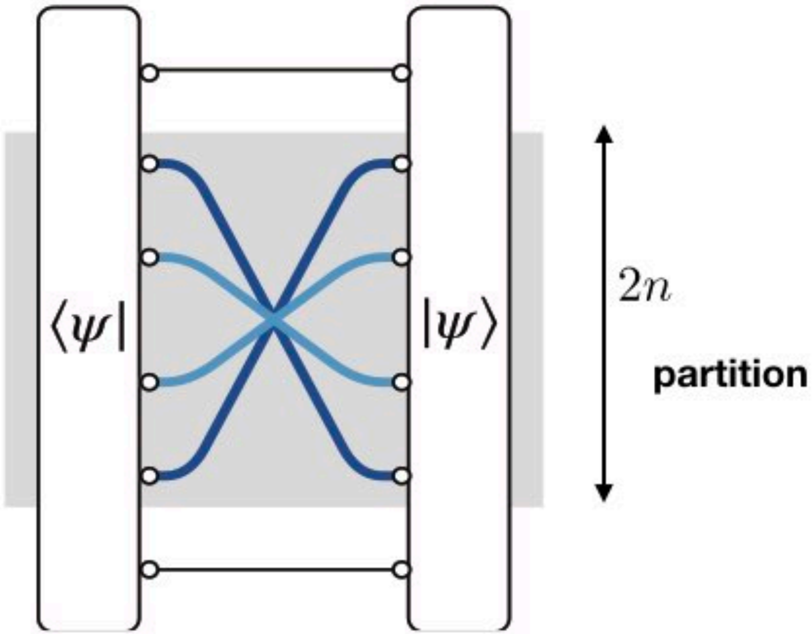
No protocols so far

Partial inversion invariant

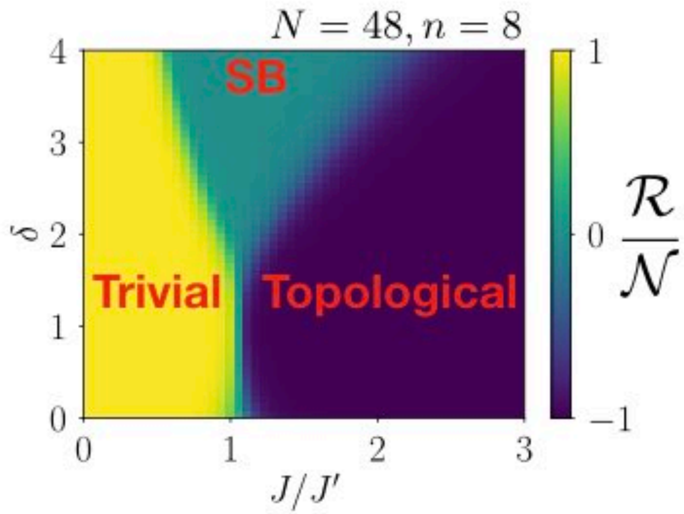
Pollmann, Turner, PRB 2012

$$\mathcal{R}(n) = \mathrm{Tr}\left[\$_{I_1,I_2} |\Psi\rangle \langle\Psi|\right]$$

$$\xrightarrow[n\to\infty]{\text{MPS theory}} \pm\mathrm{Tr}\left[\rho_{HP}^2\right] \leftarrow \text{purity}$$



Application to the SSH model



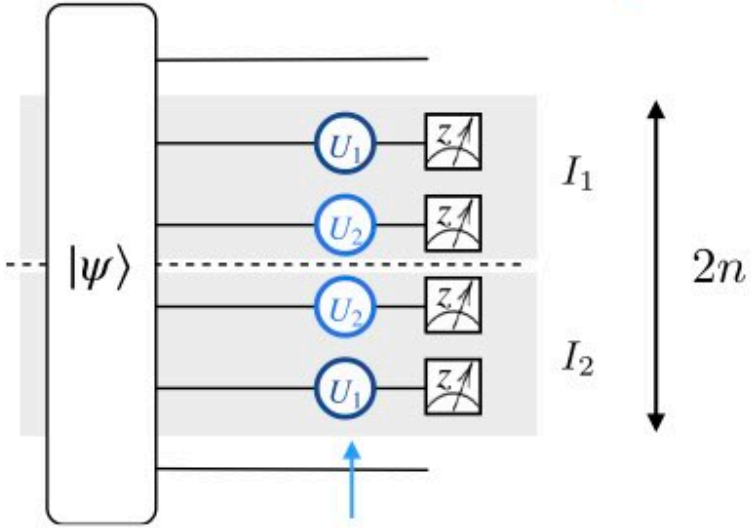
The partial inversion invariant classifies the whole SSH model

How to measure such *non-local* correlations in an experiment?

Measuring topological invariants

IQI uibk

Idea: Correlate random unitaries *in space*

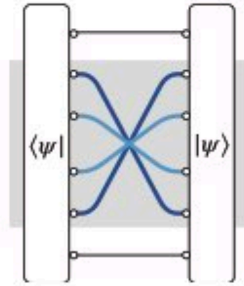


single qubit
random unitaries:

$$
d^n \sum_{\mathbf{s}_{I_1}, \mathbf{s}'_{I_2}} (-d)^{-D[\mathbf{s}_{I_1}, \mathbf{s}'_{I_2}]} \overline{P_{U\otimes U}(\mathbf{s}_{I_1}, \mathbf{s}'_{I_2})}
$$

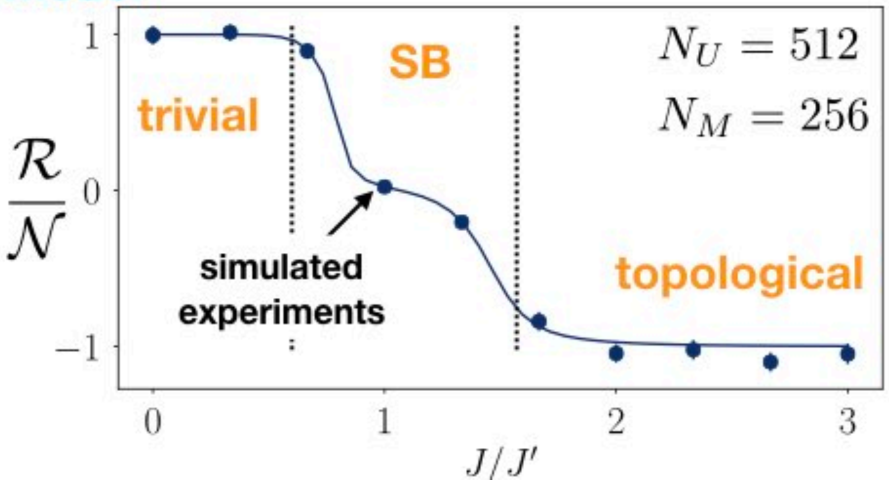
$$
= \mathrm{Tr}\left[\$_{I_1,I_2} |\Psi\rangle \langle\Psi|\right]
$$

$$
\xrightarrow[n\to\infty]{} \pm \mathrm{Tr}\left[\rho_{HP}^2\right]
$$



Classification via random measurements:
apply a distribution with encodes the symmetry to characterize

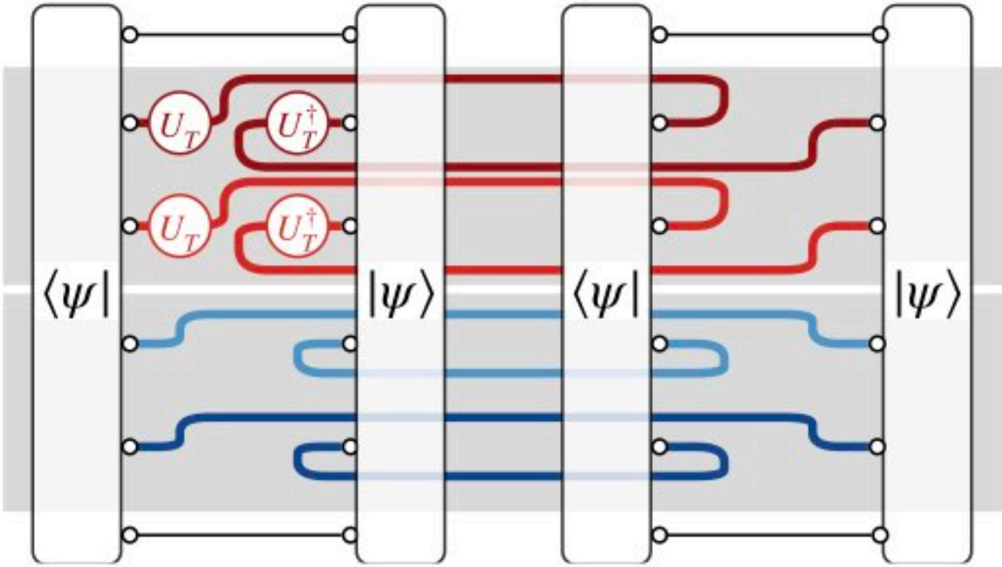
SSH - model



Error bars and bias correction with Jackknife resampling

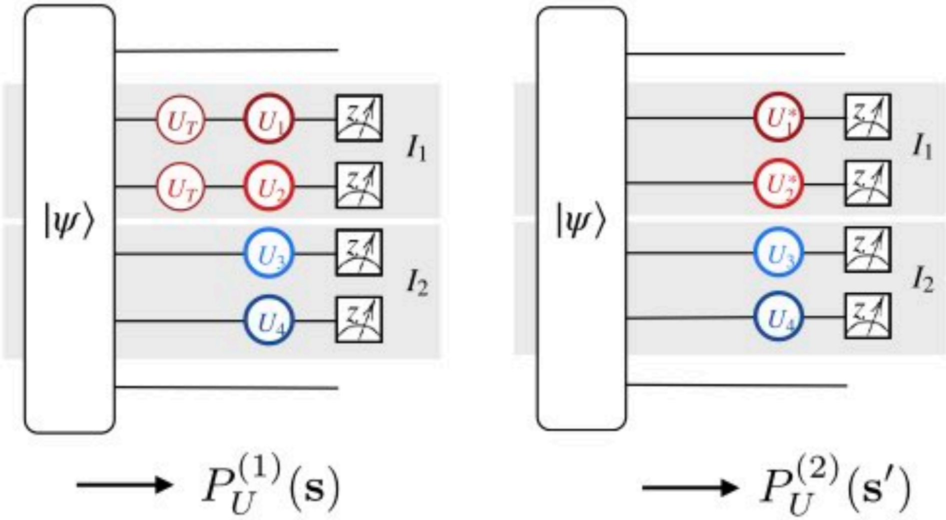
$$N = 24, n = 6, \delta = 2.5$$

Partial transpose invariant



$$\mathcal{T}(n) = \mathrm{Tr}\left[R_{I_1} S_{I_2} |\Psi\otimes\Psi\rangle\langle\Psi\otimes\Psi|\right] \xrightarrow[n\to\infty]{\text{MPS theory}} \pm\mathrm{Tr}\left[\rho_{HP}^2\right]^3$$

Protocol: Correlate *two* experiments



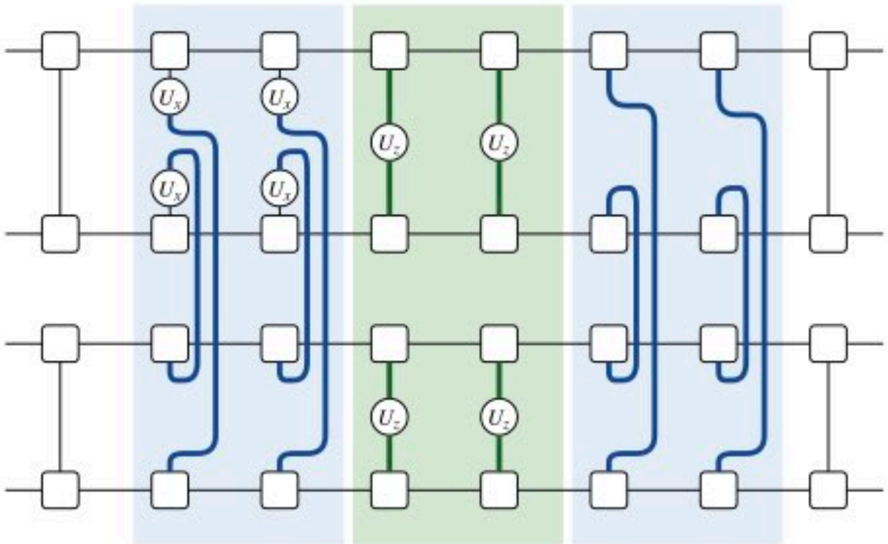
$$d^{2n}\sum_{\mathbf{s},\mathbf{s}'}(-d)^{-D[\mathbf{s},\mathbf{s}']}\overline{P_U^{(1)}(\mathbf{s})P_U^{(2)}(\mathbf{s}')}$$

$$= \mathrm{Tr}\left[R_{I_1} S_{I_2} |\Psi\otimes\Psi\rangle\langle\Psi\otimes\Psi|\right]$$

Other Symmetries

Onsite unitary symmetry

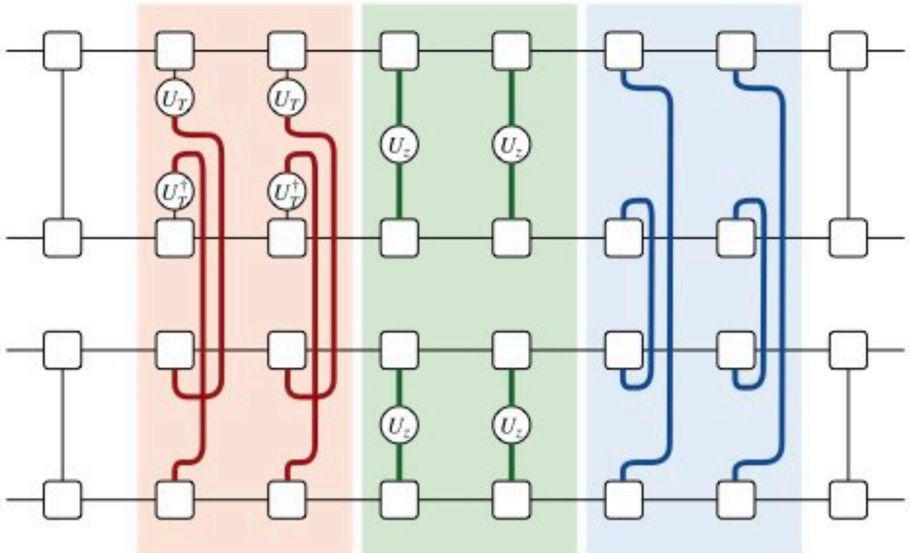
SSH: $\mathbb{Z}_2 \times \mathbb{Z}_2 \qquad e^{i\pi/2\sigma_x}, e^{i\pi/2\sigma_z}$



Haegemann et al., PRL 2012

Time reversal + Onsite symmetry

SSH: Time reversal + $U(1)$



Shiozaki, Ryu, JHEP 2017

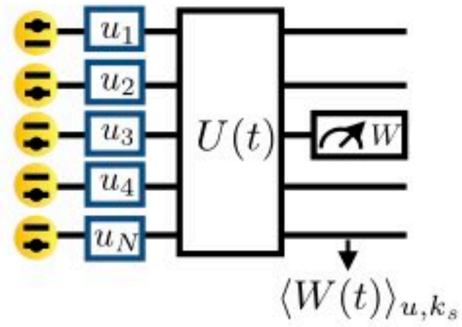
are accessible with a specific distribution of *local* random unitaires

Direct applications:

Bosonic SPT phases can be classified and tested in the lab now

Statistical correlations of randomized measurements

- a tool to probe quantum states beyond standard observables
- applicable in any state-of-the art quantum simulation platform with high repetition rate
- **A tool to verify the quantum features of quantum simulators**



Rényi entropies

Elben, Vermersch et al. PRL, PRA 2018

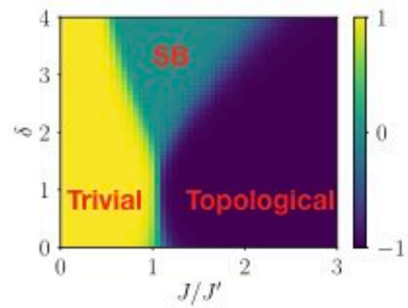
Brydges, Elben et al., arXiv:1806.05747

Out-of-time ordered correlation functions

Vermersch et al., arXiv:1807.09087

Topological invariants

with A.Elben, J. Yu, G. Zhu, M. Hafezi and P. Zoller



Prospects

- Detection/Classification of true topological order
- Protocols for Hubbard models (MBL as ressource?)
- Theory of random measurements
- Verification

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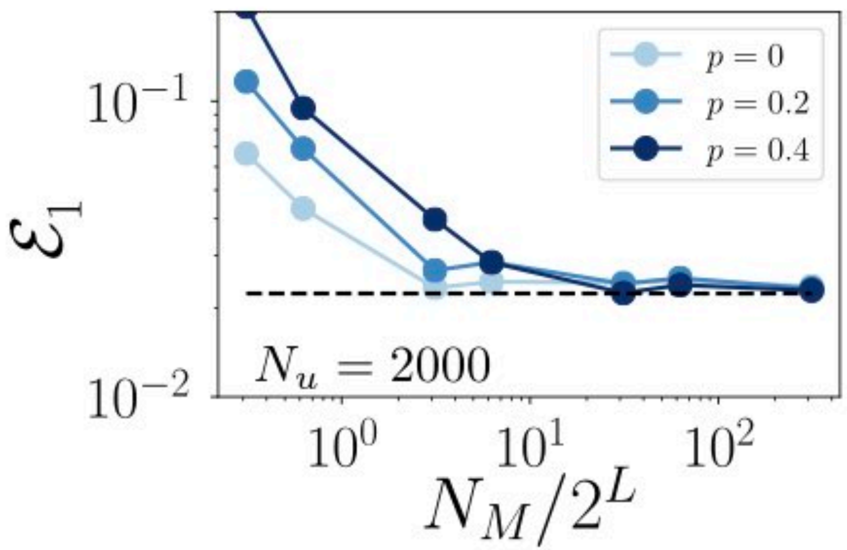
ÖAW

UQUAM ERC Synergy Grant

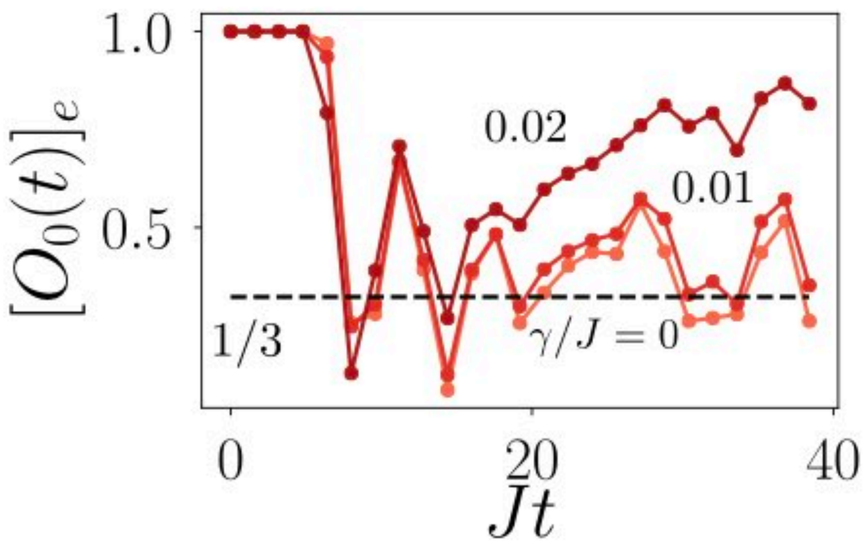
PASQuanS

Robust against local unitary errors depolarization



Spontaneous emission

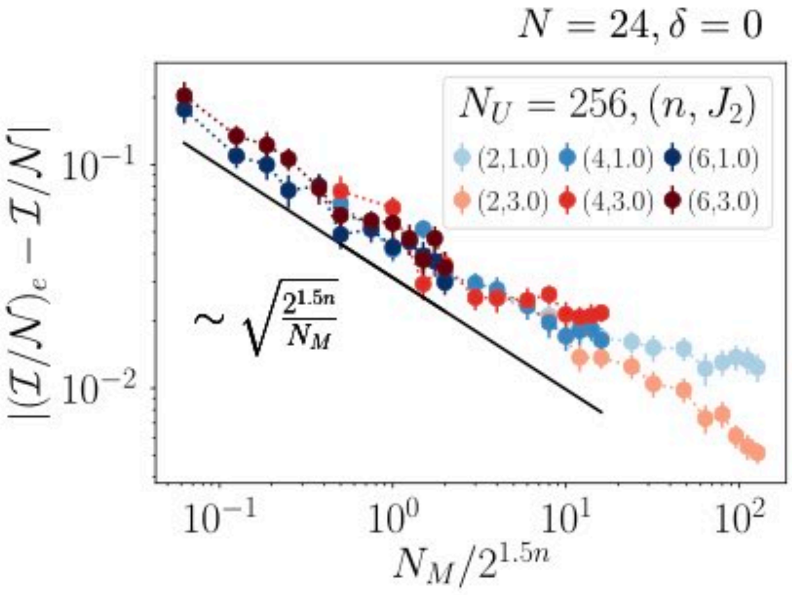
In contrast to time-reversal methods, decoherence and scrambling have opposite signatures



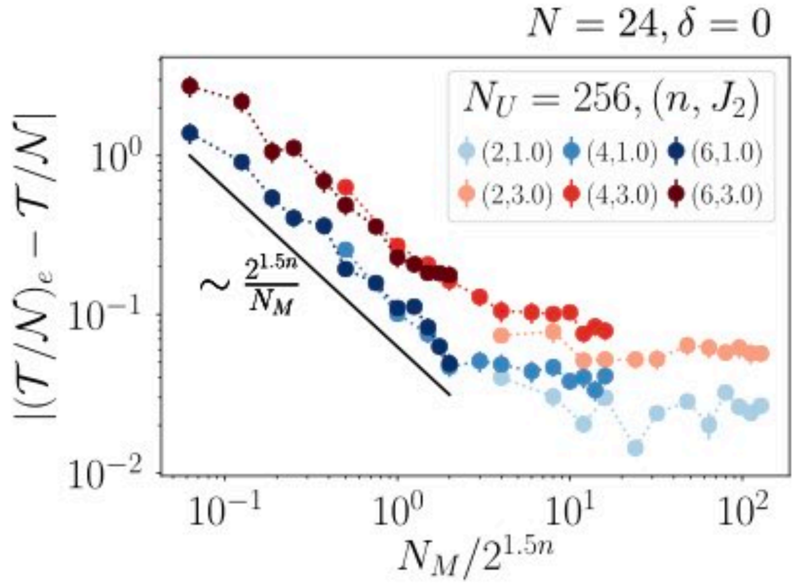
Average statistical errors

How does the required number of measurements scale with swapped sites n ?

Inversion invariant



Time reversal invariant



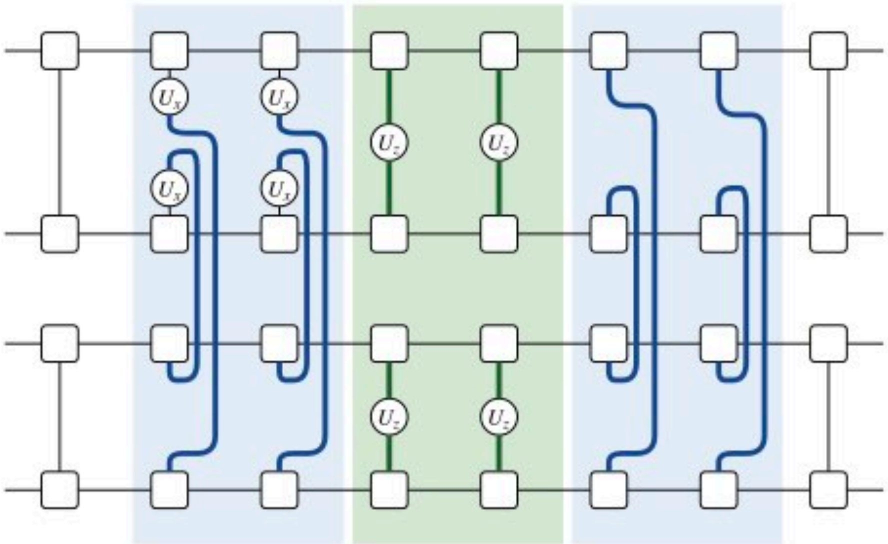
$$\Delta_{\mathcal{I}_n} \sim \frac{1}{\sqrt{N_U}} \left(C_I(n) + \sqrt{\frac{2^{1.5n}}{N_M}} \right)$$

$$\Delta_{\mathcal{T}_n} \sim \frac{1}{\sqrt{N_U}} \left(C_T(n) + \frac{2^{1.5n}}{N_M} \right)$$

Exponential scaling with swapped sites - for relevant sizes within range of experimental possibilities (comparable to Renyi experiments)!

Onsite symmetry

Order parameter



$$\mathcal{C}_n = \langle \Psi \otimes \Psi | \bigotimes_{i \in I_1} (U_i^g)^\dagger \bigotimes_{i \in I_2} (U_i^h)^\dagger$$
$$S_{I_1} S_{I_3} \bigotimes_{i \in I_1} U_i^g \bigotimes_{j \in I_2} U_j^h | \Psi \otimes \Psi \rangle$$

Haegemann et al., PRL 2012

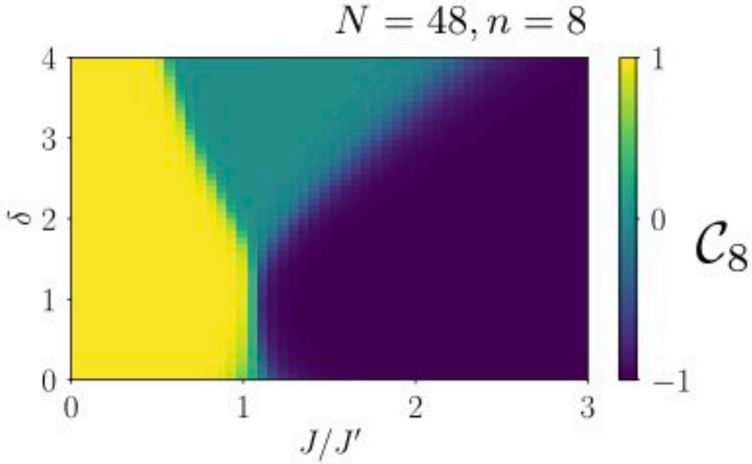
SSH model

Symmetry

$$D_2 = \mathbb{Z}_2 \times \mathbb{Z}_2$$

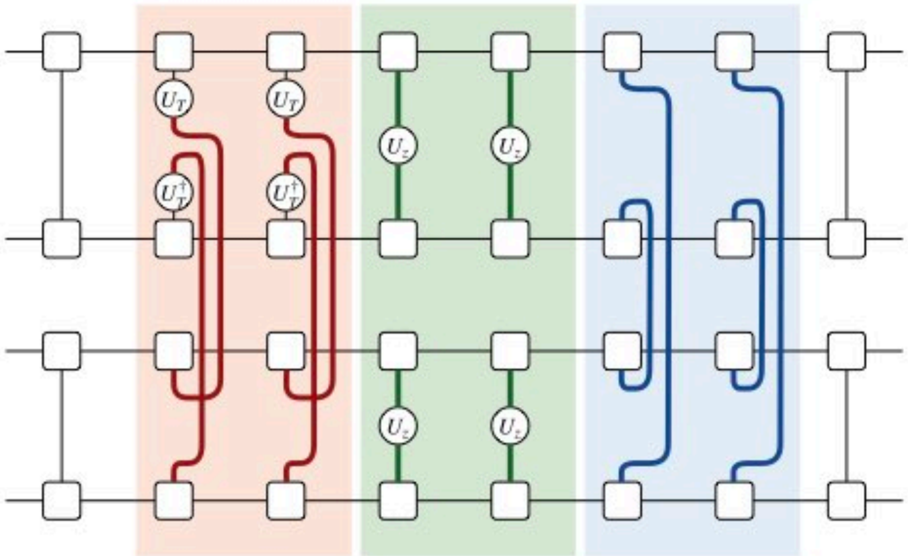
acting through

$$e^{i\pi S_x} \quad e^{i\pi S_z}$$



Klein-Bottle invariant

Time reversal + Onsite symmetry



Shiozaki, Ryu, JHEP 2017

$$\mathcal{K}_n = \langle \Psi \otimes \Psi | \bigotimes_{i \in I_1} (U_i^T)^\dagger \bigotimes_{i \in I_2} (U_i^g)^\dagger$$

$$R_{I_1} S_{I_3} \bigotimes_{i \in I_1} U_i^T \bigotimes_{j \in I_2} U_j^g | \Psi \otimes \Psi \rangle$$

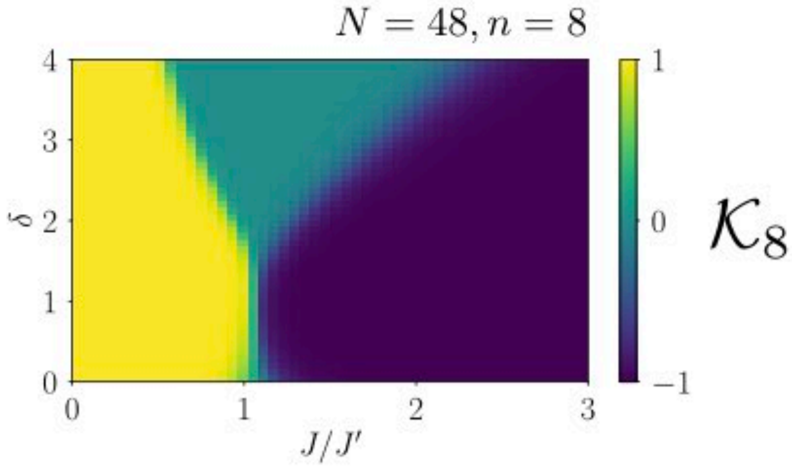
SSH model

Symmetry

$D_2 = \mathbb{Z}_2 \times \mathbb{Z}_2$ + Time reversal

acting through

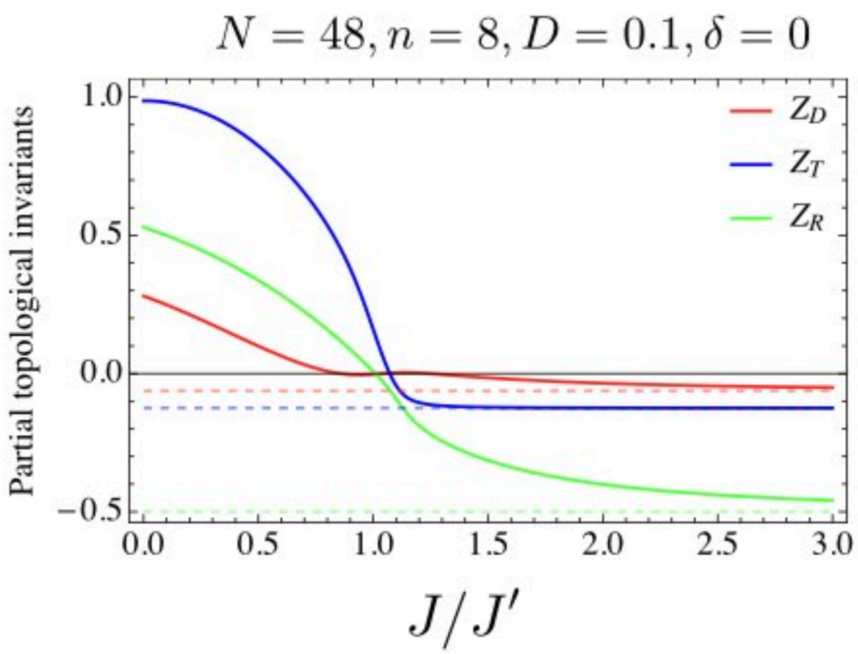
$e^{i\pi/2\sigma_y}\mathcal{K}, e^{i\pi/2\sigma_x}, e^{i\pi/2\sigma_z}$

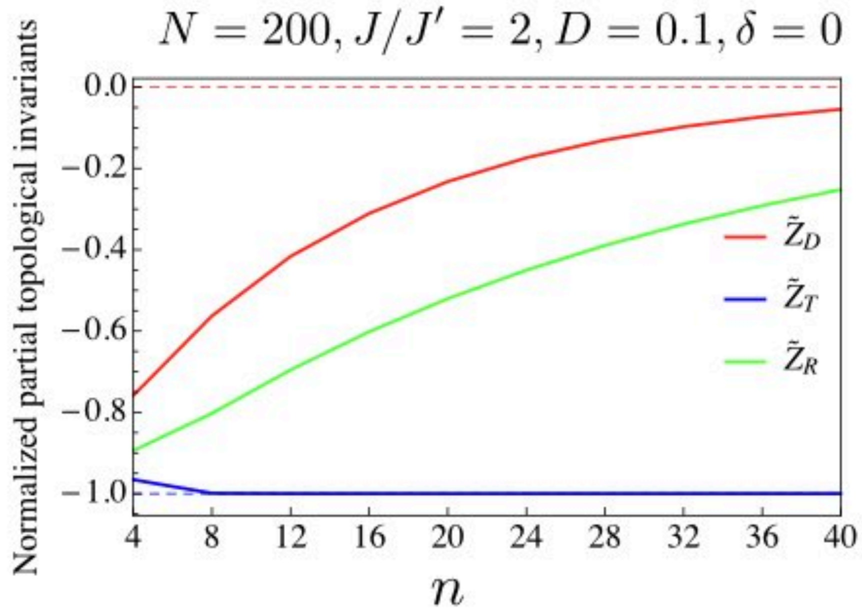


Explicit breaking of symmetries

$$H = H_{\mathrm{SSH}} + D \sum_j (\sigma_j^x \sigma_{j+1}^z - \sigma_j^z \sigma_{j+1}^x)$$

Inversion and D2 symmetry broken

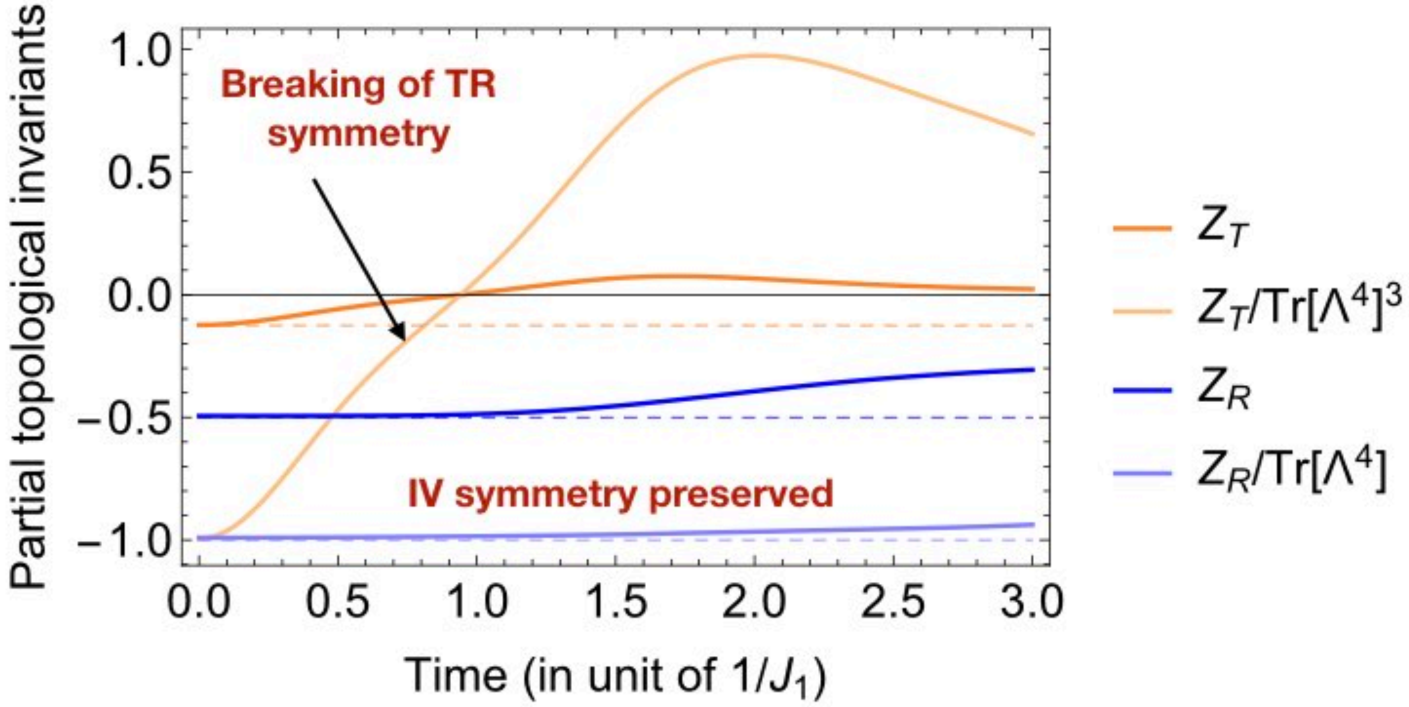




Quench dynamics - Breaking time reversal symmetry

$$H = J' \sum_{i=1}^{N} \left(\sigma_{2i-1}^{-}\sigma_{2i}^{+} + \text{h.c.} + \delta\sigma_{2i-1}^{z}\sigma_{2i}^{z}\right) + J\sum_{i=1}^{N-1}\left(\sigma_{2i}^{-}\sigma_{2i+1}^{+} + \text{h.c.} + \delta\sigma_{2i}^{z}\sigma_{2i+1}^{z}\right)$$

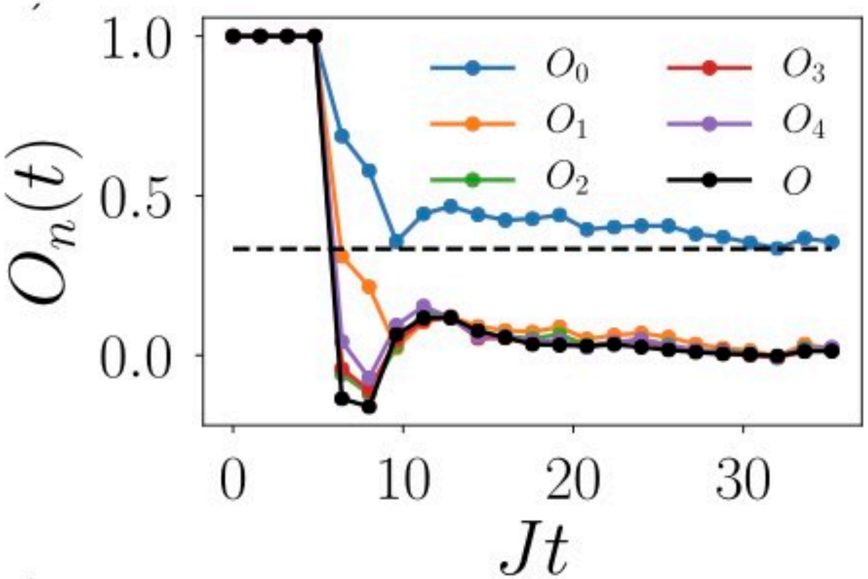
Quench from topological to trivial phase: $(J', J) = (1,2) \to (1, 1/2)$

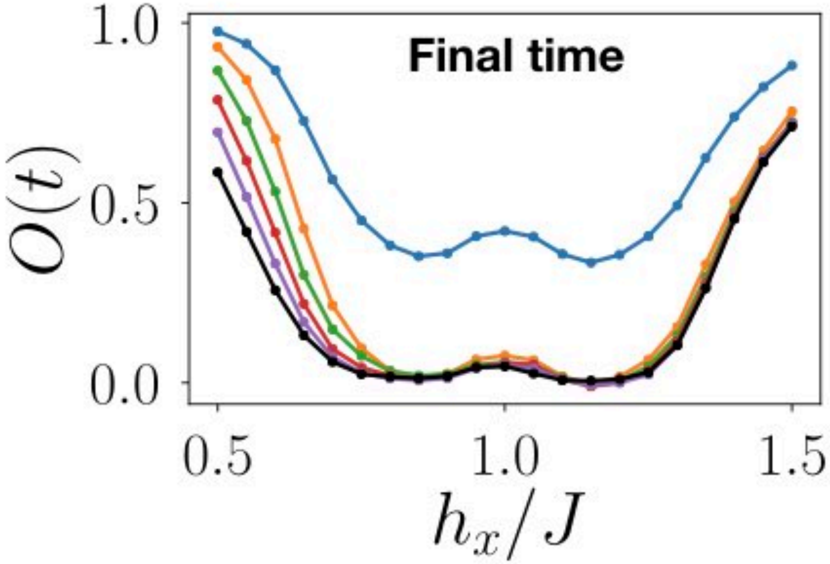


Fast convergence of modified OTOCs: chaos

IQI uibk

Example of Many-body Chaos: Kicked Ising with 8 sites





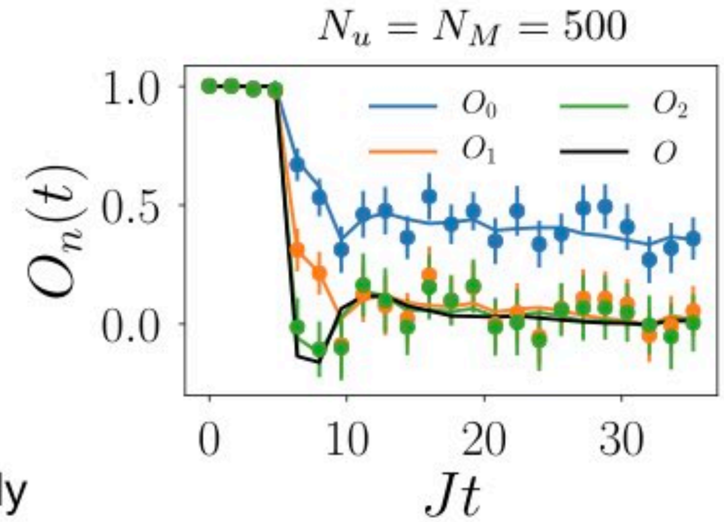
Statistical errors

$L(t) \approx v_B t$ "Scrambling length"

$N_M = 2^{L(t)} \longrightarrow$ error $1/\sqrt{N_u}$

projective measurements

unitaries



Independent of System Size

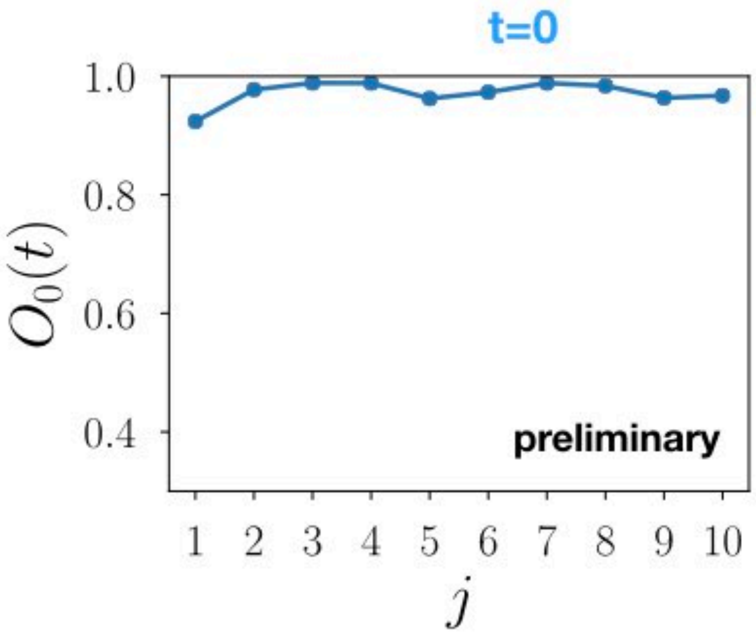
All N operators W are measured simultaneously

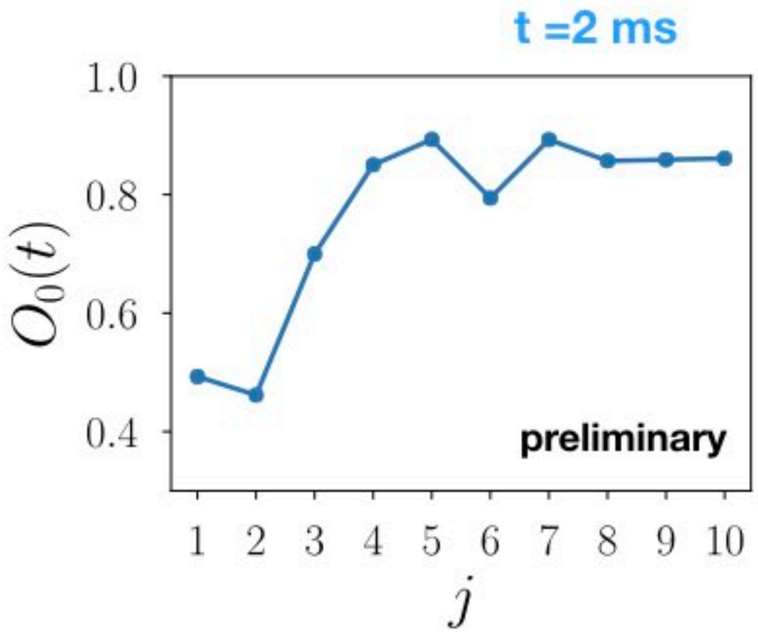
First experimental pictures

Collaboration with **M. Joshi, T. Brydges, C. Maier, C. Roos, and R. Blatt**

10 ions, evolution with long-range XY model

OTOCs in the x-basis





Classification of 1D SPT phases via Matrix-Product-States

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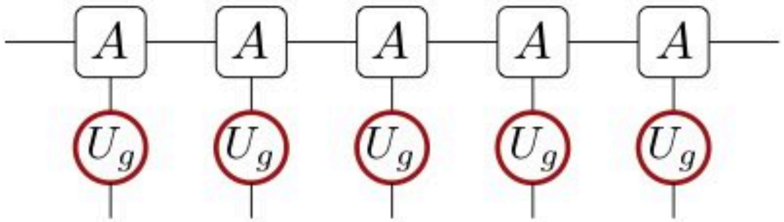
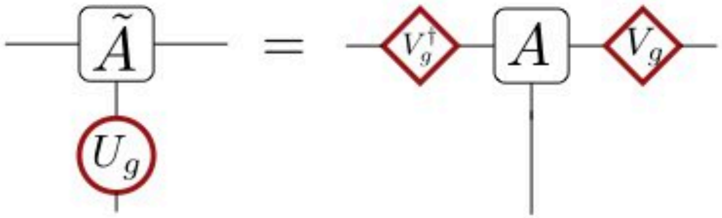
Haegeman et al., PRL, 2012
Pollmann et al. PRB 2012

IMPS: $\bigotimes_i U_g^i |\psi_{GS}\rangle =$  $= |\psi_{GS}\rangle$

Transformation under symmetry action



with

Projective representations of G

$$V_g \tilde{V}_h = e^{i\phi(g,h)} V_{gh}$$

On-site unitary: $\tilde{A} = A$
Inversion: $\tilde{A} = A^T$
Time reversal: $\tilde{A} = A^*$

Classification of SPT phases

$$\left[e^{i\phi(g,h)}\right] \in H^2(G, U(1)_\phi)$$

Chen et al., Science 2012

2nd cohomology group of G

How to access and measure $\left[e^{i\phi(g,h)}\right]$ for a given G?

Purity from random measurements

Observation 1

$$\mathrm{Tr}\left[\rho^2\right] = \mathrm{Tr}\left[\mathbb{S}\rho\otimes\rho\right]$$ with Swap operator $\mathbb{S}$

Observation 2

$$\sum_{\mathbf{s},\mathbf{s}'} A_{\mathbf{s},\mathbf{s}'}\overline{P_U(\mathbf{s})P_U(\mathbf{s}')} = \mathrm{Tr}[\underbrace{\overline{U^\dagger\otimes U^\dagger AU\otimes U}}_{\Phi_N^2(A)}\rho\otimes\rho]$$

$$P_U(\mathbf{s}) = \mathrm{Tr}\left[U\rho U^\dagger \left|\mathbf{s}\right\rangle\left\langle\mathbf{s}\right|\right]$$

$$U = \bigotimes u_i \quad u_i \in \mathrm{CUE}(d)$$

Generalized unitary twirling channel on 2-copy space

$$\xrightarrow[\text{duality}]{\text{Schur-Weyl}} \quad \Phi_N^2(A) = \sum_{\pi,\sigma\in S_2^{\otimes N}} C_{\pi,\sigma}\mathrm{Tr}\left[W_\pi A\right]W_\sigma \overset{!}{=} \mathbb{S}$$

Weingarten Matrix

Permutation operator

$$\longrightarrow \quad A = 2^N\sum_{\mathbf{s},\mathbf{s}'}(-2)^{-D[\mathbf{s},\mathbf{s}]}\left|\mathbf{s}\right\rangle\left\langle\mathbf{s}\right|\otimes\left|\mathbf{s}'\right\rangle\left\langle\mathbf{s}'\right|$$