A New Solid-State Platform for Quantum Simulation and Optimization

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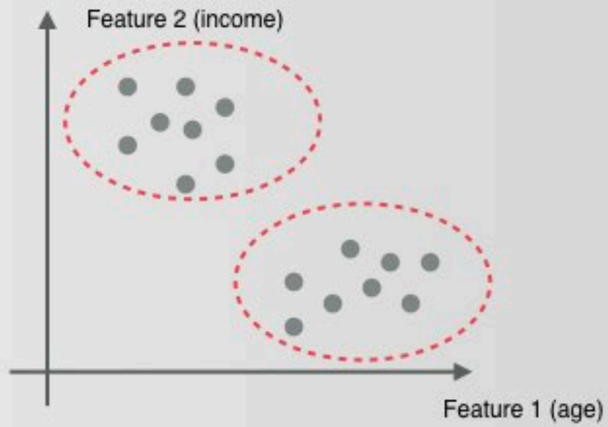
arXiv:1809.03794

Motivation: Quantum Optimization (QAOA)

Motivation: Clustering (unsupervised learning).

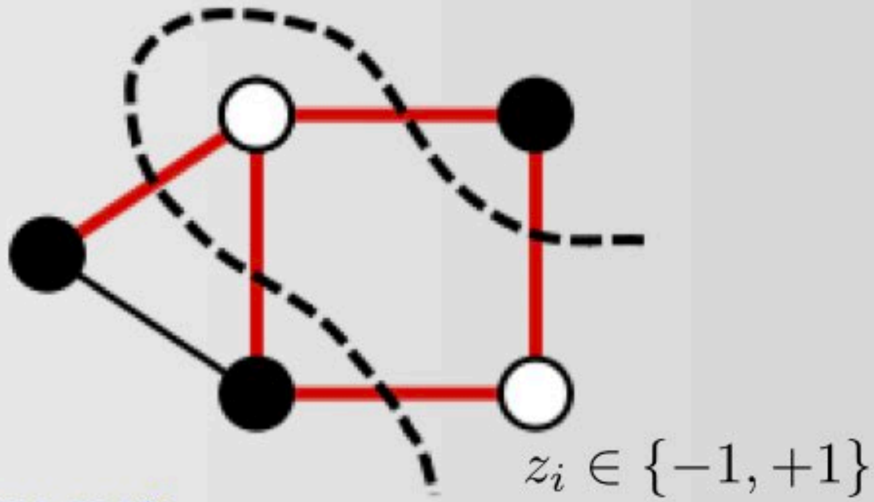
$$w_{i,j} = d(\mathbf{x}_i, \mathbf{x}_j)$$



Maximization goal:

$$\sum_{i \in S, j \in \bar{S}} w_{i,j} \to \max$$

Put the problem on a graph:



$$H_C = \sum_{i,j} \frac{w_{i,j}}{2}(1 - z_i z_j)$$

[MAXCUT: NP-complete problem]

Minimize energy of frustrated anti-ferromagnet:

$$H_{AF} = \sum_{i,j} w_{i,j} z_i z_j$$

Quantum Optimization (QAOA)

Quantum Adiabatic Algorithm:

- (Classical) problem Hamiltonian: $H_C = \sum_{\langle i,j \rangle} J_{ij} \sigma_i^z \sigma_j^z$
- **Goal: Find/approximate ground state (energy)**

$$H(s) = s \sum_{\langle i,j \rangle} J_{ij} \sigma_i^z \sigma_j^z + (1-s) \sum_i \sigma_i^x \qquad s = t/T$$

$$s = 0, H = \sum_i \sigma_i^x \qquad\qquad s = 1, H = \sum_{\langle i,j \rangle} J_{ij} \sigma_i^z \sigma_j^z$$

Slow, adiabatic

- Gap can be exponentially small (with the system size)

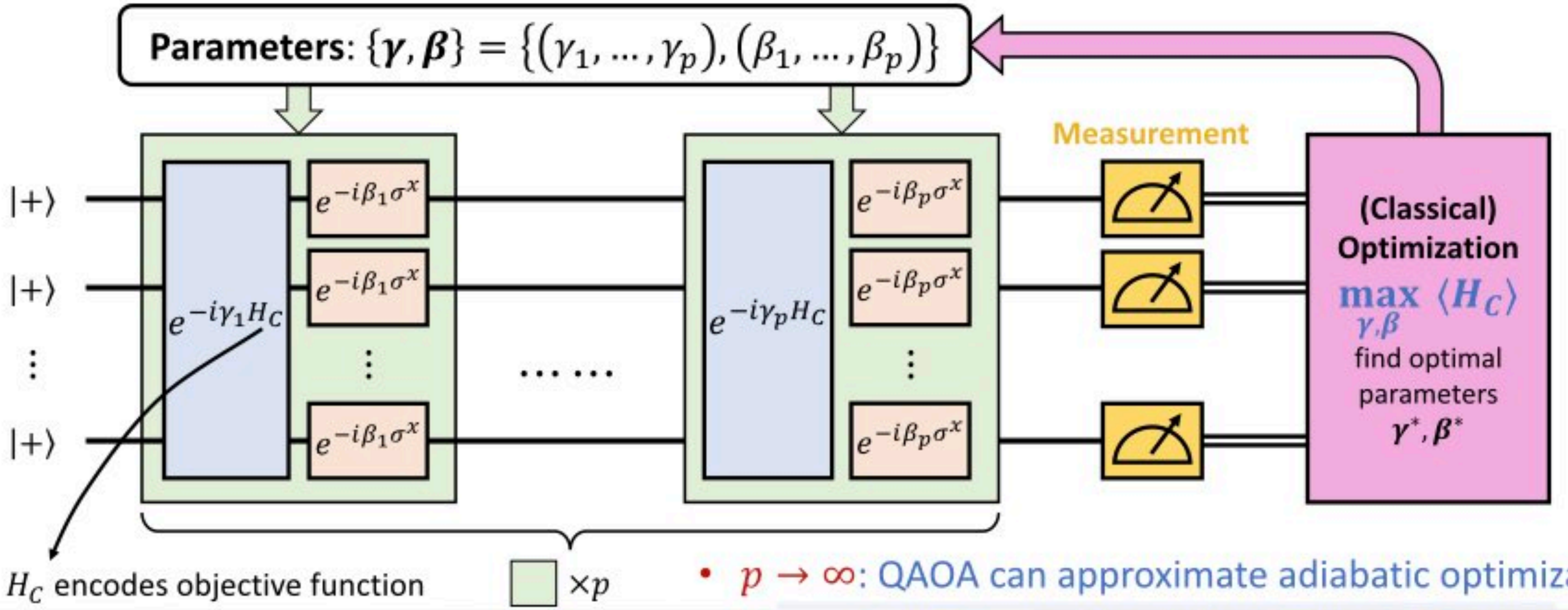
Quantum Optimization (QAOA)

Quantum Approximate Optimization Algorithm (QAOA):

$$H_C = \sum_{\langle i,j \rangle} J_{ij}\sigma_i^z\sigma_j^z \quad H_B = \sum_i \sigma_i^x$$

$$|\psi_p(\boldsymbol{\gamma},\boldsymbol{\beta})\rangle = e^{-i\beta_p H_B} e^{-i\gamma_p H_C} \cdots e^{-i\beta_1 H_B} e^{-i\gamma_1 H_C} |+\rangle^{\otimes n}$$

$$\langle H_C \rangle = \langle \psi_p(\boldsymbol{\gamma},\boldsymbol{\beta})|H_C|\psi_p(\boldsymbol{\gamma},\boldsymbol{\beta})\rangle$$



- $p \to \infty$: QAOA can approximate adiabatic optimization

arXiv:1411.4028

- QAOA: Heuristic *variational* ansatz (Edi Farhi, MIT).
- Apply iteratively pair of unitaries before measuring classical string.
- Probability of success approaches unity for $M \to \infty$.

Requirements for a solid-state QAOA architecture

Single qubit rotations

$$H_C = \sum_{\langle i,j \rangle} J_{ij} \sigma_i^z \sigma_j^z \quad H_B = \sum_i \sigma_i^x$$

Arbitrary All to All Ising interactions

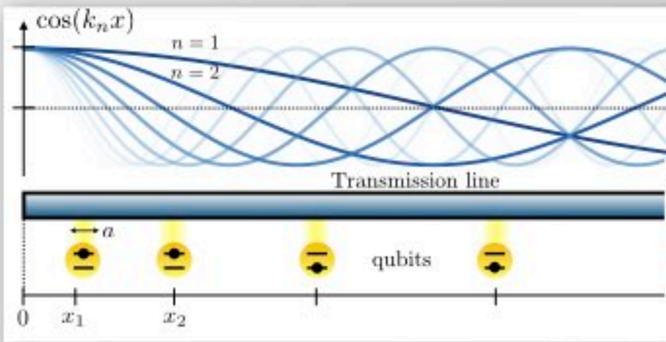
→ **Programmable**

→ Scalable

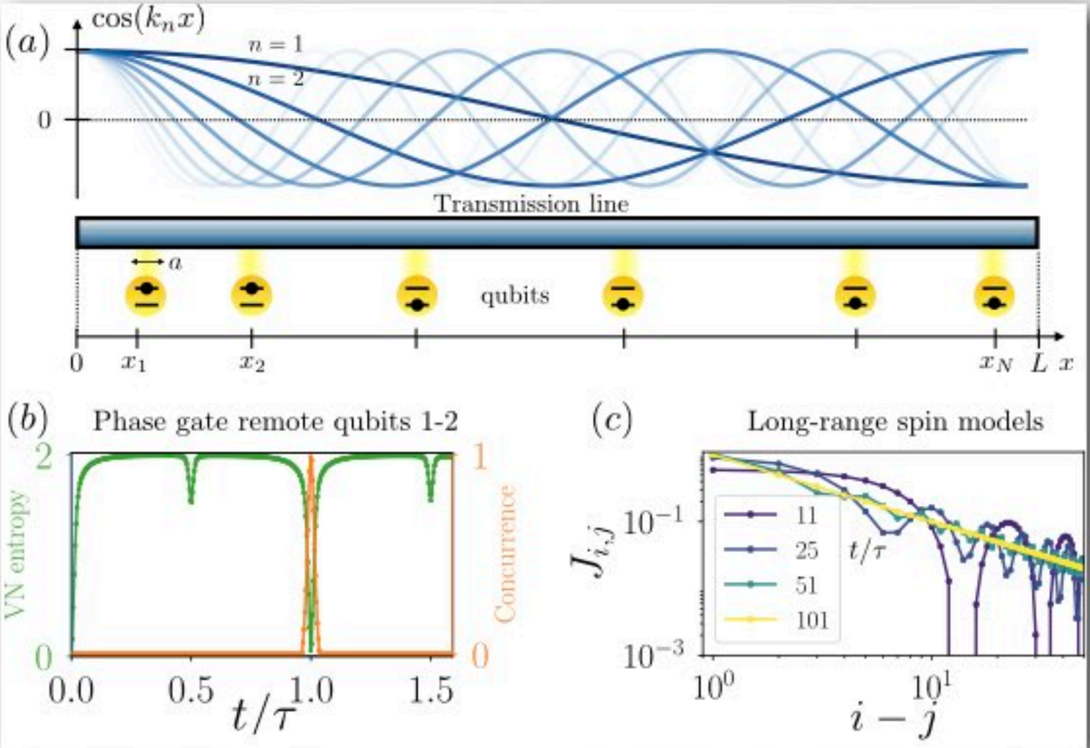
→ Non-Perturbative

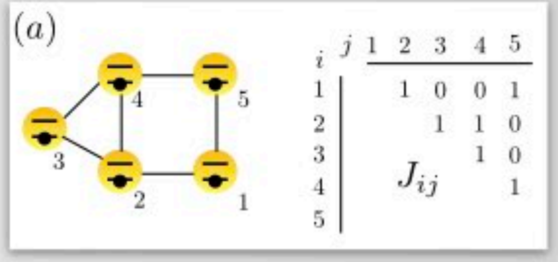
→ Robust

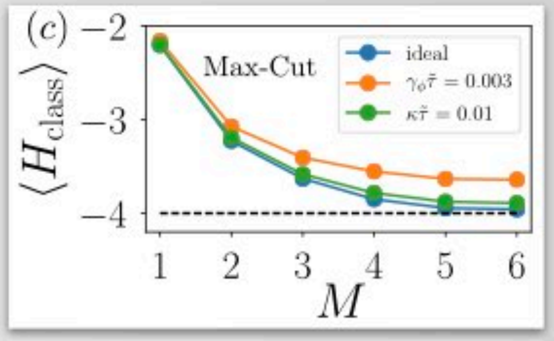
- Quantum Simulation and Optimization in Hot Quantum Networks:
  - Flexible and robust platform for QAOA.
  - arXiv: 1809.03794.



Take-Home Message







Long-range entanglement and quantum optimization in hot quantum networks:

- Long (*multi-mode*) transmission line as quantum bus.
- No ground-state cooling required, data bus can be hot.
- Approach not based on perturbative argument for qubit-photon coupling.
- Qubits do *not* have to be identical.
- Recipe to generate desired spin-spin interactions.
- Robust and flexible implementation for QAOA.

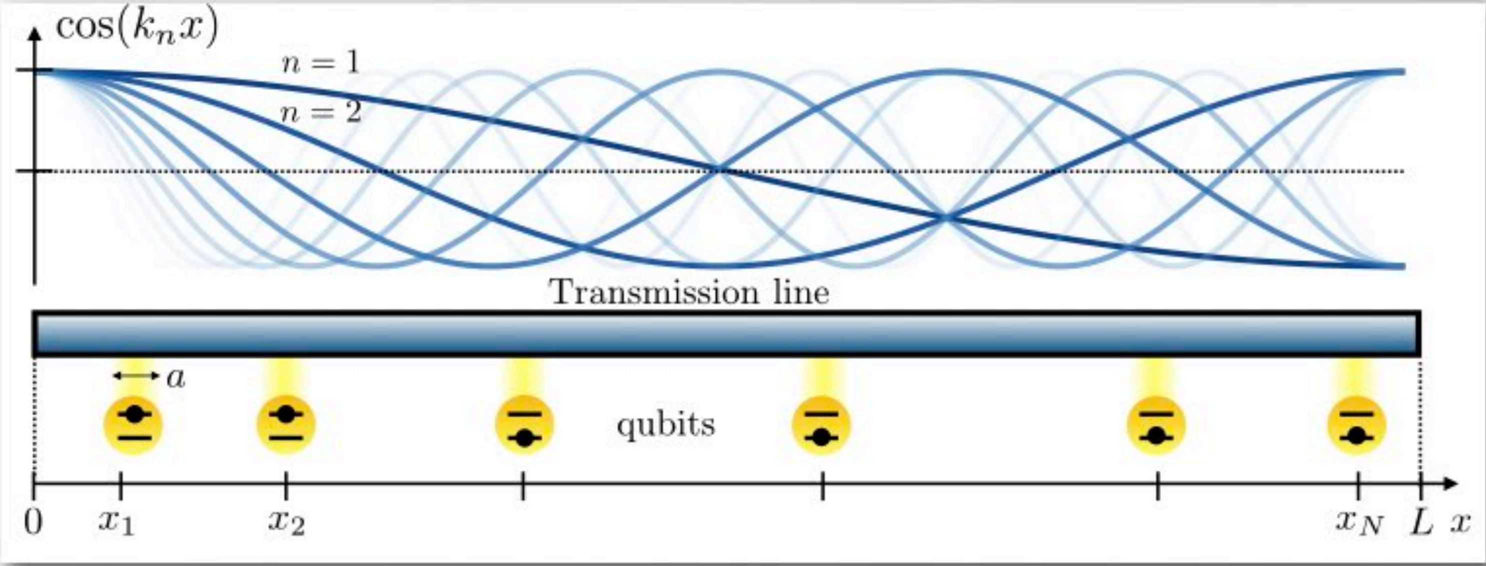
Outline

Model and exact Solution

Illustrations

- Two-qubit gate
- Quantum Simulation
- Quantum optimization

Implementations with Superconducting qubits



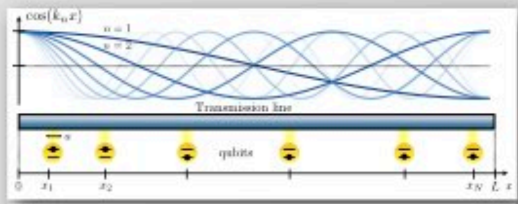
$$H = \sum_i \frac{\omega_i}{2}\sigma_i^z + \sum_{n=1}^{\infty} \omega_n a_n^\dagger a_n + \sum_{i,n} g_{i,n}\sigma_i^z \left(a_n + a_n^\dagger\right)$$

Spin-resonator system with *longitudinal* coupling:

- Linear resonator spectrum: $\omega_n = k_n c = n\omega_1$, where $\omega_1 = \pi c/L$

The Model

$$H = \sum_i \frac{\omega_i}{2}\sigma_i^z + \sum_{n=1}^{\infty} \omega_n a_n^\dagger a_n + \sum_{i,n} g_{i,n}\sigma_i^z \left(a_n + a_n^\dagger\right)$$



Multi-mode (polaron) transformation

$$H = U_{\text{pol}} \tilde{H} U_{\text{pol}}^\dagger$$

$$U_{\text{pol}}^\dagger = \exp[\textstyle\sum_{n,i} \frac{g_{i,n}}{\omega_n}\sigma_i^z \left(a_n^\dagger - a_n\right)],$$

$$\tilde{H} = \sum_i \frac{\omega_i}{2}\sigma_i^z + \sum_n \omega_n a_n^\dagger a_n + \sum_{i<j} J_{ij}\sigma_i^z\sigma_j^z,$$

[exact result]

$$J_{ij} = -2\sum_n \frac{g_{i,n}g_{j,n}}{\omega_n}.$$

[effective spin-spin interactions]

Analytical Solution of Lab-Frame Dynamics

$$H = U_{\rm pol}\tilde{H}U_{\rm pol}^{\dagger}$$

$$\tilde{H} = \sum_i \frac{\omega_i}{2}\sigma_i^z + \sum_n \omega_n a_n^{\dagger} a_n + \sum_{i<j} J_{ij}\sigma_i^z\sigma_j^z,$$

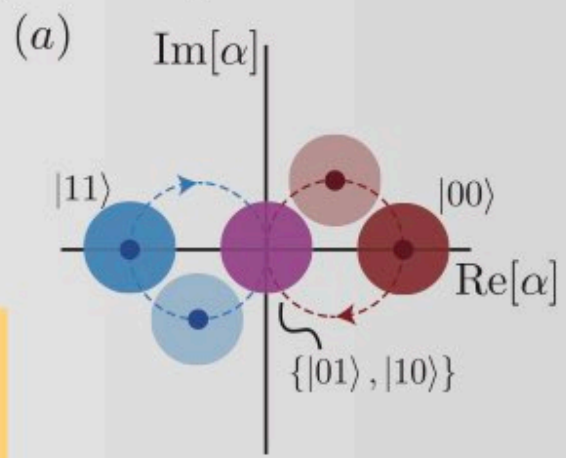
$$e^{-iHt} \;=\; U_{\rm pol}e^{-i\tilde{H}t}U_{\rm pol}^{\dagger}$$

$\omega_n t_p \;=\; 2\pi p_n$ Stroboscopic, not perturbative.

$$\exp\left[-it_p \textstyle\sum_n \omega_n a_n^{\dagger}a_n\right] = \exp\left[-2\pi i \textstyle\sum_n p_n a_n^{\dagger}a_n\right] = \mathbb{1}$$

$t_p = p_1\tau$ (with $p_n = np_1$)

$\tau \equiv 2L/c$



$$U_{\rm lab}(t_p) = e^{-it_p\sum_i(\omega_i/2)\sigma_i^z}\,e^{-it_p\sum_{i<j}J_{ij}\sigma_i^z\sigma_j^z}.$$

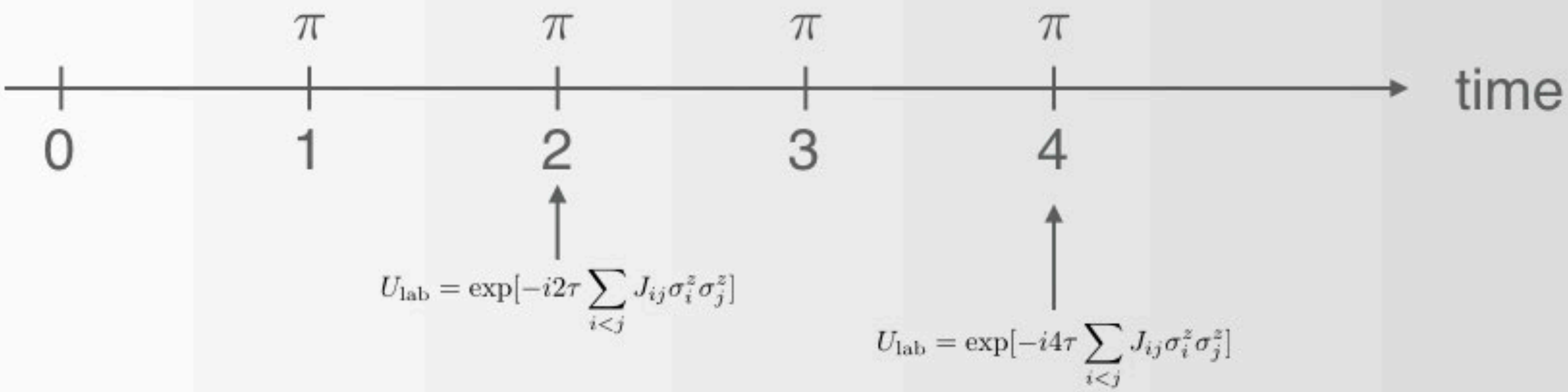
[unitary generates qubit-qubit interactions, independent of state of resonator modes]

Analytical Solution of Lab-Frame Dynamics

$$U_{\mathrm{lab}}(t_p) = e^{-it_p \sum_i (\omega_i/2)\sigma_i^z} e^{-it_p \sum_{i<j} J_{ij}\sigma_i^z\sigma_j^z}.$$

Pure spin Hamiltonian: Independent of resonator modes.

- Approach not based on perturbative arguments.
- For certain times, evolution in polaron and lab frame fully coincide.
- Scheme insensitive to the state of the resonator, allowing for *thermally* robust gate, without need for ground-state cooling.
- No (resonance) conditions on qubit frequencies ω_i .



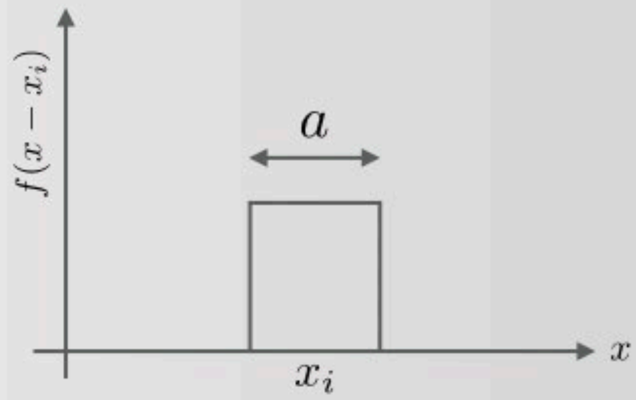
Effective Spin-Spin Interactions: Frequency Cutoff

$$J_{ij} = -2\sum_n \frac{g_{i,n}g_{j,n}}{\omega_n}.$$

- By definition, effective spin-spin interactions involves all modes n=1,2,...
- Unphysical divergencies? Example: $g_{i,n} = g_i\sqrt{n}\cos(k_n x_i)$
- Microscopic length-scale introduces frequency cutoff: $k_{\max} \sim \pi/a$.

$$g_{i,n} = g_i\sqrt{n}\int_0^L \cos(k_n x) f(x - x_i) dx$$

$$J_{ij} = g_i g_j/\omega_1$$



$$g_{i,n} = g_i\sqrt{n}\left(\sin\left[k_n(x_i + a)\right] - \sin\left[k_n x_i\right]\right)/(k_n a)$$

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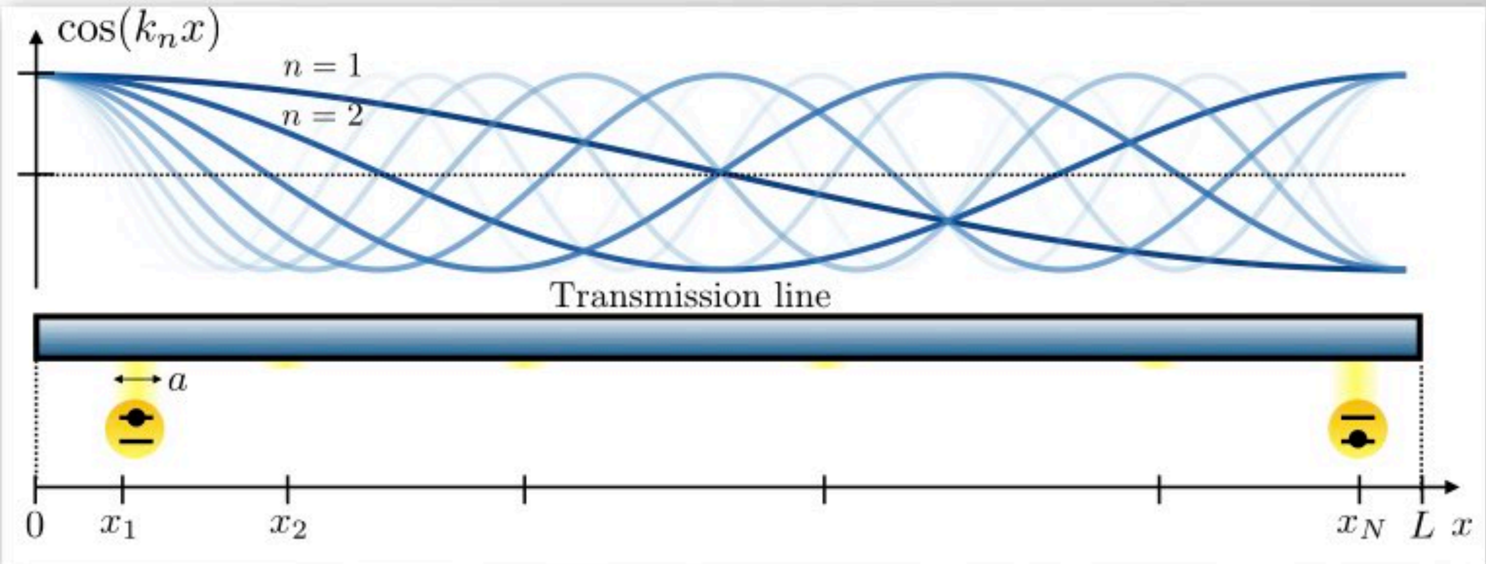
Hot qubit Gate between two remote qubits



$$\rho_0 = |\Psi_0\rangle \langle\Psi_0| \otimes_n \rho_n$$

$$|\Psi_0\rangle = \otimes_i(|0\rangle + i|1\rangle)/\sqrt{2}$$

ρ_n Arbitrary

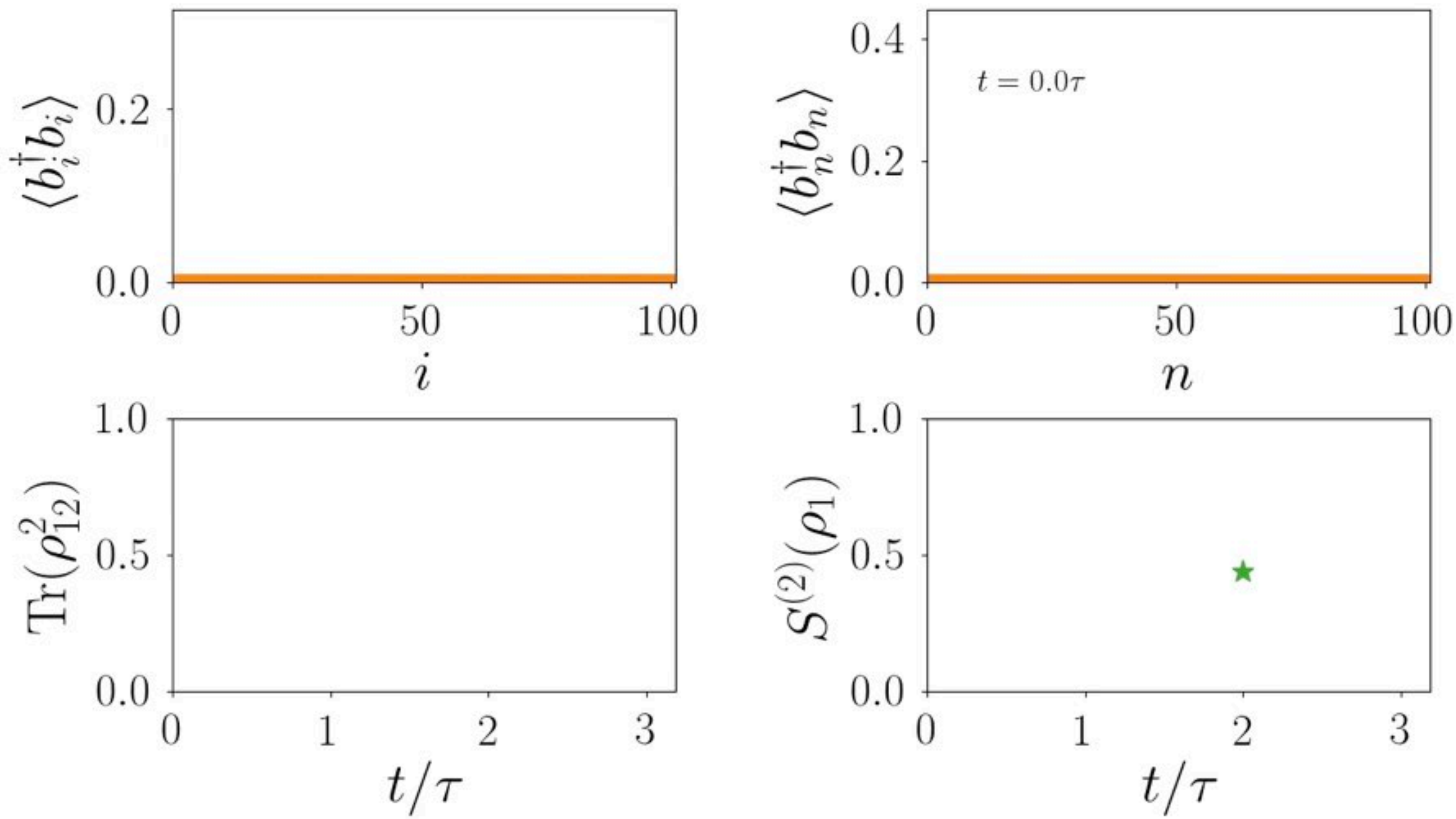
- Max. entangled target state:

$$|\Psi_{\text{target}}\rangle = \exp(-i\frac{\pi}{4}\sigma_1^z\sigma_2^z)|\Psi_0\rangle$$

Example 1

Matrix-Product State (101 Bosonic Modes)

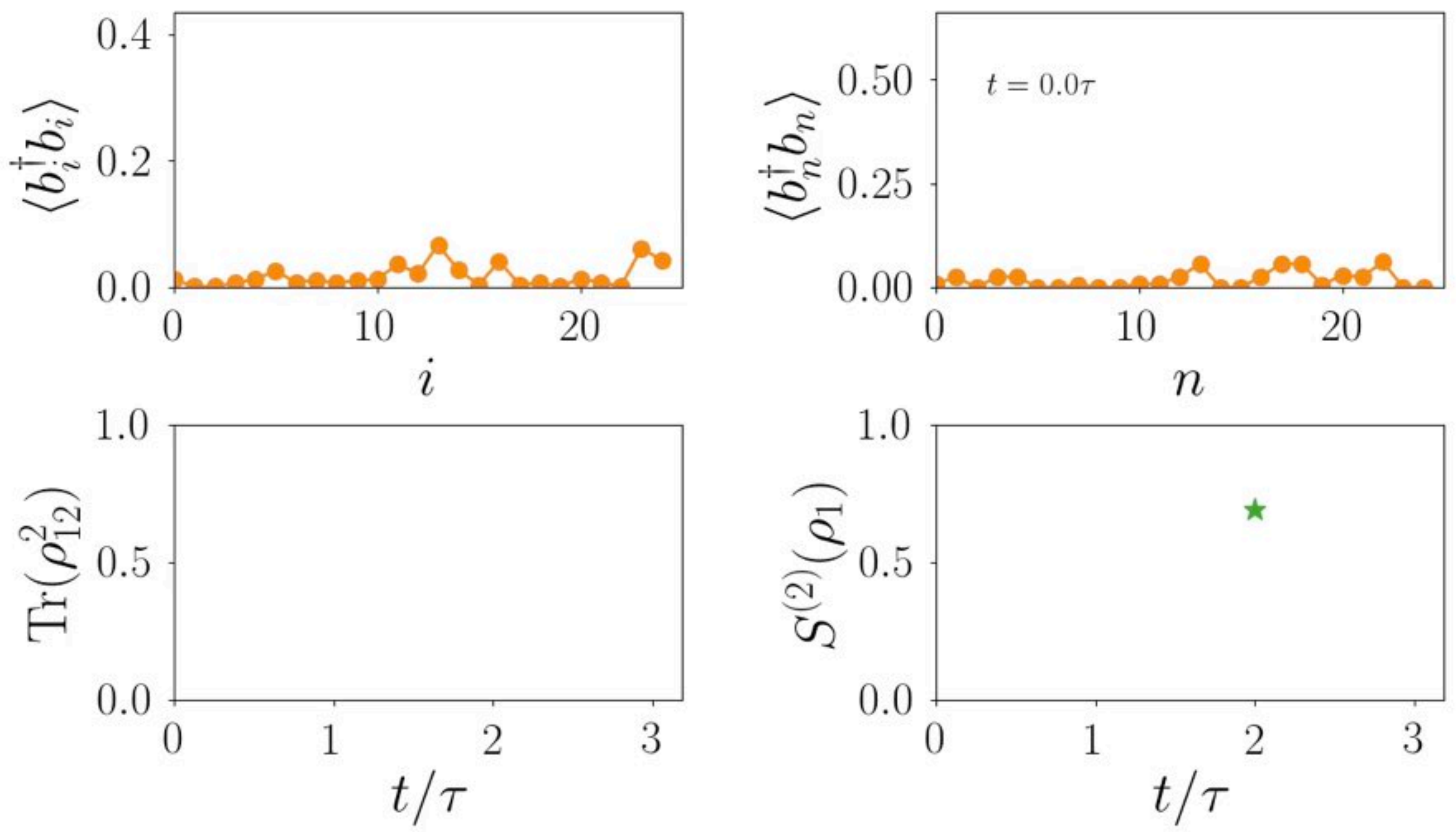
Zero-Temperature



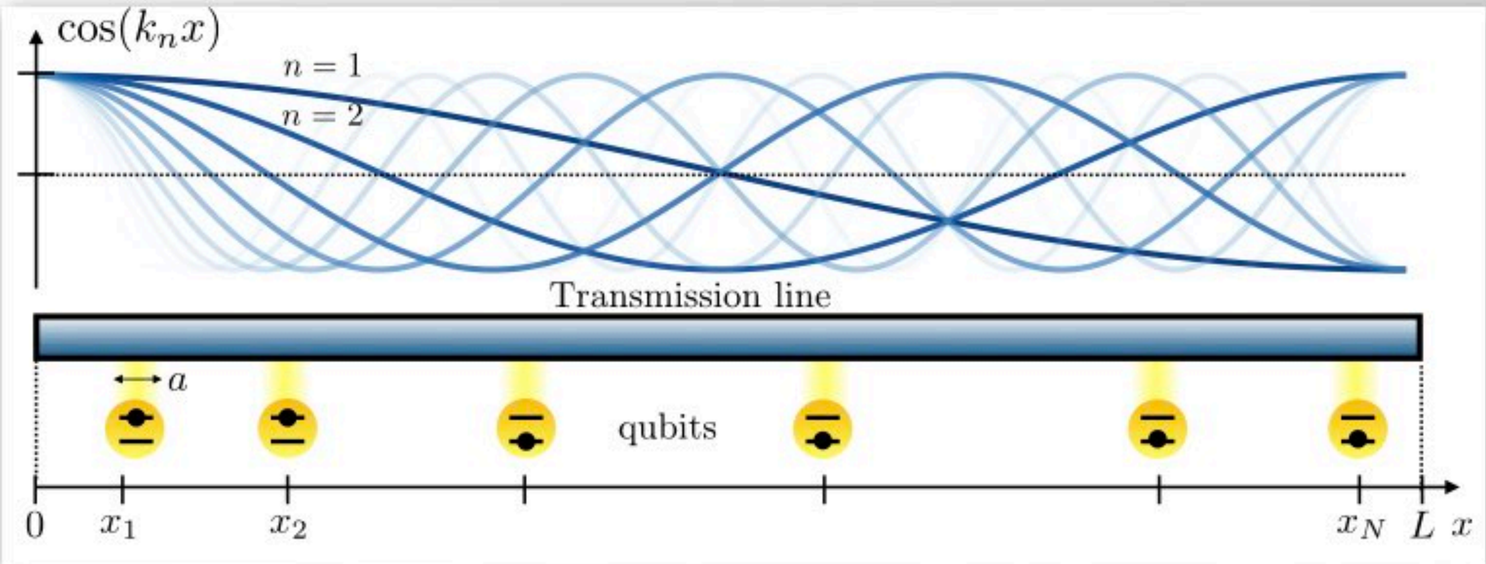
Example I

Matrix-Product State (25 Bosonic Modes)

Random noise in the transmission line



Quantum Simulation of Spin Models



$$U_{\text{lab}}(t_p) = e^{-it_p \sum_i (\omega_i/2)\sigma_i^z} e^{-it_p \sum_{i<j} J_{ij}\sigma_i^z \sigma_j^z}.$$

$$J_{ij} = -2 \sum_n \frac{g_{i,n} g_{j,n}}{\omega_n}.$$

How to program an arbitrary interaction matrix?

Engineering of Spin Models

Generate targeted and scalable time-evolution:

$$W = \exp(-i \sum_{i<j} w_{ij} \sigma_i^z \sigma_j^z)$$

$$w_{ij} = \sum_{q=1}^{N} w_q u_{i,q} u_{j,q}$$

Consider sequence of successive cycles: $q = 1, \ldots, \eta$

- Adjustable parameters: $g_i \to g_i^{(q)}$
- Evolution at end of sequence:

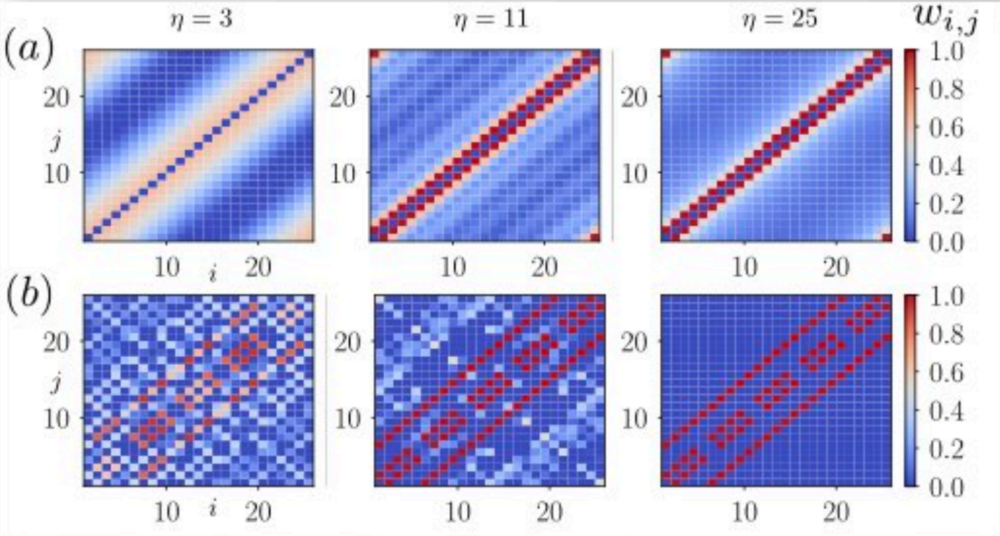
$$U_\eta = \exp(-it_p \sum_{i<j} J_{i,j}^{(\eta)} \sigma_i^z \sigma_j^z) \qquad J_{i,j}^{(\eta)} = \sum_{q=1}^{\eta} \frac{g_i^{(q)} g_j^{(q)}}{\omega_1}$$

- Identify: This is our recipe for physical system.

$$g_i^{(q)} = \sqrt{w_q \omega_1 / t_p} u_{i,q}$$

- Engineer arbitrary spin-spin interactions within linear runtime: $T = N t_p$

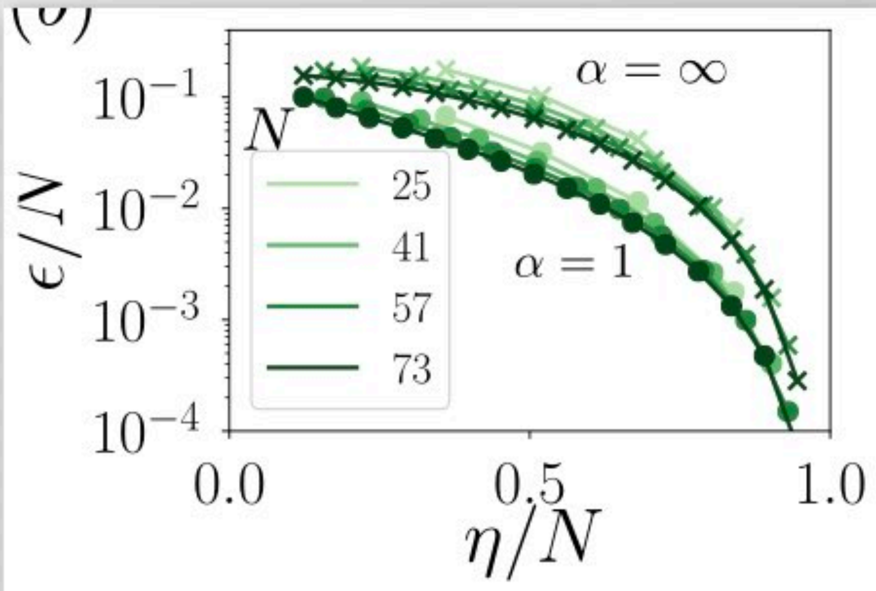
Engineering of Spin Models



$N = 25$

Figure 3: *Engineering of spin models.* (a) Long range interactions $w_{ij} = 1/|i - j|$ and periodic boundary conditions. (b) 2D nearest neighbor interactions with open boundary conditions. Here, the indices i correspond to 2D indices $\mathbf{i} = (i_x, i_y)$ of a square of 5×5 sites using the convention $i = i_x + 5i_y$.

- Can generate general Ising spin models in any spatial dimension and geometry.
- Observe progressive emergence of target matrix.



QAOA Wavefunction

$$|\boldsymbol{\gamma}, \boldsymbol{\beta}\rangle = U_x(\beta_p)U_{zz}(\gamma_p)\cdots U_x(\beta_1)U_{zz}(\gamma_1)\,|s\rangle$$

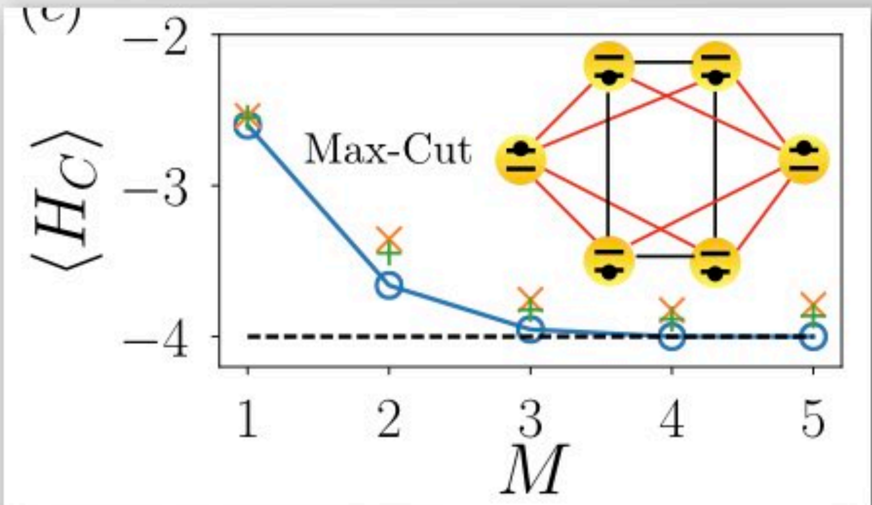
[Hot-gate protocol]

$$U_x(\beta_m) = \exp[-i\beta_m \textstyle\sum_i \sigma_i^x]$$

$$U_{zz}(\gamma_m) = \exp[-i\gamma_m H_C]$$

Quantum Algorithm

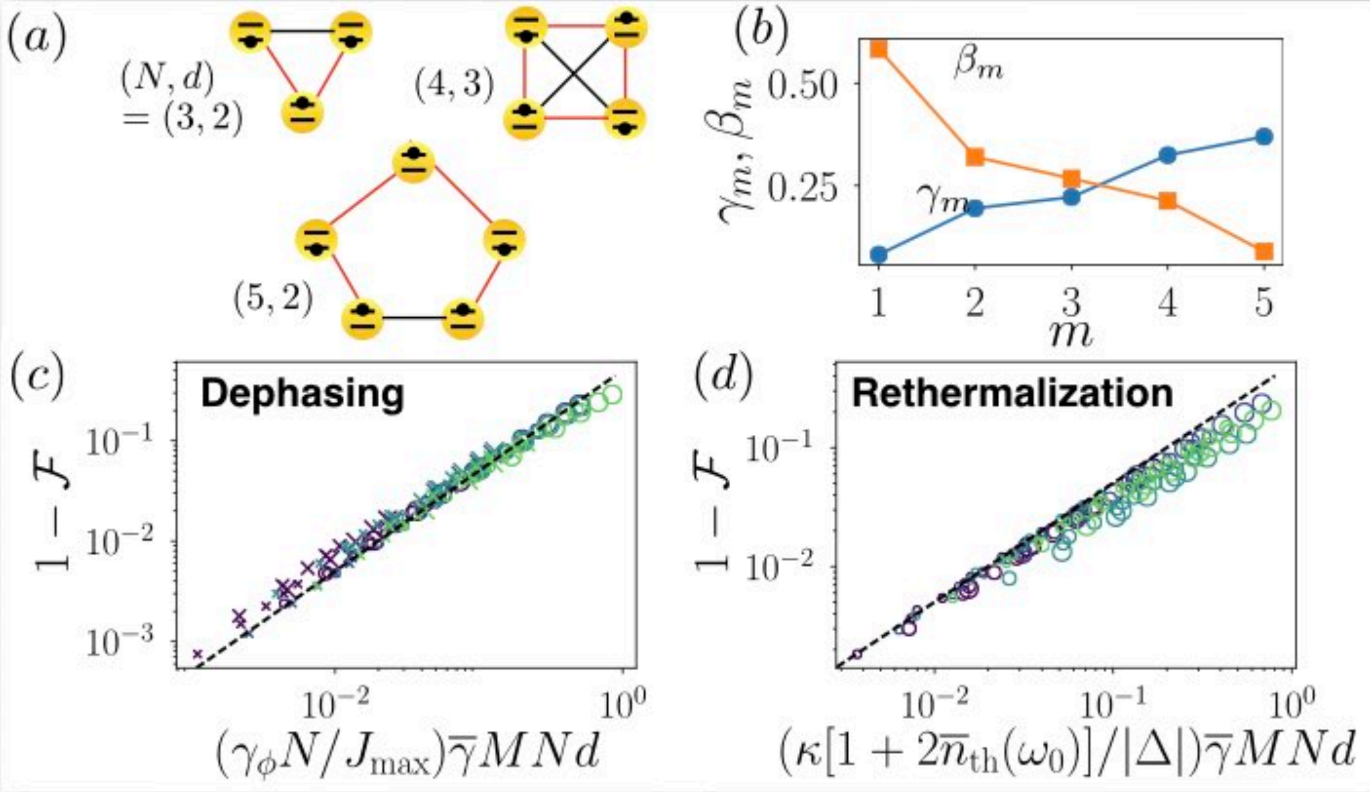
Implement QAOA wavefunction
Measure Energy (spin configs)
Get new Parameters
Repeat until energy is conserved



Numerical simulations of our setup:

- Account for finite temperature.
- Account for decoherence.

Robustness against Noise



Total QAOA error:

$$\xi \approx \overline{\gamma} d M N^{3/2}/\sqrt{C},$$

$$C = g^2/(\gamma_\phi \kappa_{\rm eff})$$

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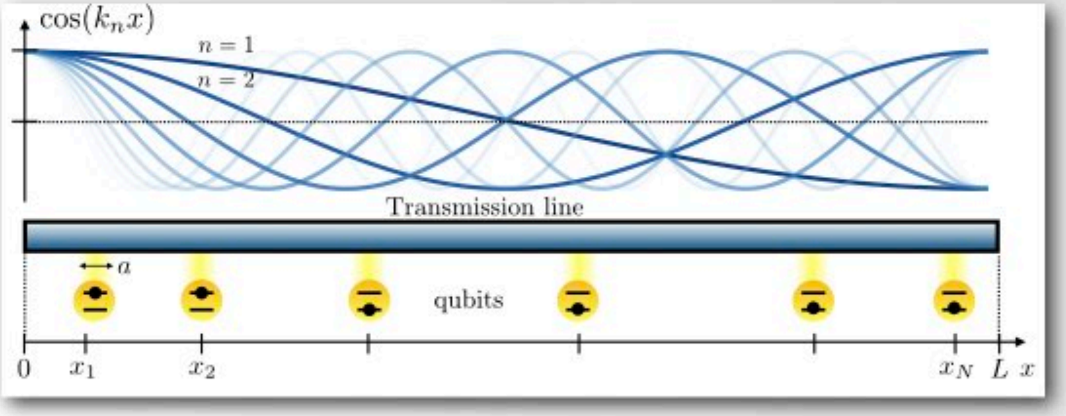
Implementation



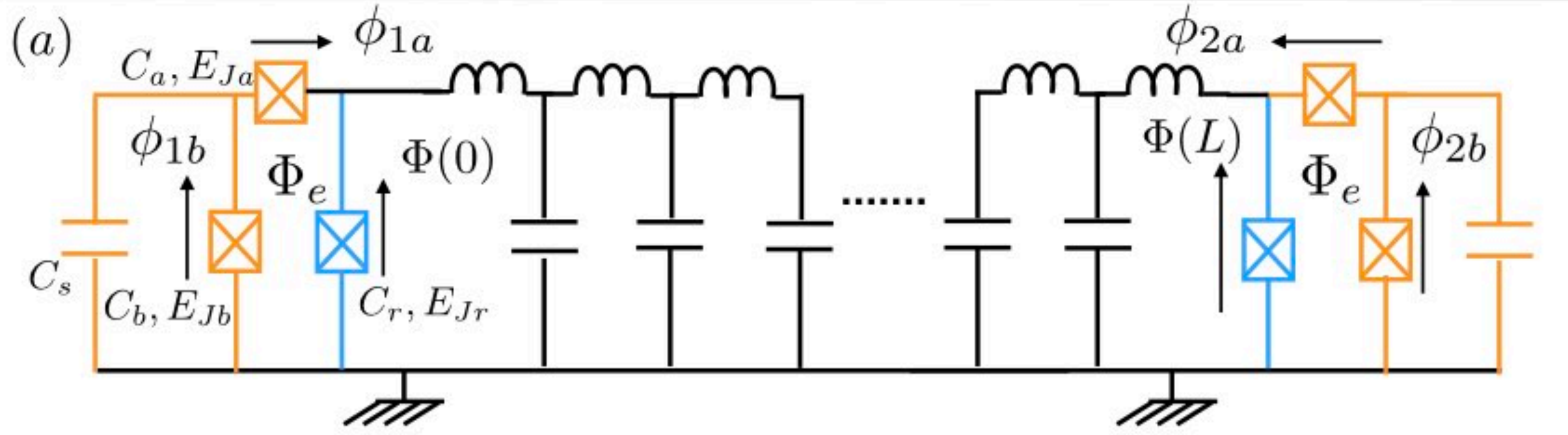
$$H = \sum_i \frac{\omega_i}{2}\sigma_i^z + \sum_{n=1}^{\infty} \omega_n a_n^\dagger a_n + \sum_{i,n} g_{i,n}\sigma_i^z \left(a_n + a_n^\dagger\right)$$

Longitudinal couplings

First ideas in SC qubits: Kerman-Nakamura-Blais (Phase Gates) - Blais (Readout)

Our implementation is just a multi-qubit, multi-mode version

Implementation with two qubits



Lagrangian

$$\mathcal{L} = \int_0^L dx \left(\frac{c\dot{\Phi}^2}{2} - \frac{(\partial_x \Phi)^2}{2\ell} \right)$$

TLine

$$+ \sum_{i=1,2} \left[E_{Jr} \cos\left(\frac{\Phi(x_i)}{\phi_0}\right) + C_r \frac{\dot{\Phi}(x_i)^2}{2} \right.$$

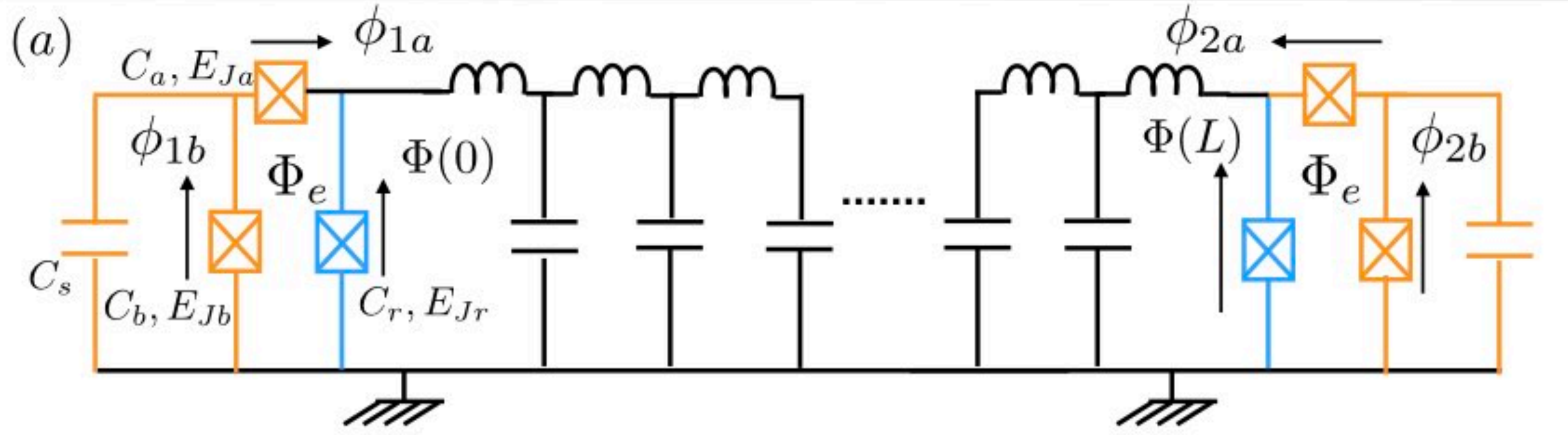
Coupling inductance

$$+ \frac{C_s + C_b}{2} \left(\frac{\dot{\Phi}(x_i)}{2} + \dot{\phi}_i \right)^2 + \frac{C_a}{2} \left(\frac{\dot{\Phi}(x_i)}{2} - \dot{\phi}_i \right)^2$$

Transmons

$$\left. + E_J \cos\left(\frac{\delta_i}{2\phi_0}\right) \cos\left(\frac{\phi_i}{\phi_0}\right) \right], \qquad \text{(S5(}$$

Implementation with two qubits

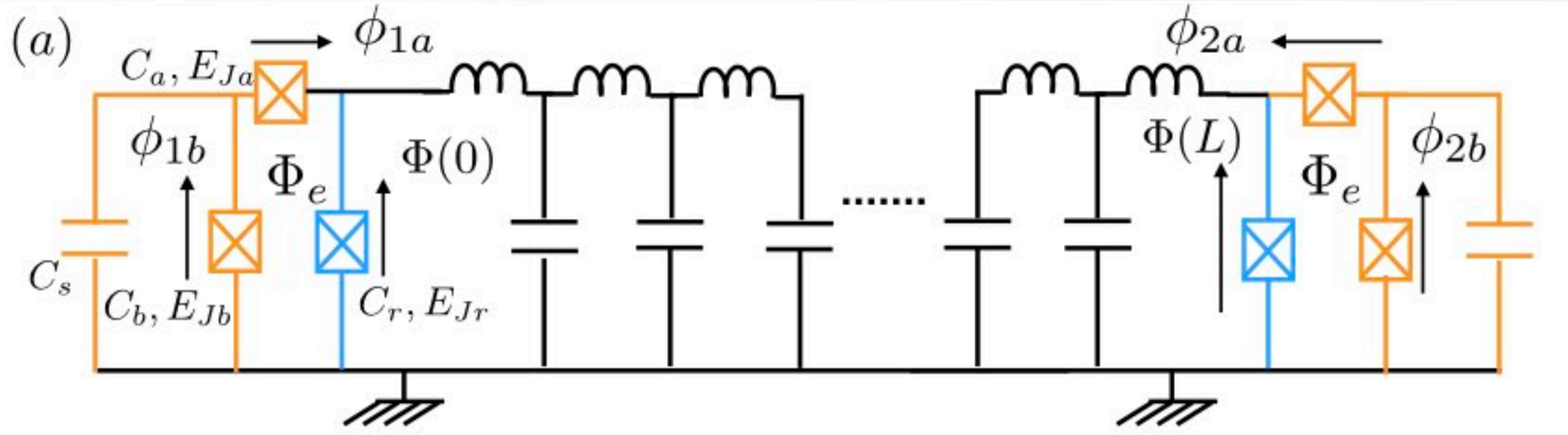


Quantization

$$\Phi_n = \sqrt{\frac{\hbar}{2c\omega_n}}(a_n + a_n^\dagger), \quad q_n = \sqrt{\frac{\hbar c\omega_n}{2}}i(a_n^\dagger - a_n)$$

$$\phi_i = \sqrt{\frac{\hbar}{2C_T\omega_z}}(a_i + a_i^\dagger), \quad q_i = \sqrt{\frac{\hbar C_T\omega_z}{2}}i(a_i^\dagger - a_i),$$

Implementation with two qubits



Hamiltonian

Driving:
can be absorbed via displacement transformations

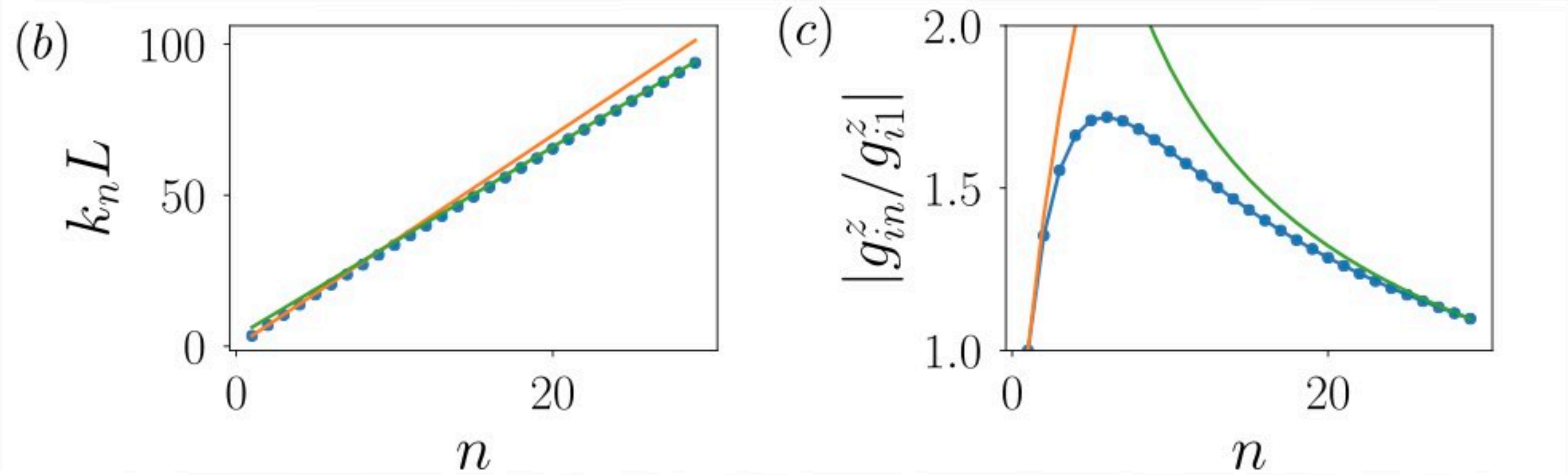
Longitudinal coupling

$$H_{\rm int} = \hbar\sum_{i,n}\Omega_{i,n}(a_n^\dagger + a_n) + \hbar\sum_{i,n} g_{in}^z \sigma_i^z (a_n^\dagger + a_n),$$

$$+ \hbar\sum_{i,n} g_{in}^y (a_i^\dagger - a_i)(a_n^\dagger - a_n), \qquad \text{(S62)}$$

Transverse coupling:
can be eliminated with condition on the capacitance

Numerical results



- Quasi-Linear Dispersion Relation
- Coupling strengths with cutoff

Typical Energy Scale (60 MHz)

Thank you!!!!

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