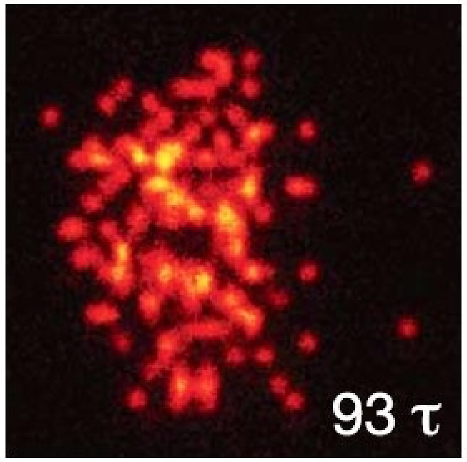
Probing mixed-state entanglement with randomized measurements

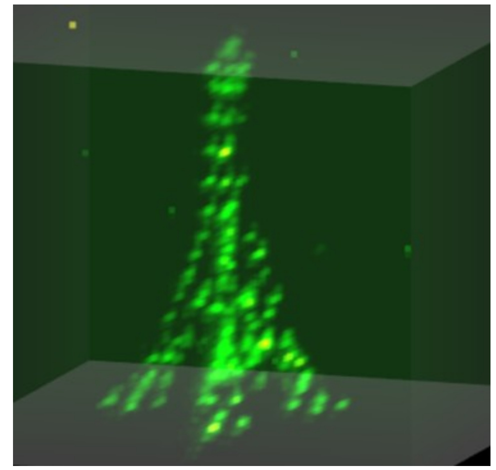
B. Vermersch (LPMMC Grenoble, & IQOQI Innsbruck)

Synthetic quantum systems

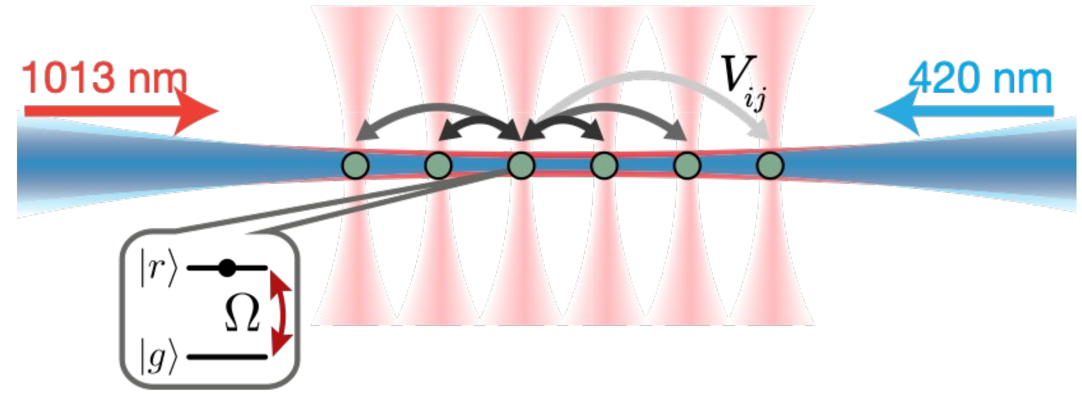
Ultracold atoms — Rydberg atoms



Choi et al., Science (2016)

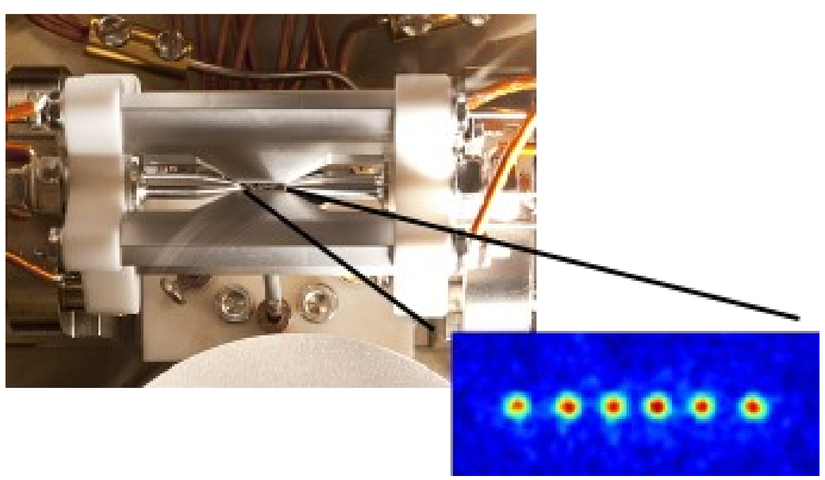


Barredo et al., Science (2016)



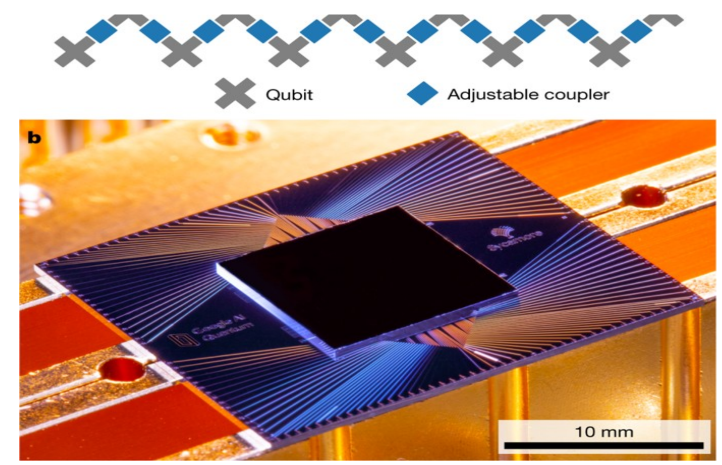
Bernien et al Nature 551, 579 (2017).

Trapped Ions



R. Blatt, Innsbruck

Superconducting circuits



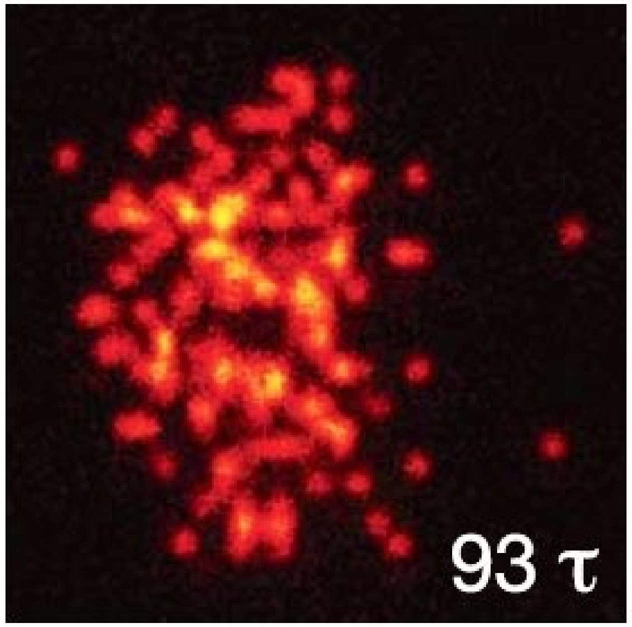
Google AI, Nature (2019)

and quantum dots, NV centers, cavity QED,..

Unique ways to create, **probe**, many-body quantum states

Applications: quantum technologies

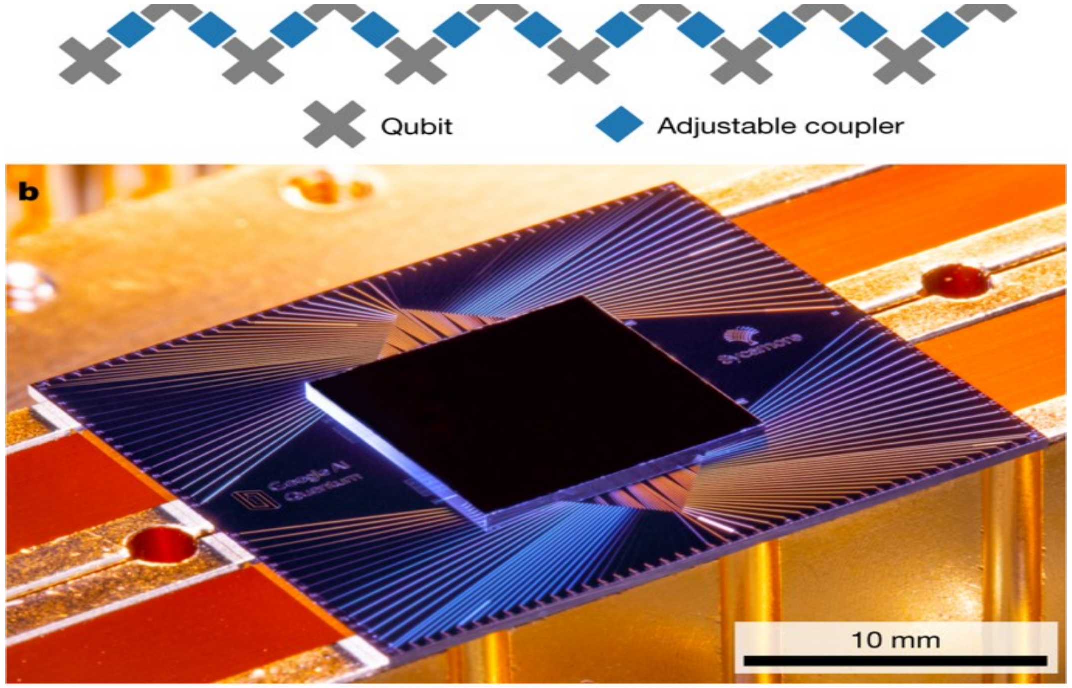
Quantum simulators



Fermi-Hubbard simulation (MPQ)

Understand quantum matter
(superconductivity, topology,
HEP,..)

Quantum computers



Google Sycamore chip

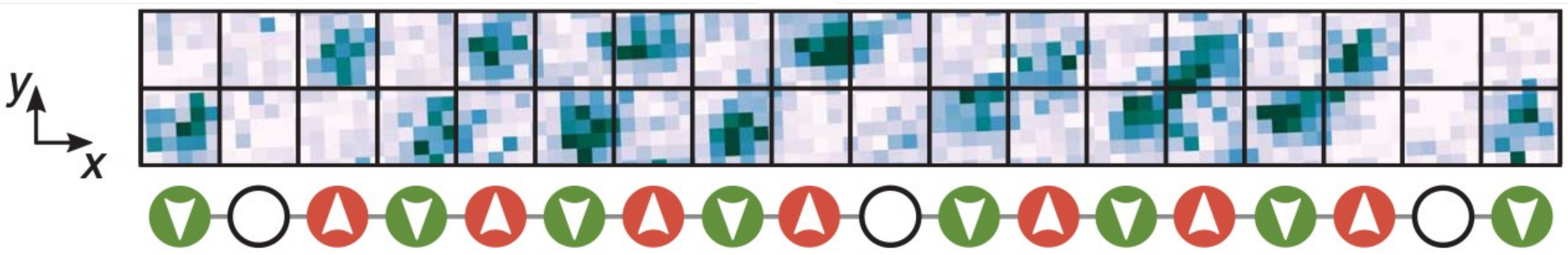
Quantum algorithms
Optimization problems (Annealing)

Key challenge: probe quantum properties of these many-body systems

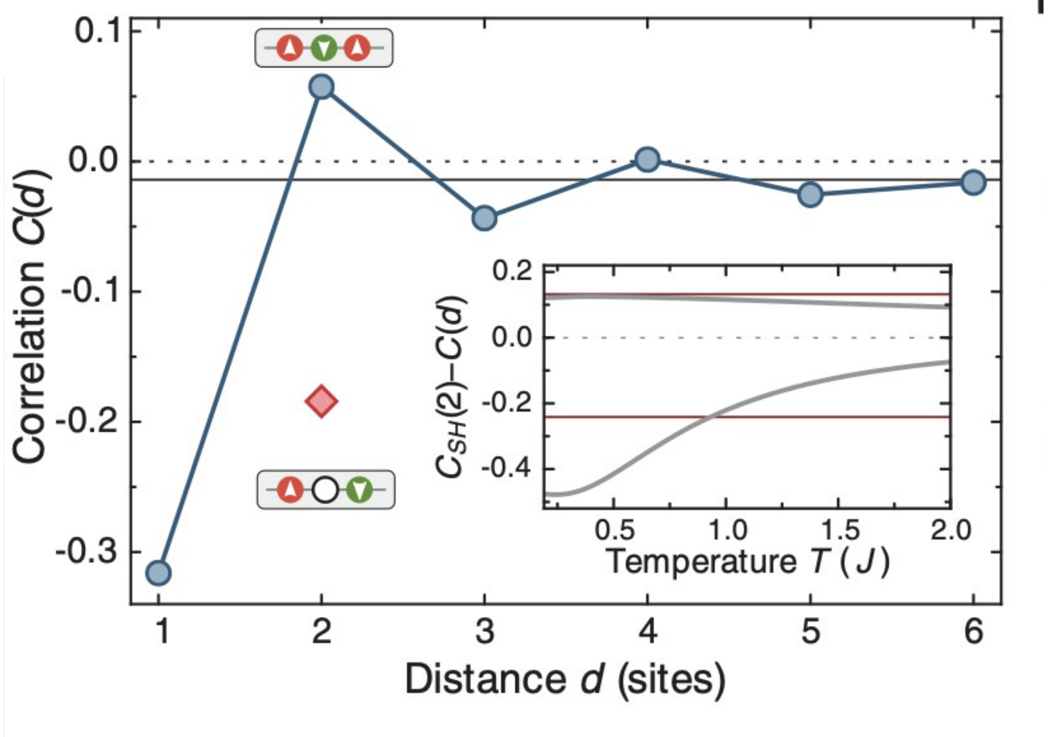
Characterizing quantum matter via correlation functions

Fermi-Hubbard system - Quantum Gas microscope

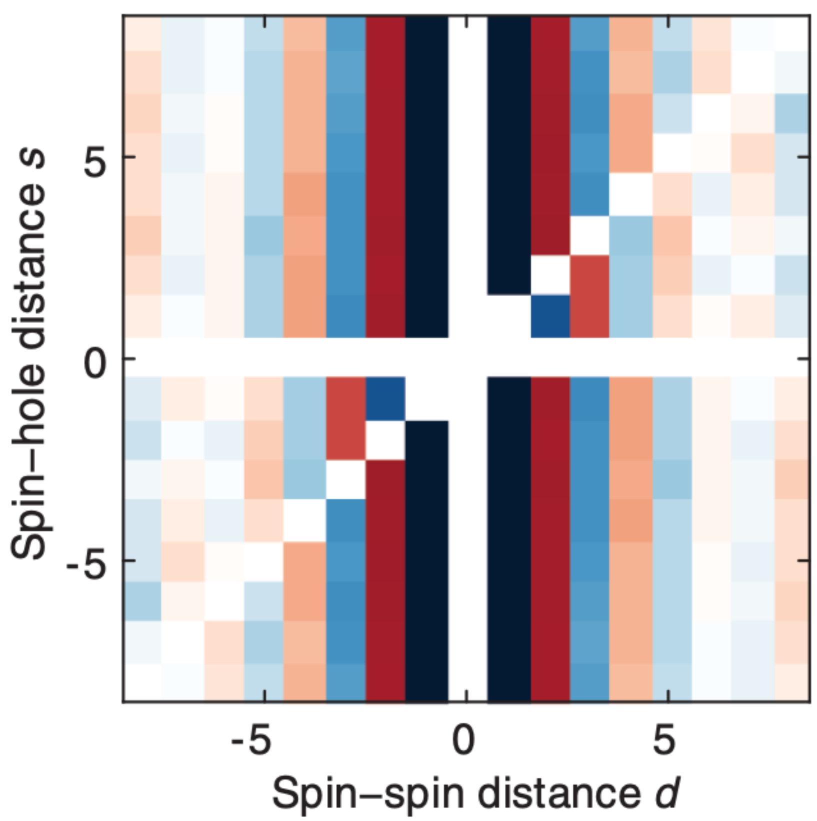
Hilker et al, Science 2017



2-point Correlation functions



Higher order (ex: String orders)

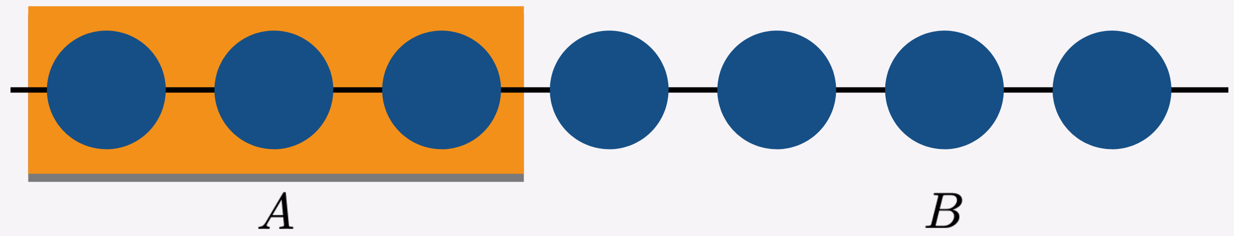


Correlations functions can be measured "directly"

$$C = \mathrm{Tr}(\rho \hat{C})$$

Most common probing tool in AMO quantum simulation experiments.

Beyond low-order correlation functions: entanglement



Two subsystems A and B are **bipartite entangled** iff

$$|\Psi\rangle \neq |\Psi_A\rangle \otimes |\Psi_B\rangle \qquad \rho \neq \sum_j p_j \rho_j^{(A)} \otimes \rho_j^{(B)}$$

Reduced density matrix $\rho_A = \mathrm{Tr}_B(\rho)$

Entanglement condition (Horodecki 1996) $\mathrm{Tr}\left[\rho_A^2\right], \mathrm{Tr}\left[\rho_B^2\right] < \mathrm{Tr}\left[\rho^2\right]$

Quantifying entanglement for pure states → Entanglement entropies

$$S_A = -\mathrm{Tr}_A\left[\rho_A \log \rho_A\right]$$ von-Neumann

$$S_A^{(n)} = \frac{1}{1-n} \log \mathrm{Tr}_A\left[\rho_A^n\right] \quad \leq S_A$$ **Nth Rényi**

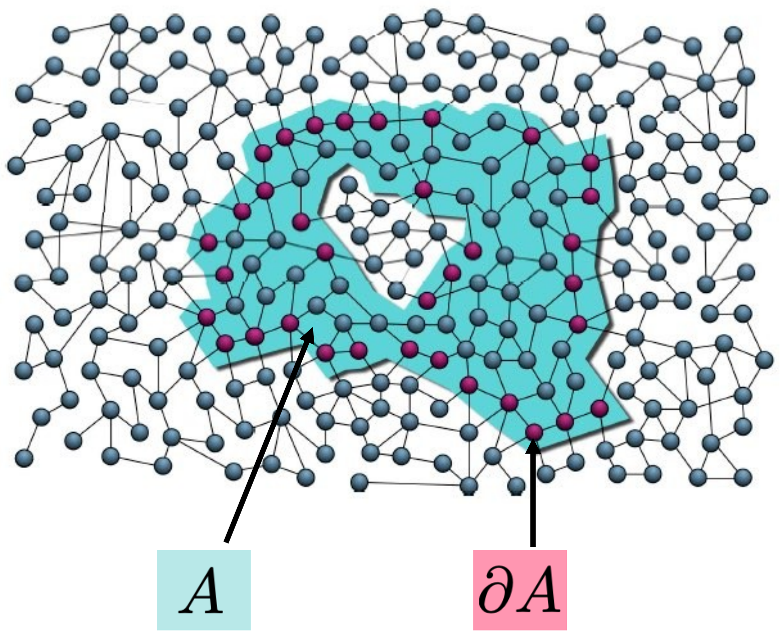
purity

$$S_A^{(2)} = -\log(\mathrm{Tr}_A(\rho_A^2))$$ **2nd Rényi**

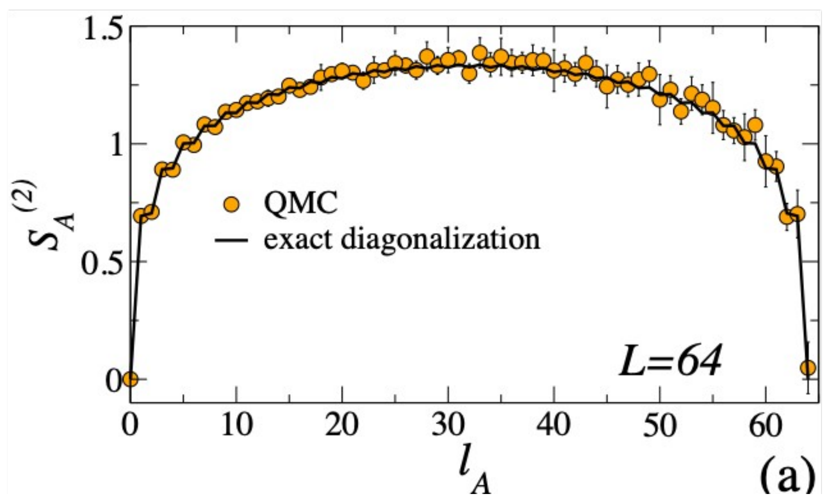
Measuring entanglement entropies: so what?

Measuring Entanglement entropies is fundamental for **Quantum Simulation**

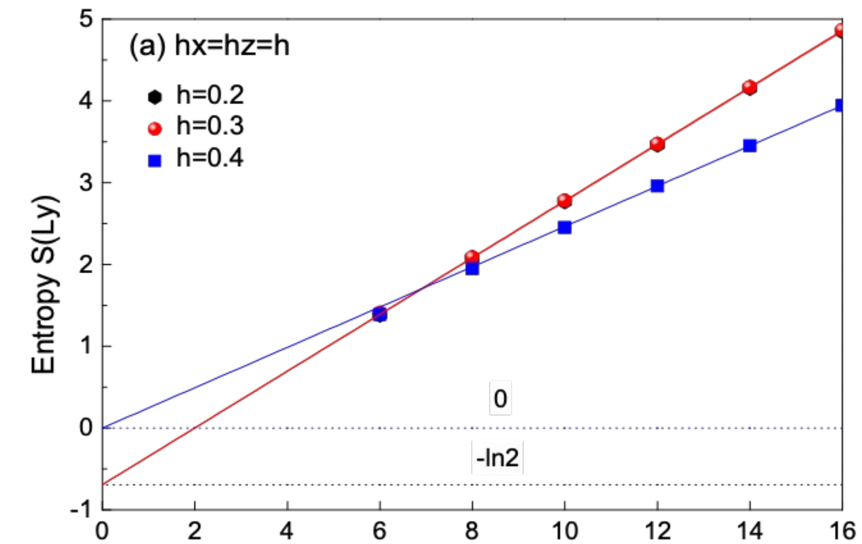
Many-body ground states **Quantum Phase transitions** **Topological order**



Amico et al., Rev.Mod.Phys, 80, 517 (2008)
Eisert et al., Rev. Mod. Phys. 82, 277 (2010)



P. Calabrese and J. Cardy, J. Stat. Mech (2004).
Humeniuk, Roscilde PRB (2012)



Kitaev, Preskill, PRL 2006
Levin, Wen, PRL 2006
Jian et al, NP 2012

Area law: $S_A^{(2)} \propto L_A^{D-1}$

$S_A^{(2)} \approx (c/4)\log(L_A)$

central charge

$S_A^{(n)} \approx \alpha_n L_A - \gamma$

Topological entanglement Entropy

Quantum Thermalization

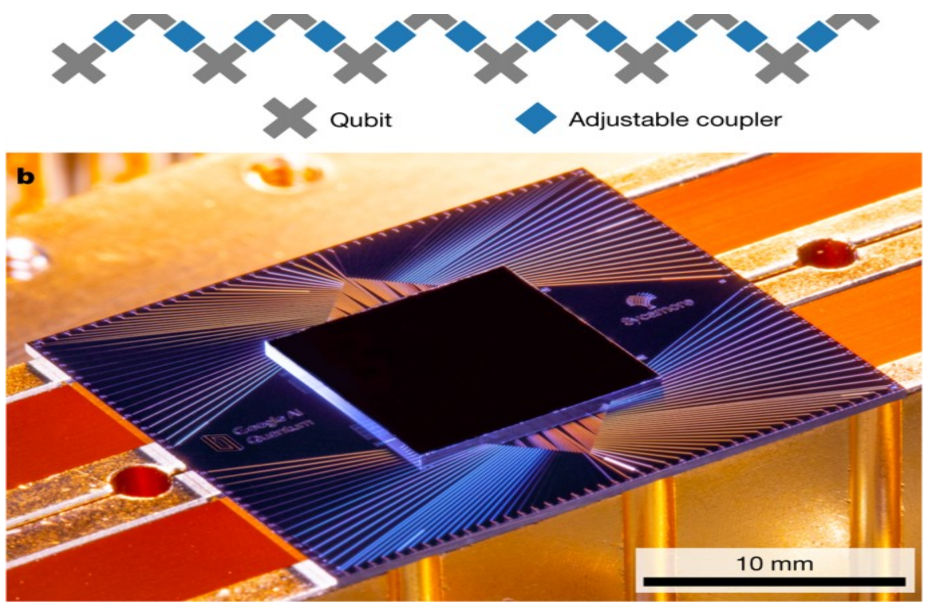
P. Calabrese and J. Cardy, PRL 2006
Badarson et al, PRL 2012

Measuring entanglement entropies: so what?

Measuring the entanglement “power” of quantum computers

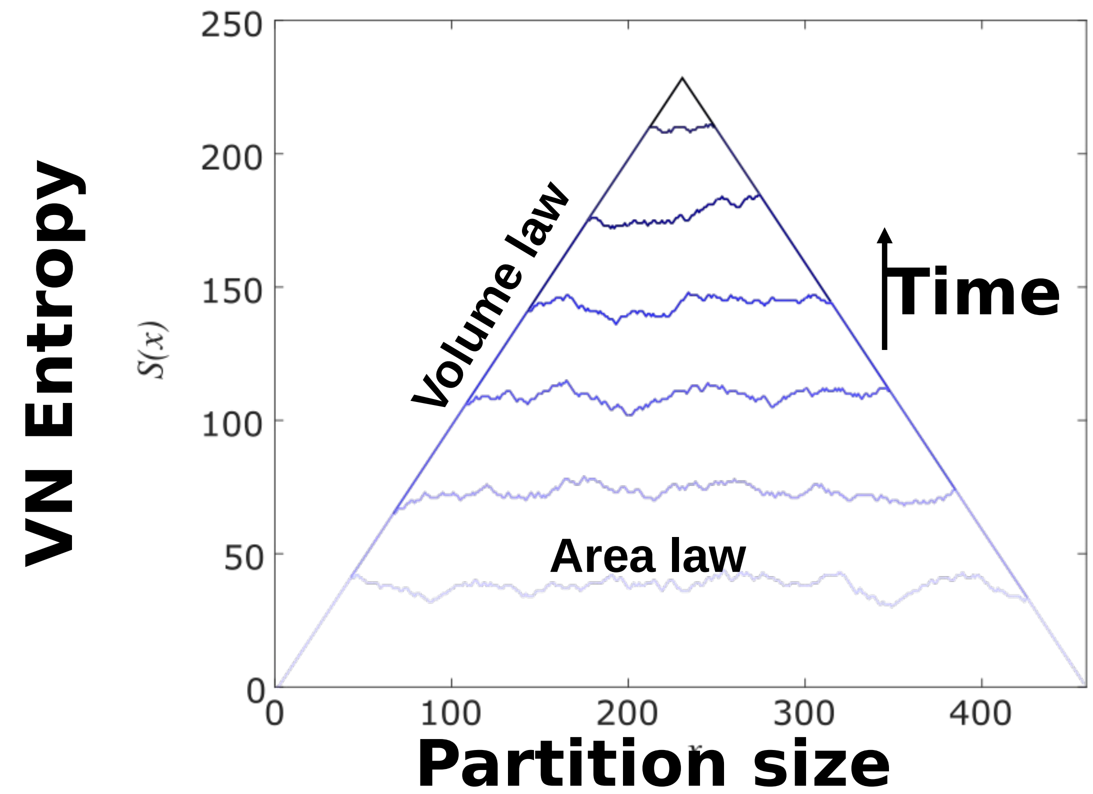
“Checks”

Purity checks
Entanglement checks



Google Sycamore chip

Universal behaviors

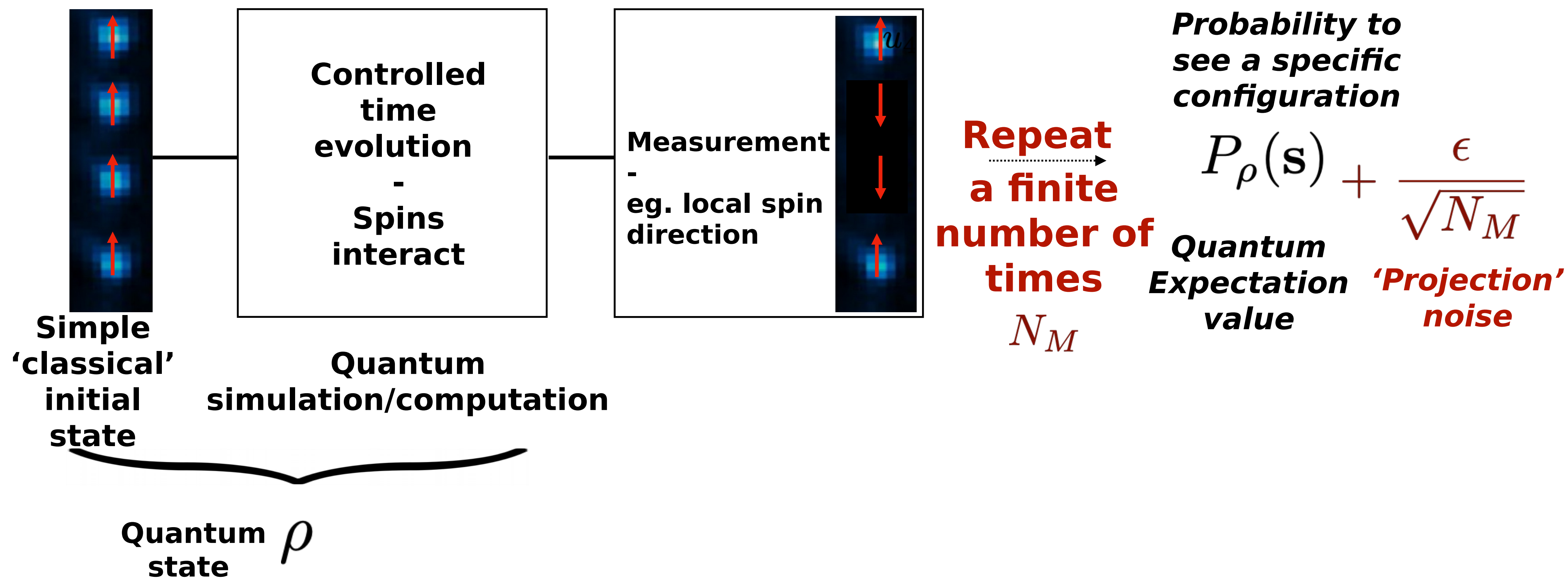


Nahum et al, Phys. Rev. X 7, 031016 (2017)

How to measure entanglement in such many-body quantum systems?

A new tool: randomized measurements

A standard measurement protocol

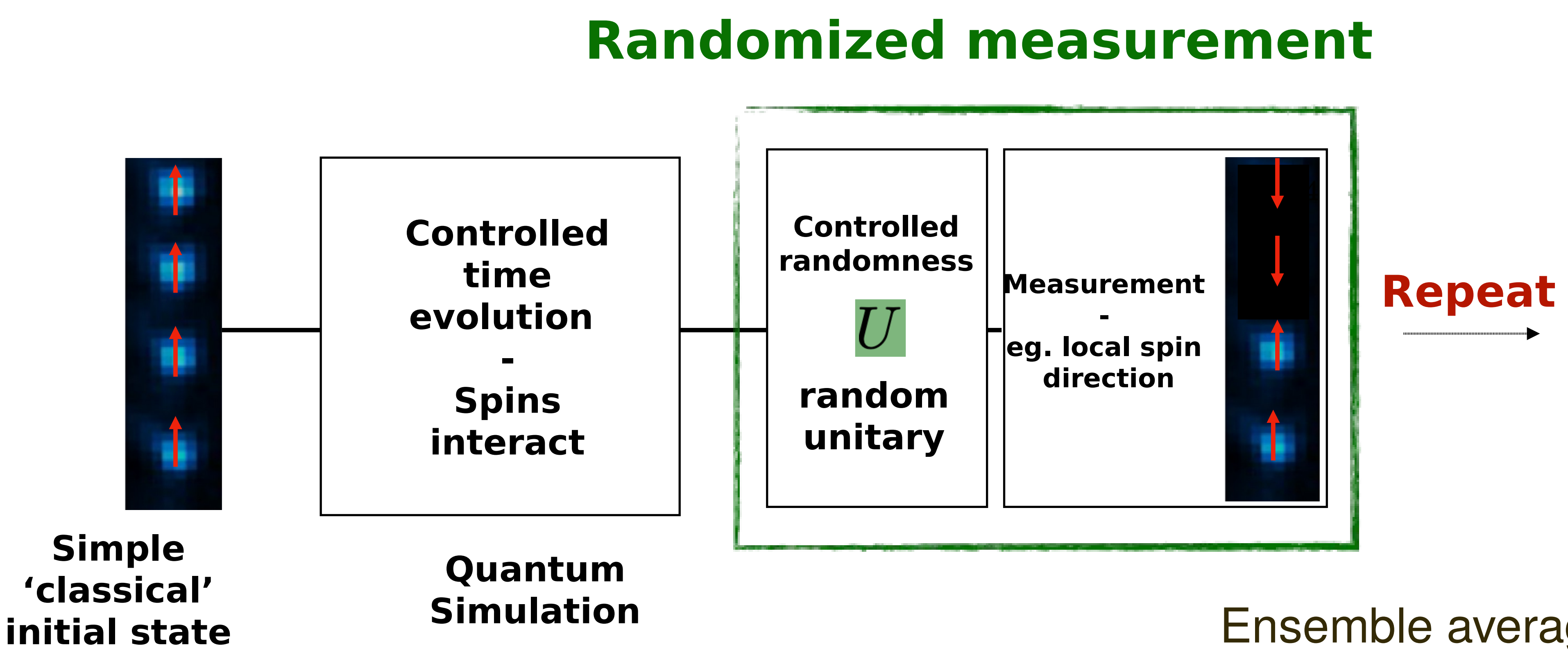


Limited to `observables', correlation functions, etc

Not applicable to Entanglement-related quantities, nonlinear functions w.r.t the density matrix

Ex: purity $\mathrm{tr}(\rho^2)$

A new tool: randomized measurement protocols



Measurement → random number

$$P_{\rho,U}(\mathbf{s})$$

Ensemble average over random unitaries

Correlations of probabilities $\overline{P_{\rho,U}(\mathbf{s}_1)P_{\rho,U}(\mathbf{s}_2)}$

Purity-Entanglement entropies

Van Enk, Beenakker 2012
A.Elben, BV, et al.
PRL 2018, PRA 2018, PRA 2019
Brydges,..,BV,.., Science 2019
Huang, Kueng, Preskill, Nature Physics 2020
Elben et al, aarXiv:2101.07814

Cross-platform verification

Elben, BV, et al.,
PRL 2020

Scrambling of quantum information

BV et al., PRX 2019
Joshi, BV, et al PRL 2020

Mixed-state entanglement (PPT condition)

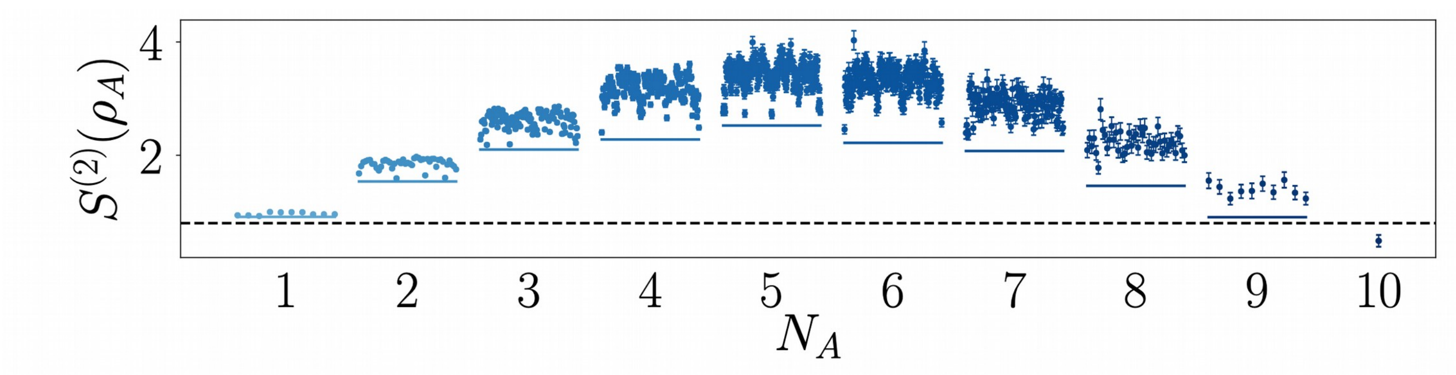
Zhou et al, PRL 2020
Elben, .., BV PRL 2020

See also works by Knips
Ketterer

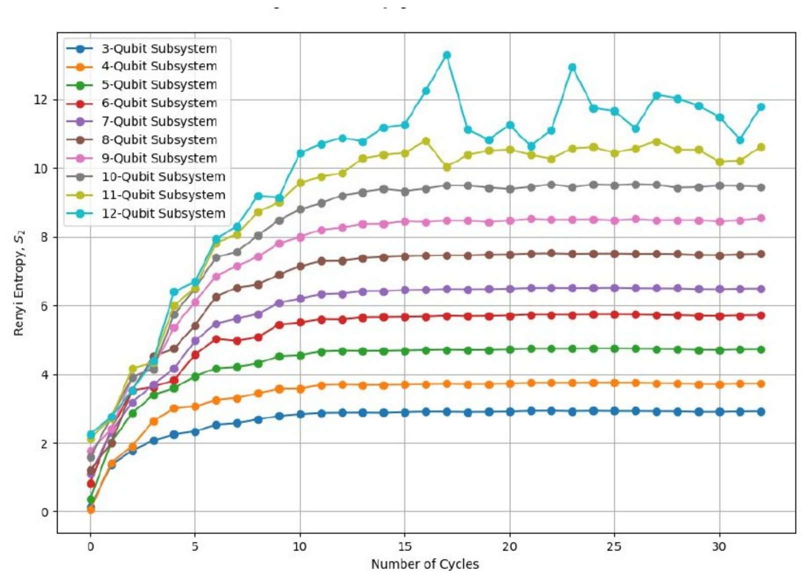
Many-body Topological invariants

Elben,..,BV
Sci. Adv. 6 (2020)
ZP Cian, BV, et al
Arxiv:2005.13543

Part 1: Tutorial on randomized measurements: Measuring the purity and 2nd Renyi entropy



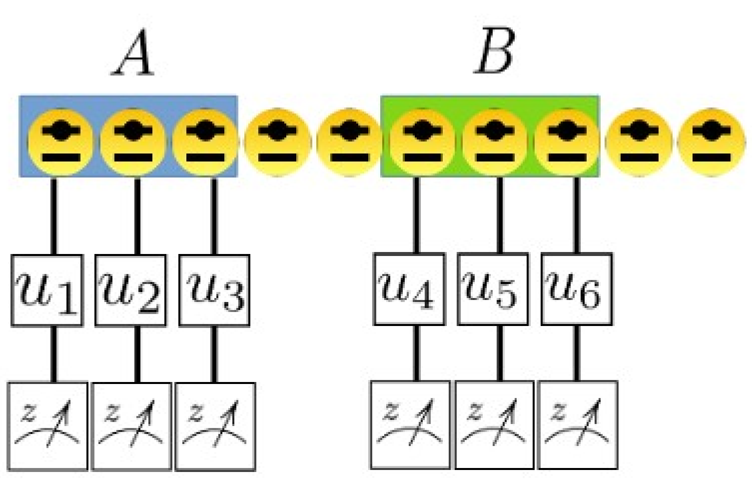
Brydges et al, Science 2019



X. Mi et al, in preparation

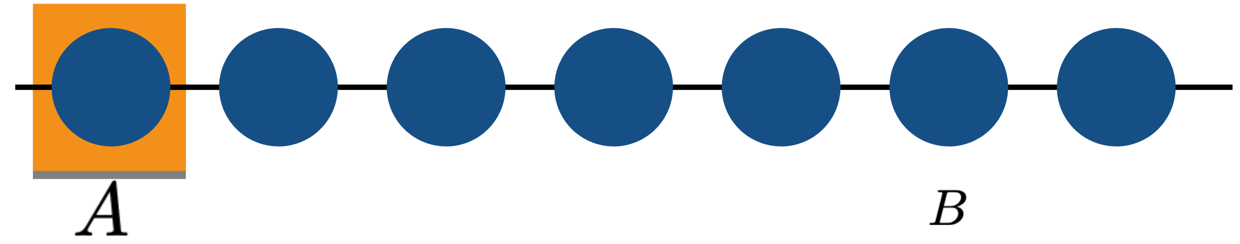
Part 2: Probing mixed-state entanglement via randomized measurements

Elben et al, PRL 2020



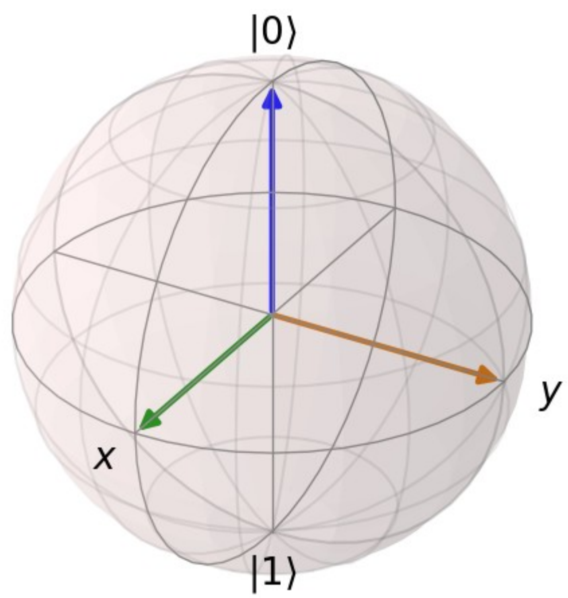
Single Hilbert-space approach [van Enk, PRL 2012]

Ex: Measuring a Single qubit purity



Projective measurement:

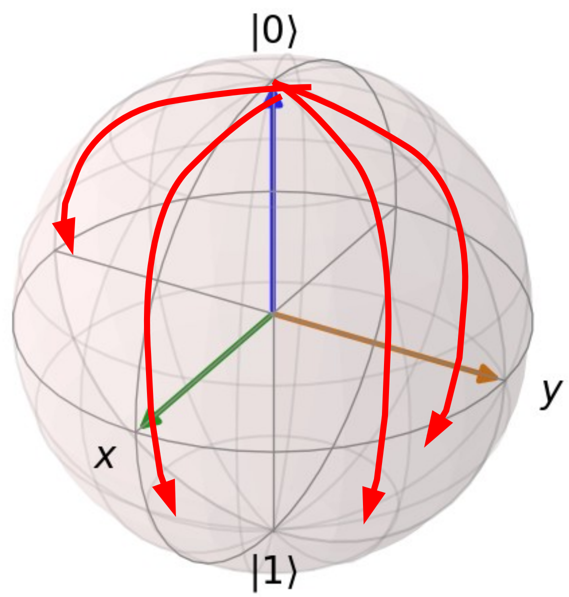
One measurement setting, for example z basis



$$P(0) = \langle 0|\,\rho|0\rangle = 1$$

Randomized measurements:

Random unitary before measurements

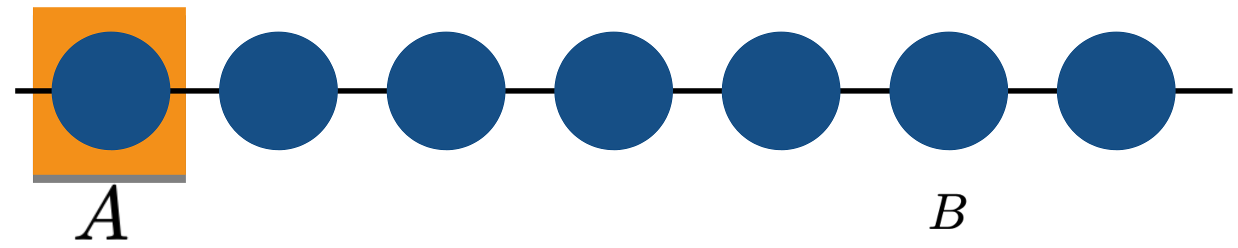


$$P_u(0) = \langle 0|\, u\rho u^\dagger |0\rangle \in [0,1]$$

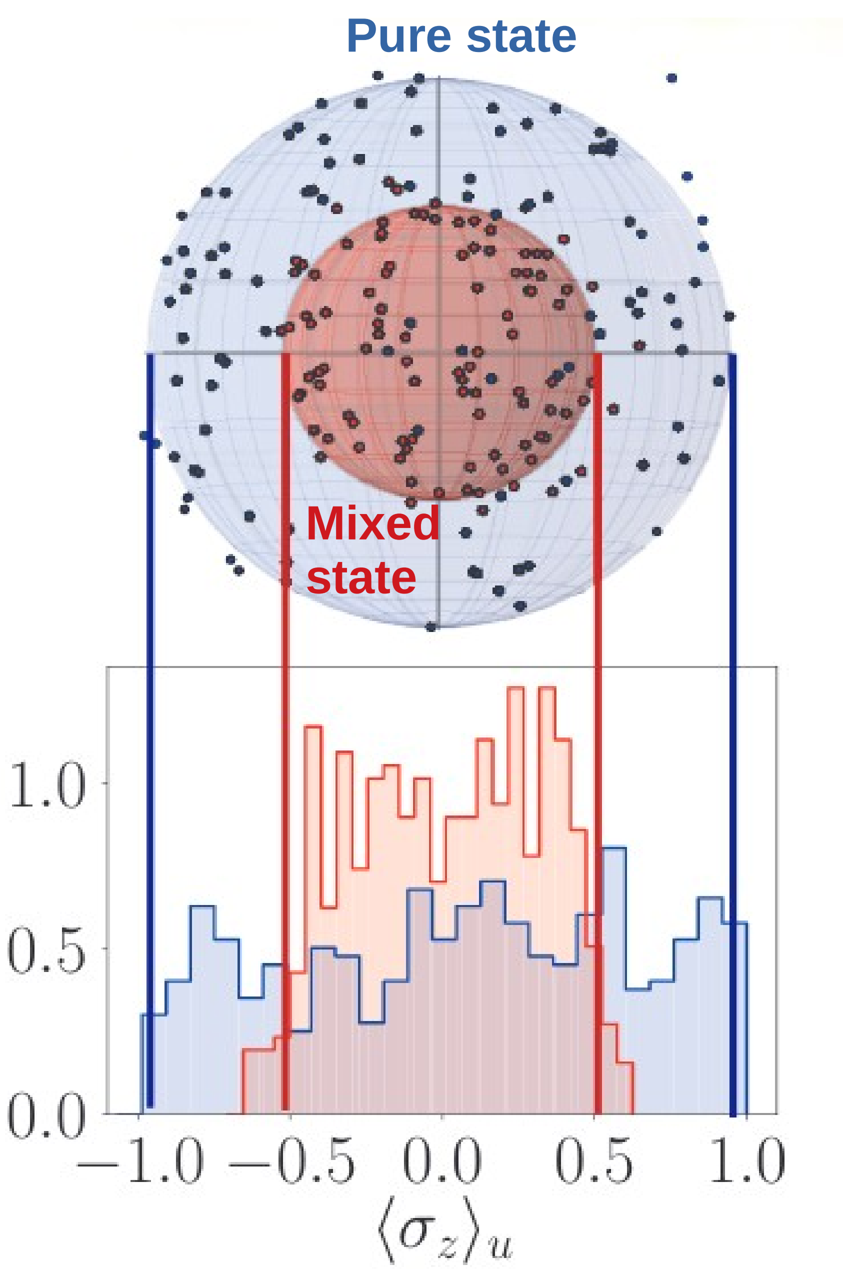
$u \in \mathrm{CUE}$ (Circular unitary ensemble)

Single Hilbert-space approach [van Enk, PRL 2012]

Ex: Measuring a Single qubit purity



Statistics of randomized measurements → purity



$$P_u(s) = \langle s | \, u \rho u^\dagger \, | s \rangle$$

Ensemble variance CUE

$$\mathrm{tr}(\rho^2) = (d+1) \sum_s \mathrm{var}(P_u(s)) + \frac{1}{d}$$

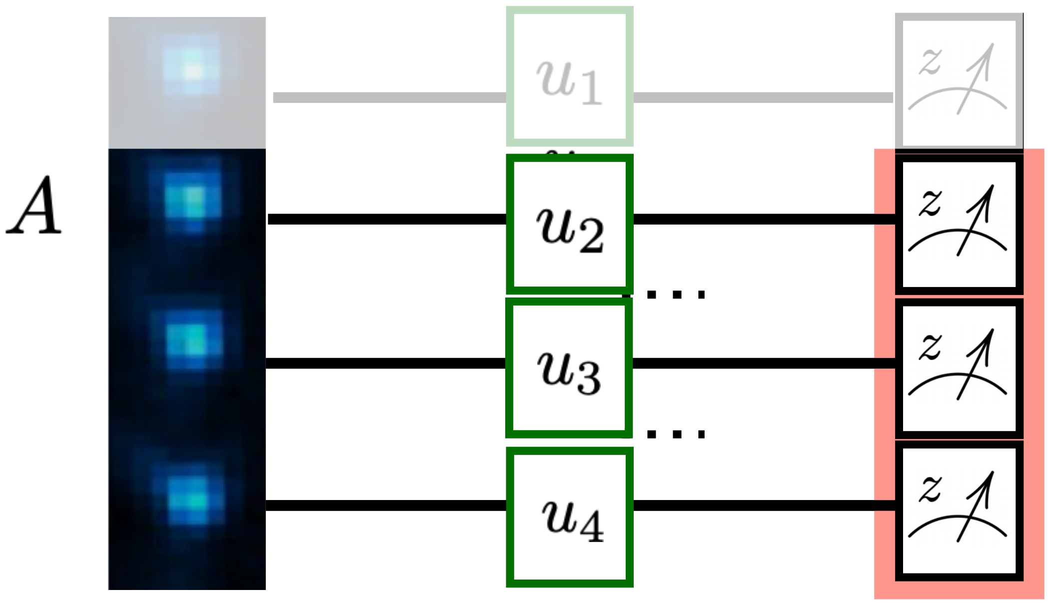
Message: The purity can be understood as statistical fluctuations over randomized measurements

Limitation: Requires ``global random unitaries'' for a many-body system

Randomized Measurement Protocols as "experimental recipe"

Protocol for spin systems with local random unitaries

Elben, BV et al. (PRL 2018, PRA 2019)



$u_i \in \mathrm{CUE}(d)$

Local random unitaries

$U_A = \bigotimes u_i$

Average over random unitaries

Hamming distance

$\mathrm{Tr}\left[\rho_A^2\right] = \overline{X_U}$ with $X_U = 2^{N_A} \sum_{s_A, s'_A} (-2)^{-D[s_A, s'_A]} P_U(s_A) P_U(s'_A)$

Proof: average over local unitary 2-designs

Cross correlation

Number of measurements to overcome stat. errors : ~ $\mathbf{2^{N[A]}}$

Randomized Measurement Protocols as "experimental recipe"

$$\mathrm{Tr}\left[\rho_A^2\right] = \overline{X_U} \qquad \text{with} \qquad X_U = 2^{N_A} \sum_{s_A,s'_A} (-2)^{-D[s_A,s'_A]} P_U(s_A) P_U(s'_A)$$

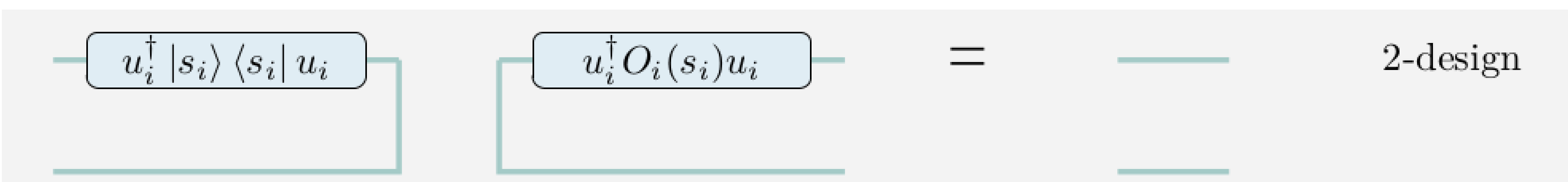
Proof: 2 design properties of the CUE

$$\overline{(u_i)_{s_i,s_i^{(1)}} (u_i)^*_{s_i,s_i^{(2)}}} = \frac{\delta_{s_i^{(1)},s_i^{(2)}}}{d}$$

$$\overline{(u_i)_{s_i,s_i^{(1)}} (u_i)^*_{s_i,s_i^{(2)}} (u_i)_{s_i,s_i^{(3)}} (u_i^*)_{s_i,s_i^{(4)}}} = \frac{\delta_{s_i^{(1)},s_i^{(2)}} \delta_{s_i^{(3)},s_i^{(4)}} + \delta_{s_i^{(1)},s_i^{(4)}} \delta_{s_i^{(3)},s_i^{(2)}}}{d(d+1)}$$

$$\overline{\langle s_i^{(2)} | u_i^\dagger | s_i \rangle \langle s_i | u_i | s_i^{(1)} \rangle \langle s_i^{(4)} | u_i^\dagger O_i(s_i) u_i | s_i^{(3)} \rangle} = \delta_{s_i^{(1)},s_i^{(4)}} \delta_{s_i^{(2)},s_i^{(3)}}$$

$$O_i(s_i) = d(d+1) |s_i\rangle \langle s_i| - dI_i$$

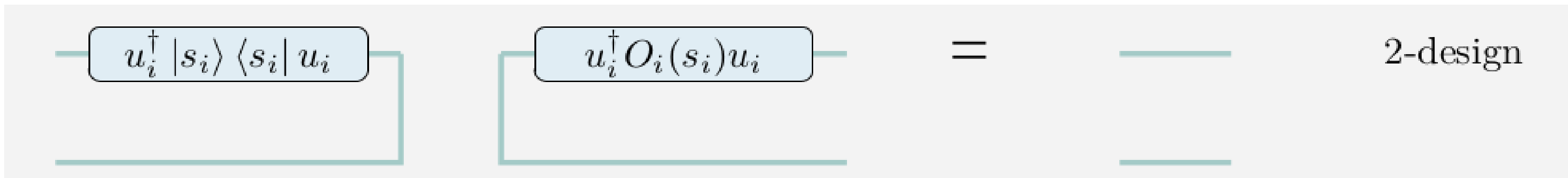


Take-Home-Message: Average terms involving randomized unitaries lead to very simple relation between matrix indices

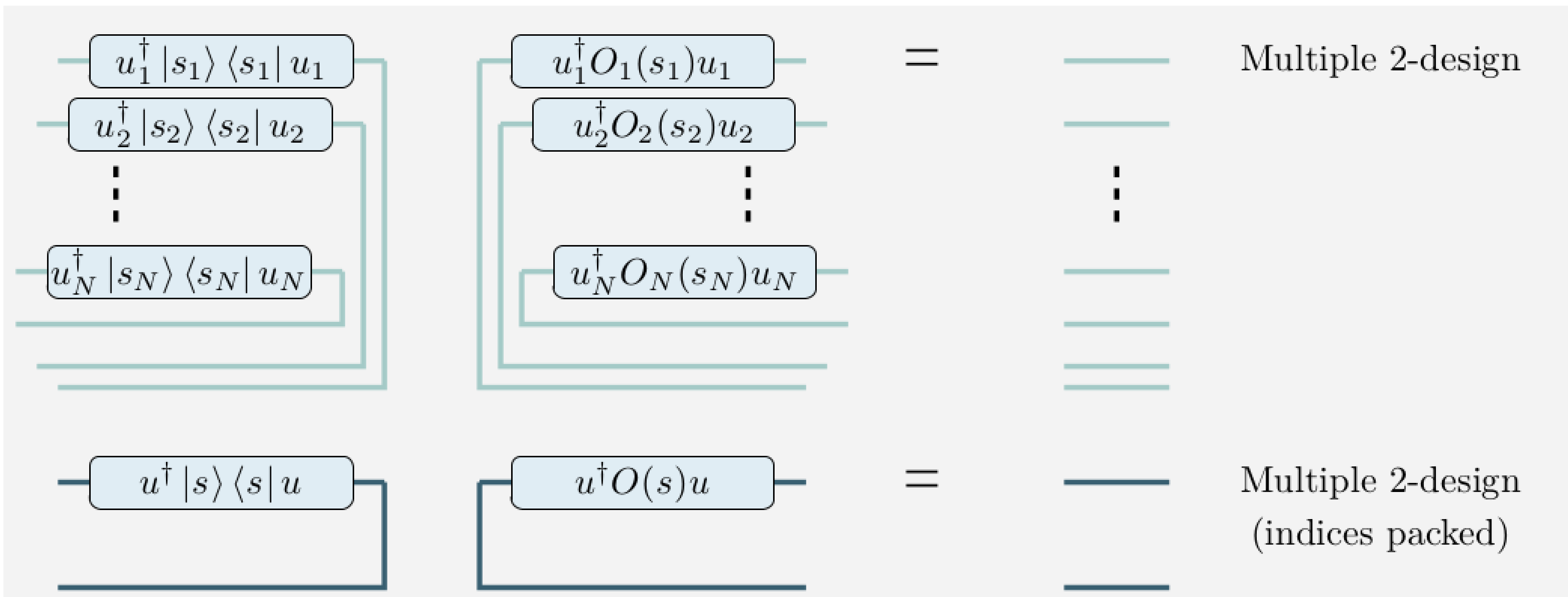
Randomized Measurement Protocols as "experimental recipe"

$$\mathrm{Tr}\left[\rho_A^2\right] = \overline{X_U} \quad \text{with} \quad X_U = 2^{N_A} \sum_{s_A, s'_A} (-2)^{-D[s_A, s'_A]} P_U(s_A) P_U(s'_A)$$

1 unitary

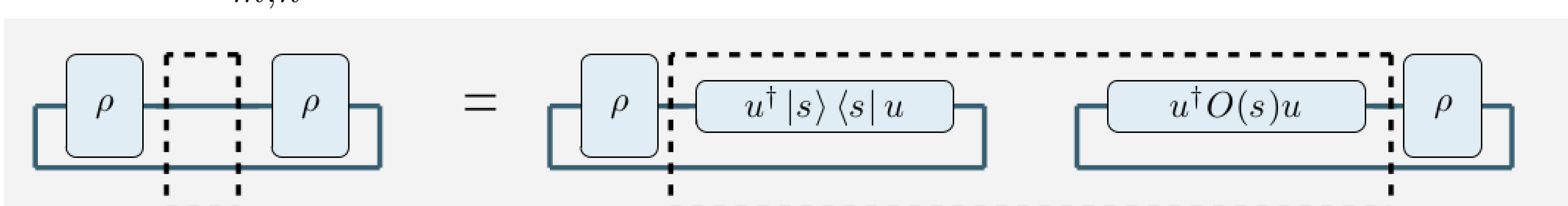


N unitaries



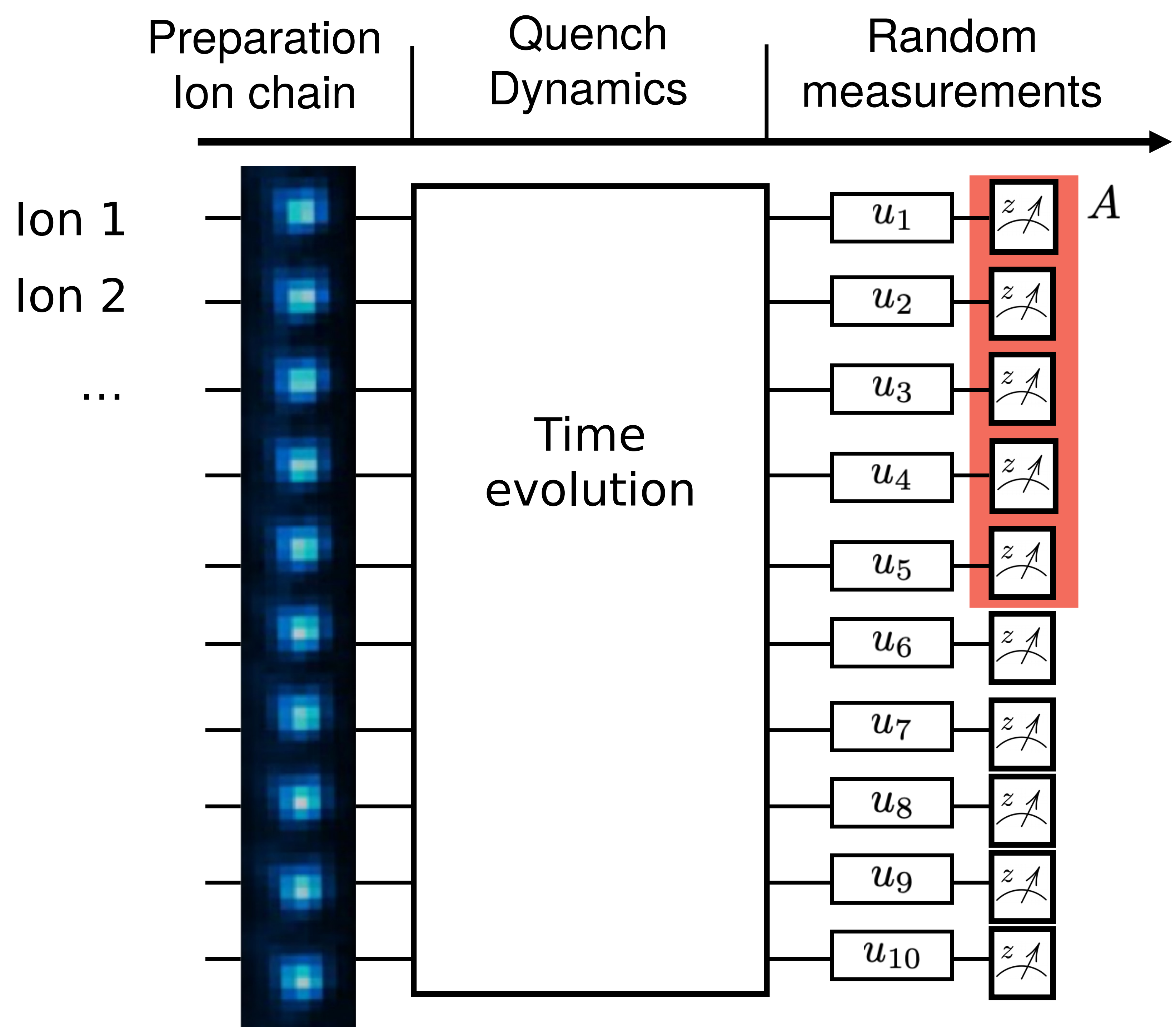
Randomized measurement Surgery: Plugging the statistical correlations to make a quantity measurable

$$\mathrm{Tr}(\rho^2) = \sum_{m,n} \rho_{m,n} \rho_{n,m}$$



Experimental demonstration with trapped ions

Brydges et al, Science 2019



Goal: Study the emergence of entanglement from a product state (quantum thermalization)

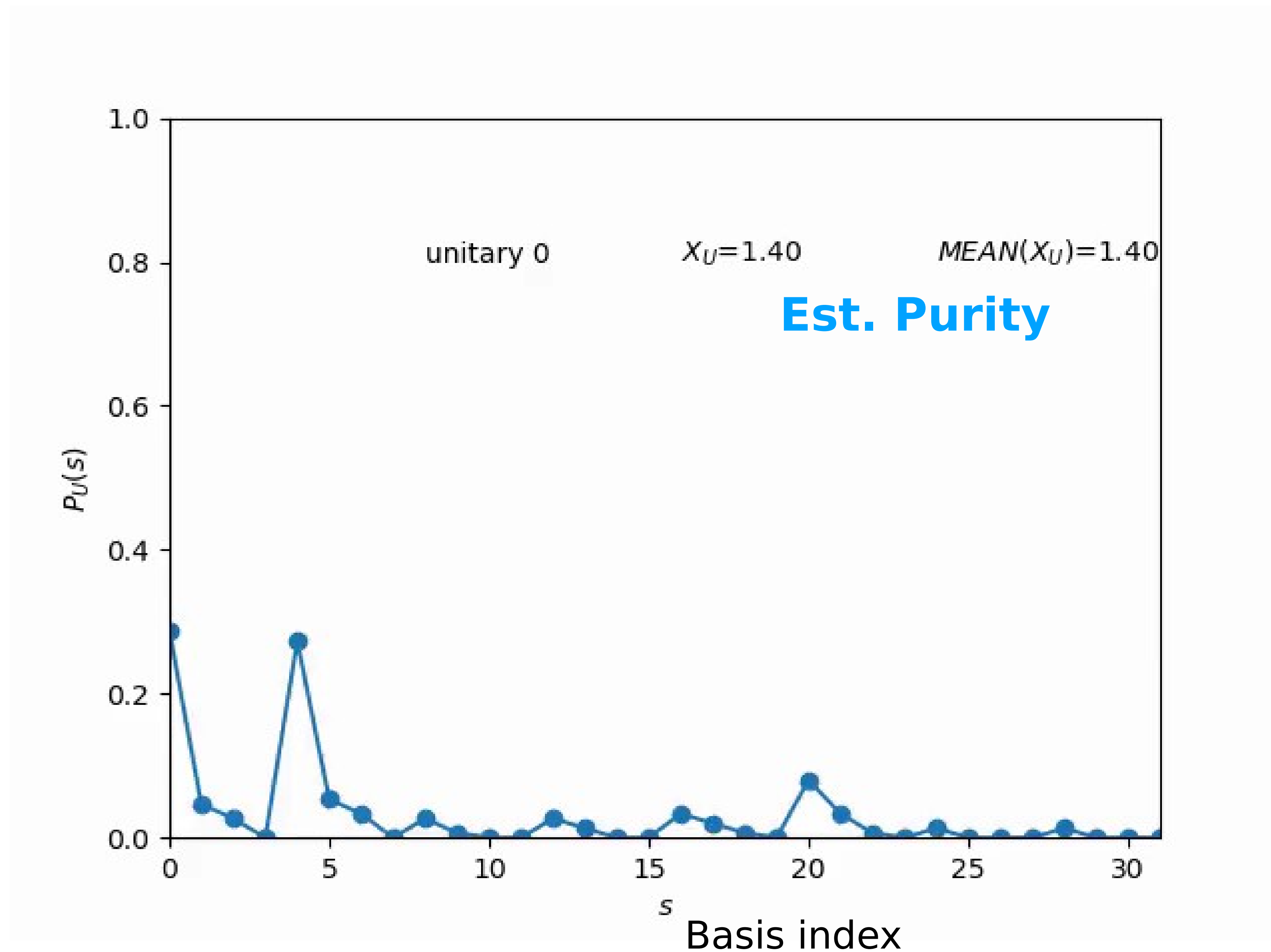
$$|\psi(t)\rangle = e^{-iH_{XY}t}|01\ldots01\rangle \qquad H_{XY} = \hbar\sum_{i<j} J_{ij}(\sigma_i^+\sigma_j^- + \sigma_i^-\sigma_j^+) + \hbar\sum_j (B+b_j)\sigma_j^z$$

Experimental demonstration with trapped ions Brydges et al, Science 2019

t=0 **State is a product state (0101010101)→ large stat. flucttutations**

Experimental data for 5 qubits

Probability



Basis index

$$\boxed{\mathrm{Tr}\left[\rho_A^2\right] = \overline{X_U}} \quad \textbf{with} \quad X_U = 2^{N_A} \sum_{s_A, s'_A} (-2)^{-D[s_A, s'_A]} \boxed{P_U(s_A) P_U(s'_A)}$$

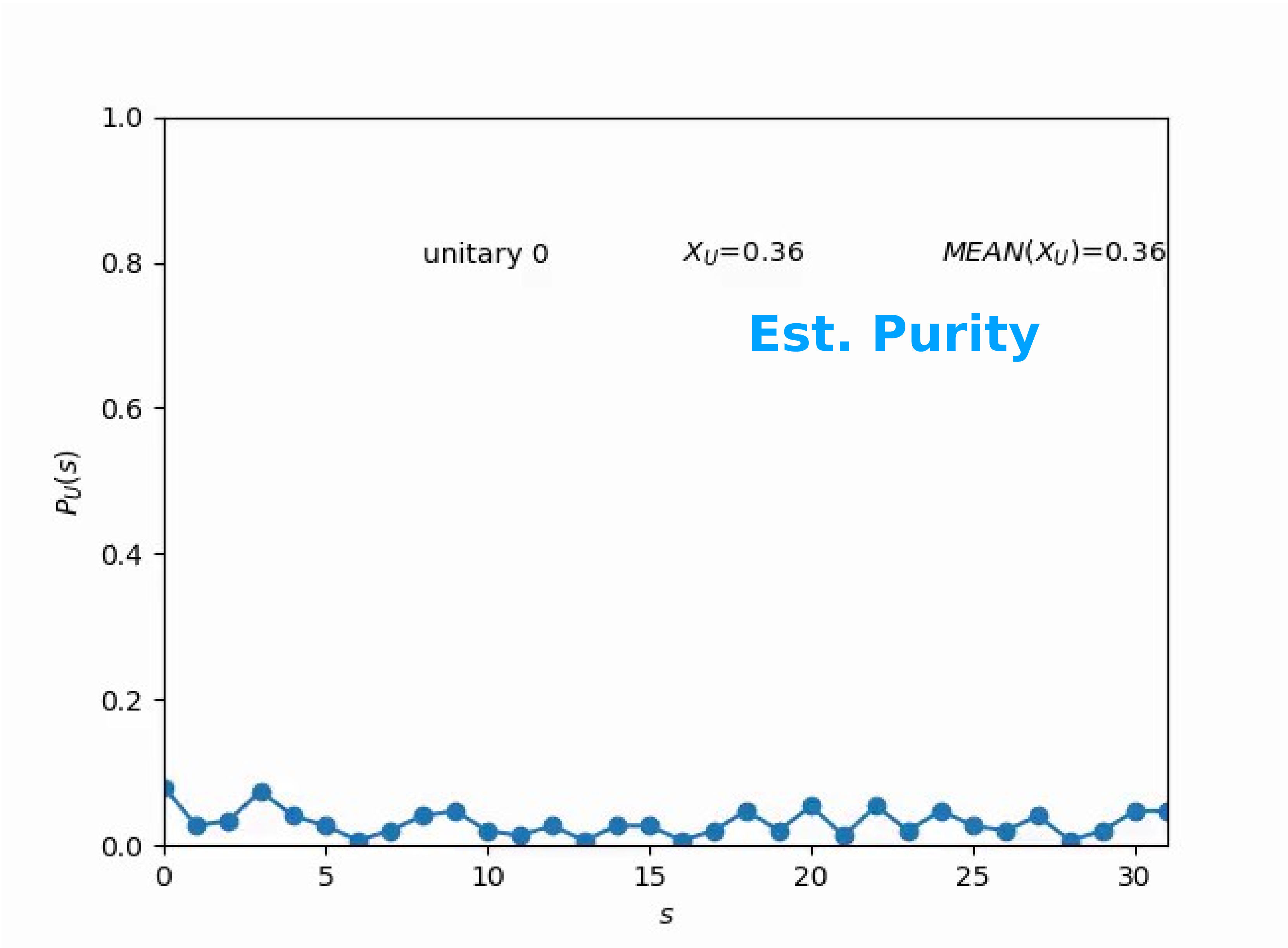
Experimental demonstration with trapped ions

Brydges et al, Science 2019

t=5 ms We have entanglement, i.e a reduced mixed state…

Experimental data for 5 qubits

Probability

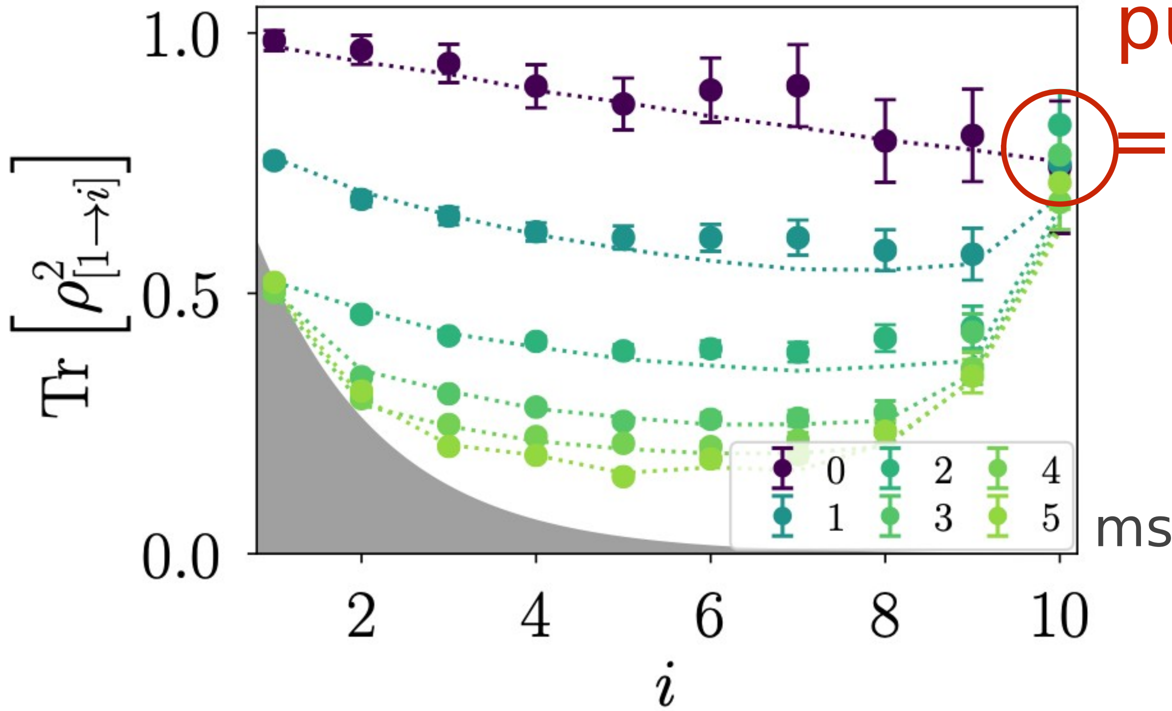


$\mathrm{Tr}\left[\rho_A^2\right] = \overline{X_U}$ **with** $$X_U = 2^{N_A} \sum_{s_A, s'_A} (-2)^{-D[s_A, s'_A]} P_U(s_A) P_U(s'_A)$$

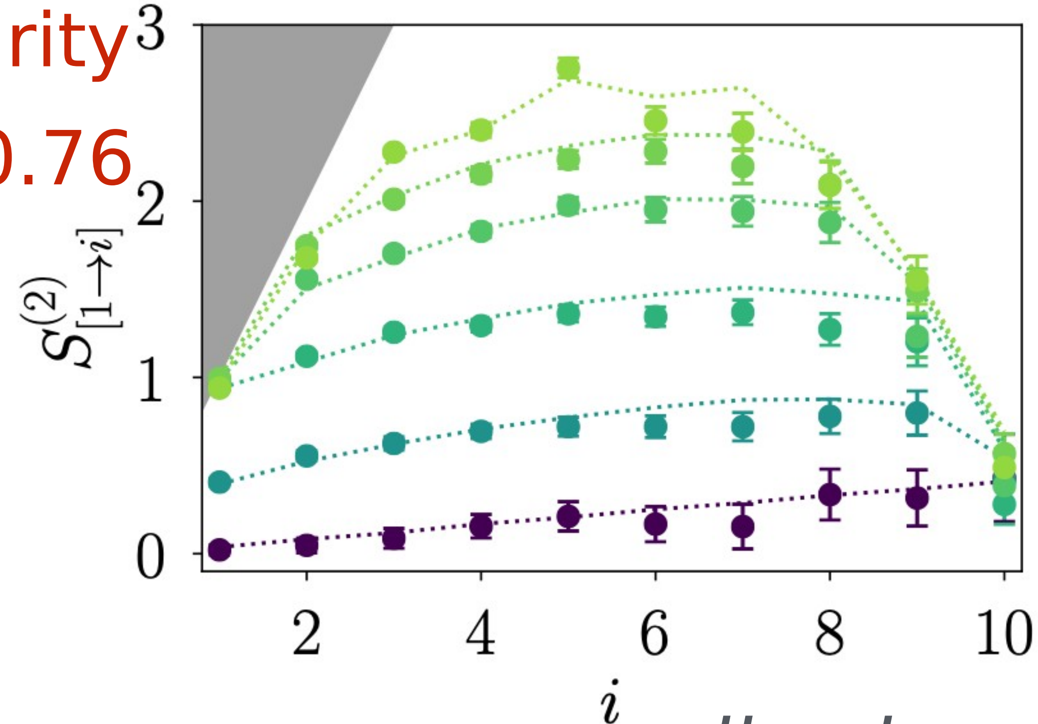
Following the growth of entanglement as a function of time

purity $\mathrm{Tr}[\rho_A^2]$

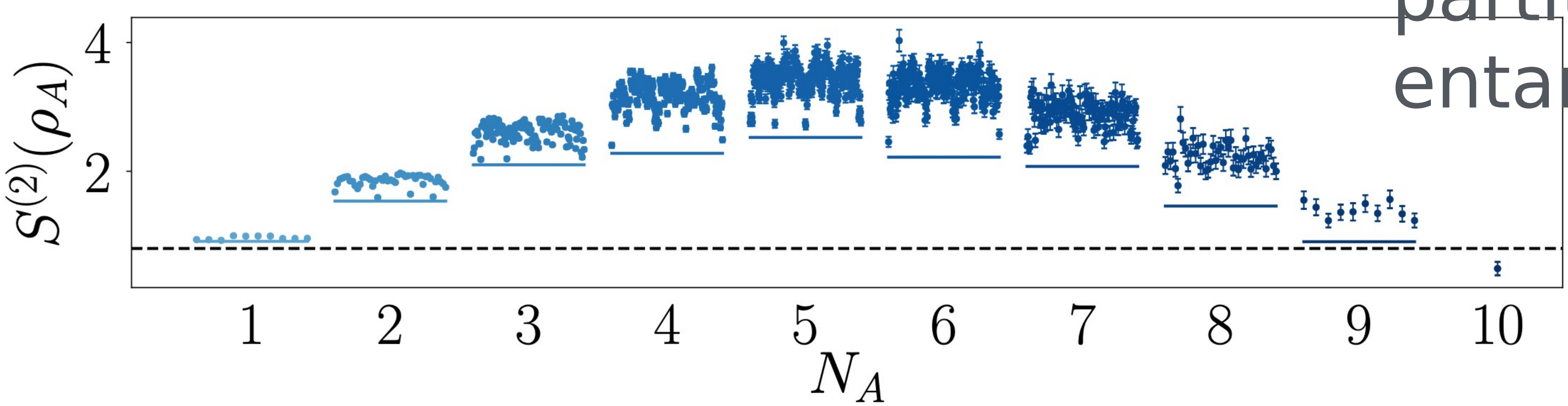
total purity =0.76



Renyi entropy



all sub partitions … entangled



See also pionnering works by Jaksch, Pichler, Zoller, Daley, etc with multiple copies in Hubbard systems

Renyi entropy measurements in quantum computers (Work in progress)

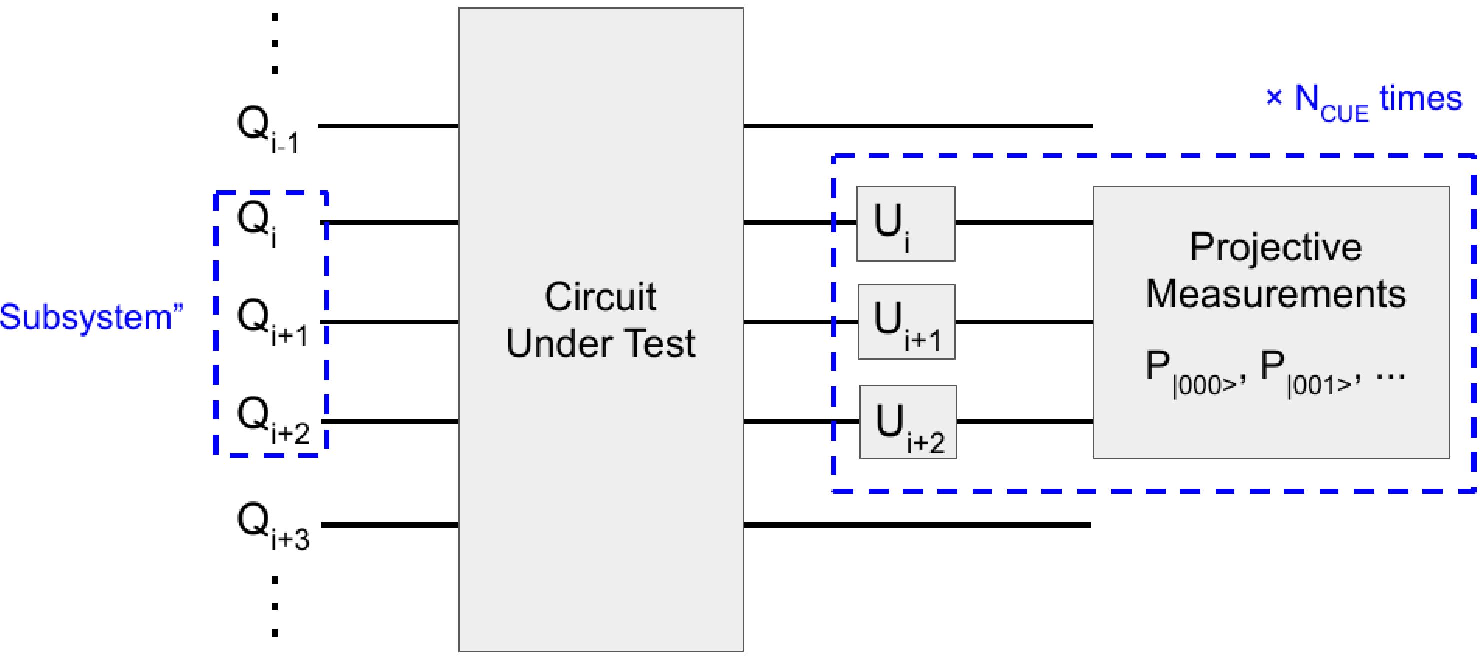
With Andreas Elben, Peter Zoller (Innsbruck),
Xiao Mi, Pedram Roushan, Yu Chen, Vadim Smelyanskiy, and Google AI quantum team

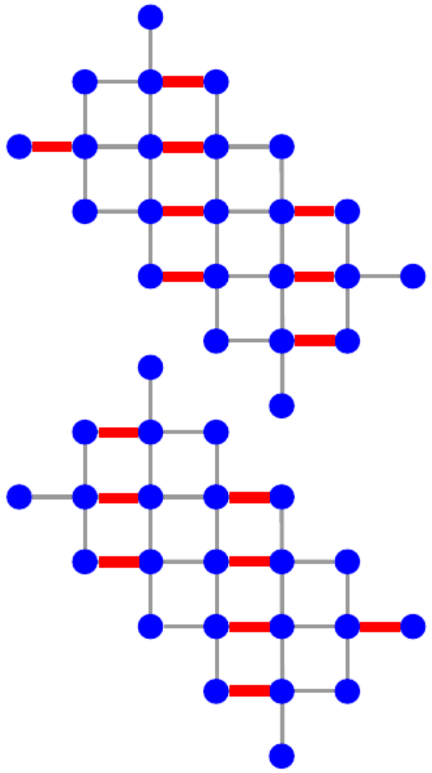
Google AI

Goal:

- **Verify quantum computing task** (decoherence+ entanglement)
- **Probe 2D entanglement growth**

Tools: Local randomized measurements

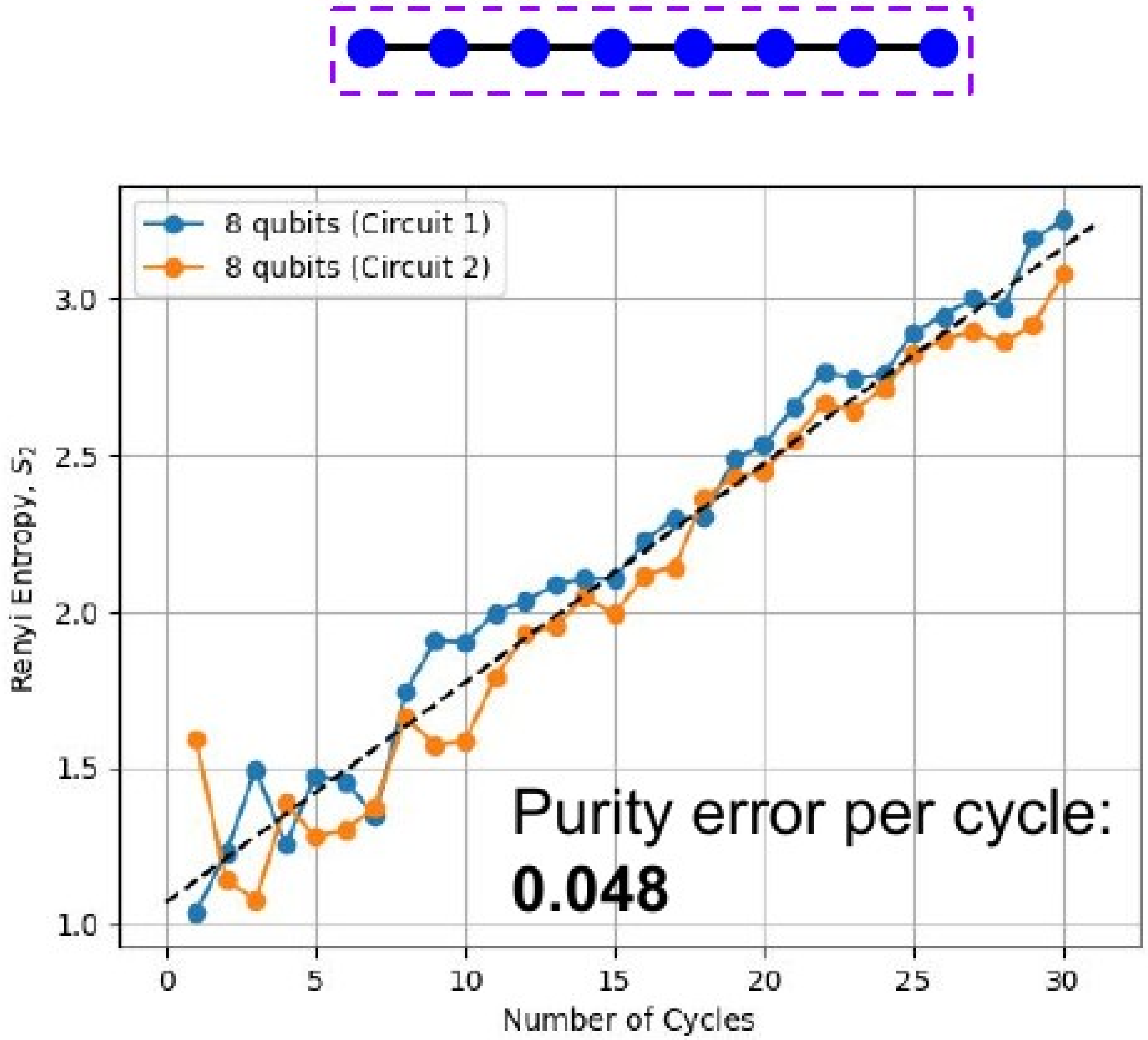




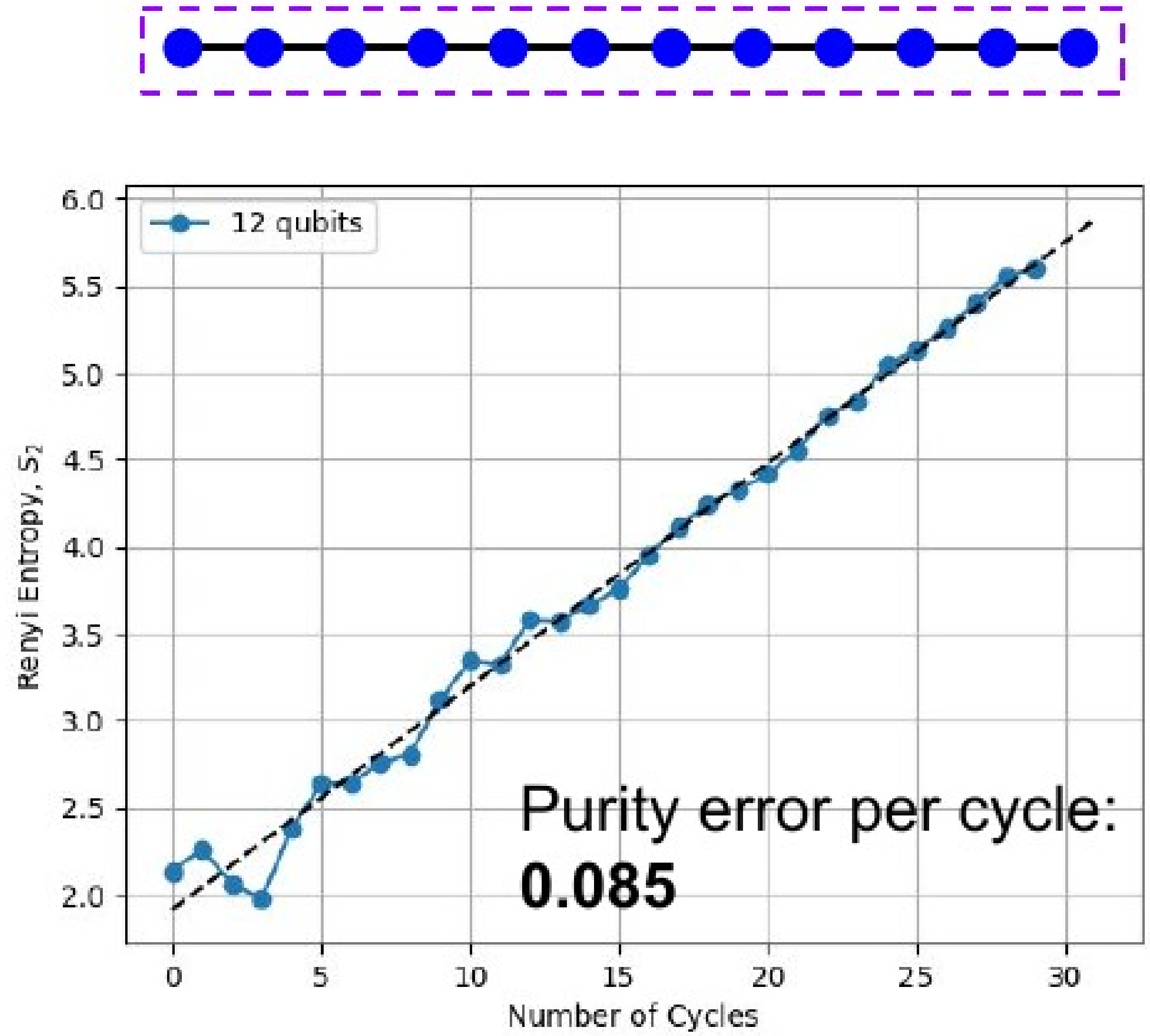
Google chips (i-swap gates)

Renyi Entropy for Closed Systems (Metrology Applications)

8 qubits

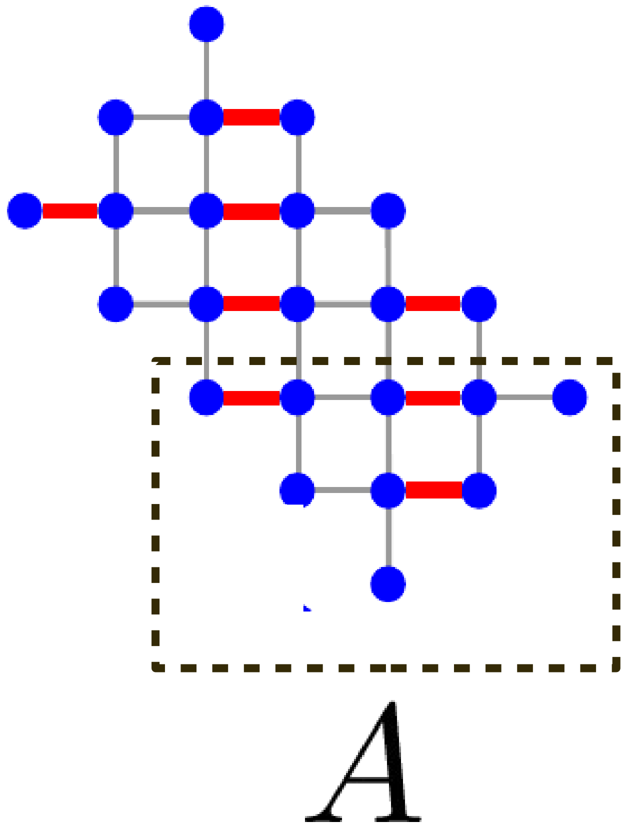


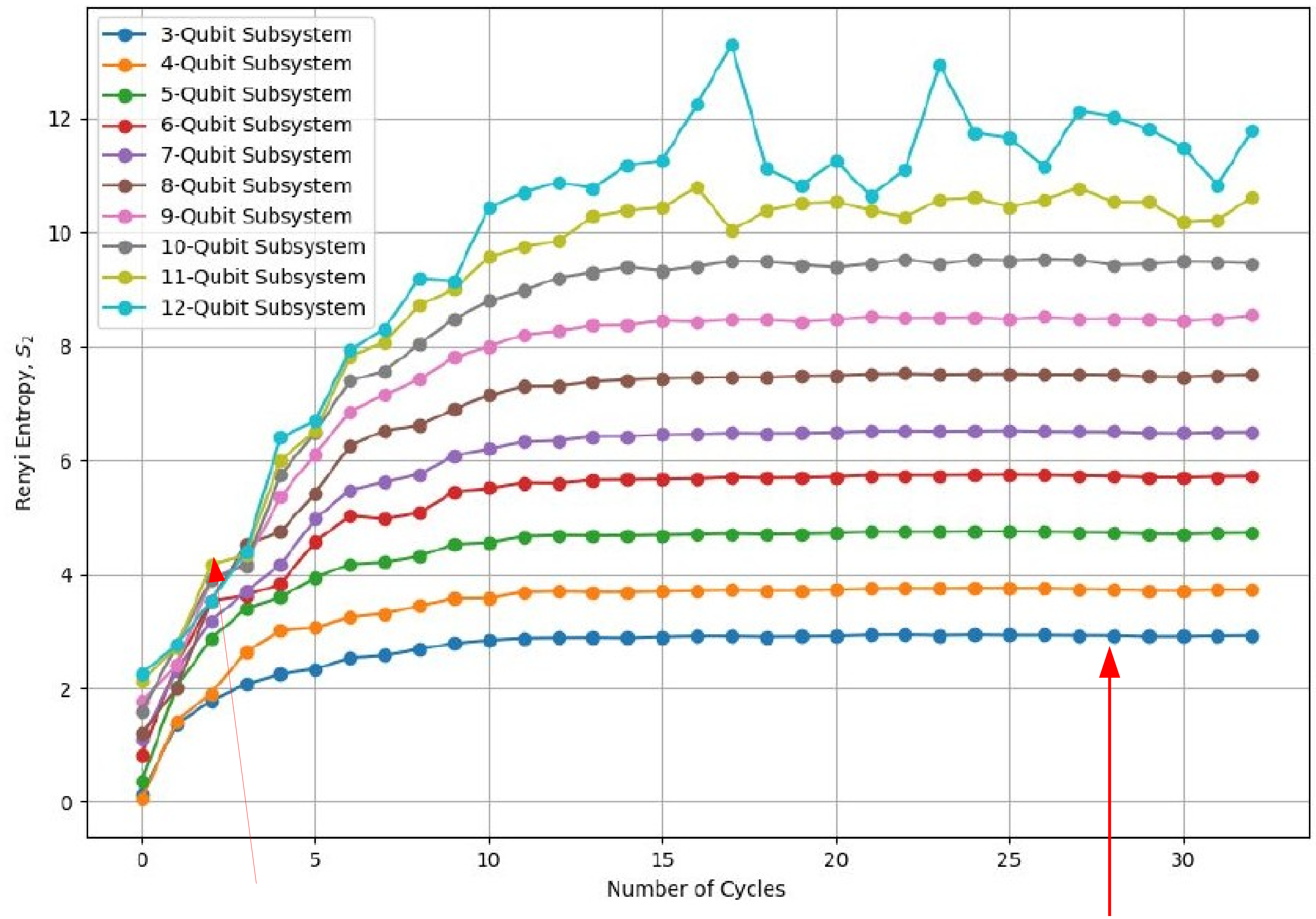
12 qubits



- Circuits are RC with microwaves and sqrt-iSWAP gates.
- Dashed lines show fits to theory with depolarization error model.
- 8 qubits: 40K measurements, 30 random gate sets.
- 12 qubits: 250K measurements, 30 random gate sets.

Observation of entanglement growth in two dimensions





Compatible with an area law:

$$S_2 \sim \ell_A$$

Volume law

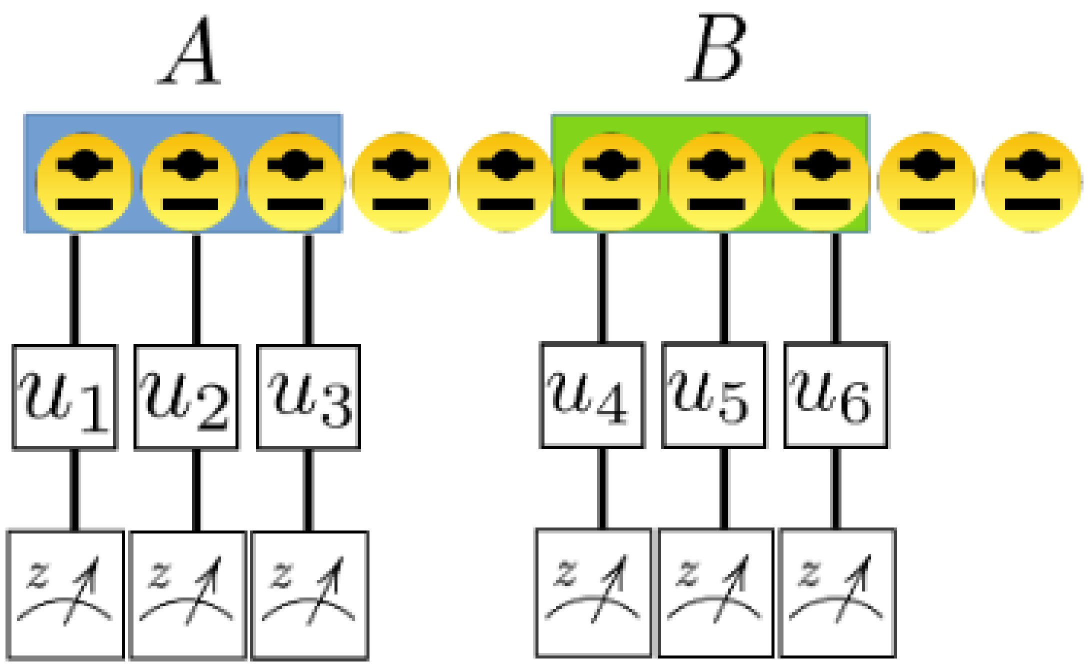
$$S_2 \sim N_A$$

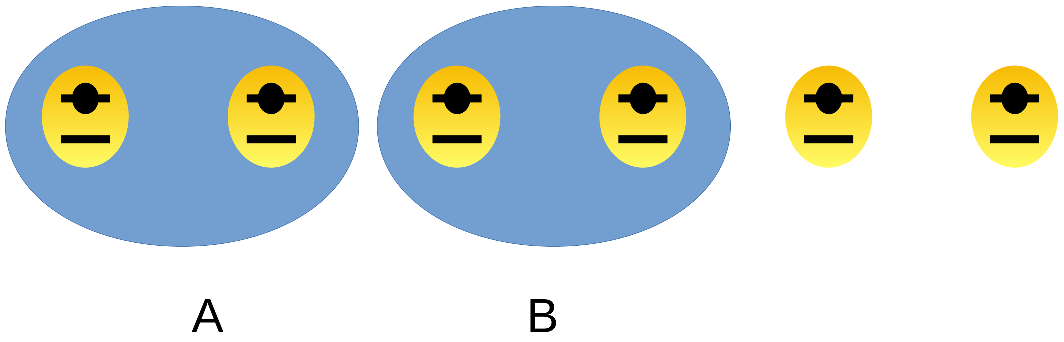
Question: How can we distinguish entanglement from decoherence?

Part II: Mixed-State Entanglement from Local Randomized Measurements

Phys. Rev. Lett. 125, 200501 (2020)

A. Elben (Innsbruck) R. Kueng (Caltech→ Linz), R. Huang (Caltech), R. van Bijnen (Innsbruck) C. Kokail (Innsbruck) , M. Dalmonte (Trieste), P. Calabrese (Trieste), B. Kraus, (Innsbruck) John Preskill (Caltech), Peter Zoller (Innsbruck), and BV





Mixed-State Entanglement

$$\rho \neq \sum_j p_j \rho_j^{(A)} \otimes \rho_j^{(B)}$$

What kind of entanglement detection?

Purity test: $\mathrm{Tr}\left[\rho_A^2\right], \mathrm{Tr}\left[\rho_B^2\right] < \mathrm{Tr}\left[\rho^2\right]$

Not very powerful for highly mixed states (Brydges 2019)

Entanglement witness: $\mathrm{Tr}(O\rho_{AB}) < 0$

The relevant operastor is state-dependent (ex: CHSH inequalities..)

Not a quantifier of mixed-state entanglement

PPT condition $\rho_{AB}^{T_A}$ is not positive semi-definite

Powerful (ex: sufficient for two qubits)
Basis-independent
Entanglement monotone: negativity
Relevant in quantum field theories

Mixed-state entanglement

Positive-Partial-Transpose (PPT) Condition for mixed state entanglement

If the state is separable $\rho_{AB} = \sum_k c_k \rho_A^{(k)} \otimes \rho_B^{(k)}$

Partial transposition

Then $\rho_{AB}^{T_A} = \sum_k c_k \left[(\rho_A^{(k)})^T \otimes \rho_B^{(k)} \right]$, Is positive semi-definite, i.e only has positive eigenvalues

Conversely, if $\rho_{AB}^{T_A}$ is not positive semi-definite → The state is not separable, I.e entangled

Example: Bell state $\rho_{AB} = |Bell\rangle\langle Bell|$

=[[0. 0. 0. 0.]
[0. 0.5 0.5 0.]
[0. 0.5 0.5 0.]
[0. 0. 0. 0.]]

00 01 10 11

$\rho_{AB}^{T_A}$ =[[0. 0. 0. 0.5]
[0. 0.5 0. 0.]
[0. 0. 0.5 0.]
[0.5 0. 0. 0.]]

Spec = (0.5,0.5,0.5,-0.5)

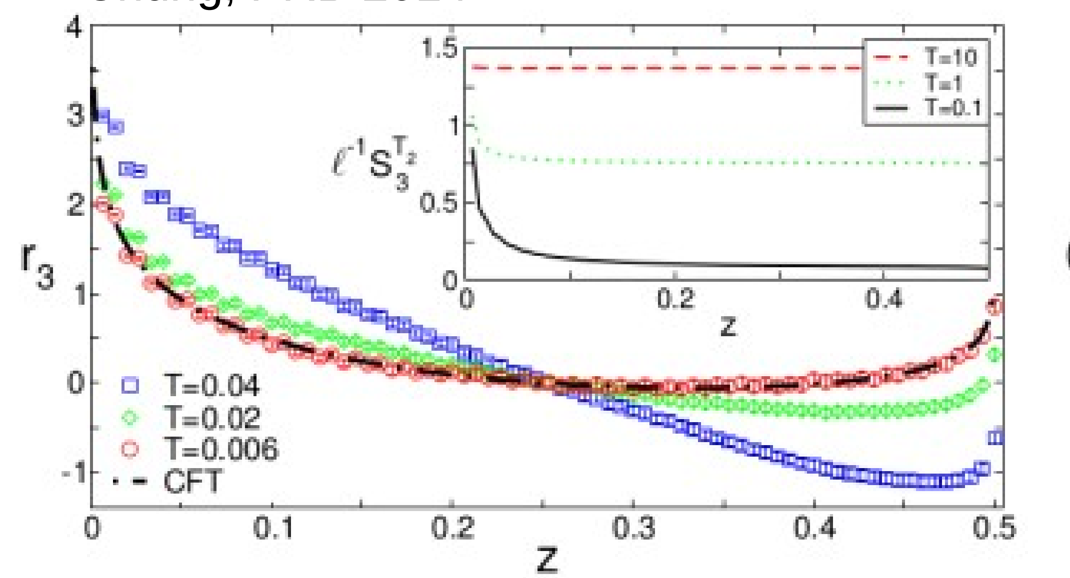
How to detect entanglement via the PPT condition in multi qubit systems??

Mixed-state entanglement

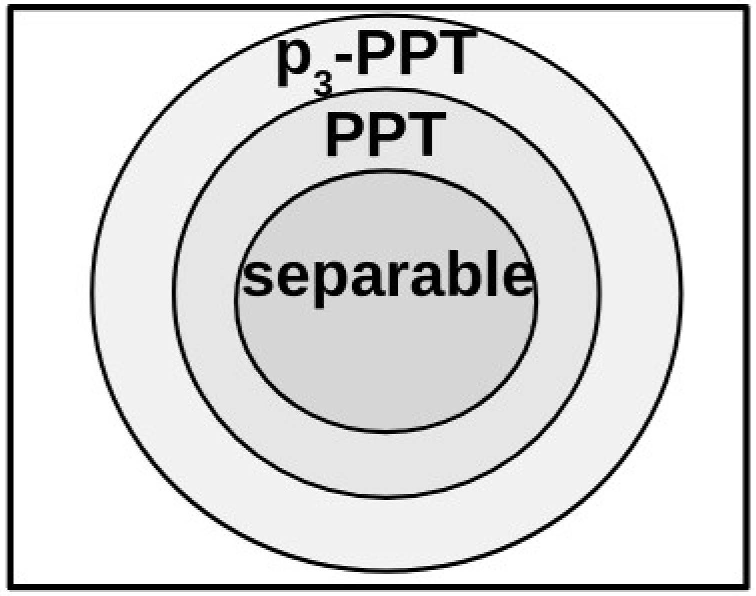
Our approach: Measuring PT moments $$p_n = \mathrm{Tr}[(\rho_{AB}^{T_A})^n] \quad \text{for } n = 1, 2, 3, \ldots.$$

→ **Quantify mixed-entanglement in quantum-field theories:**
pionnering works by P. Calabrese and co-workers

Chung, PRB 2014



→ **A measurable powerful entanglement condition**
Elben et al, PRL 2020



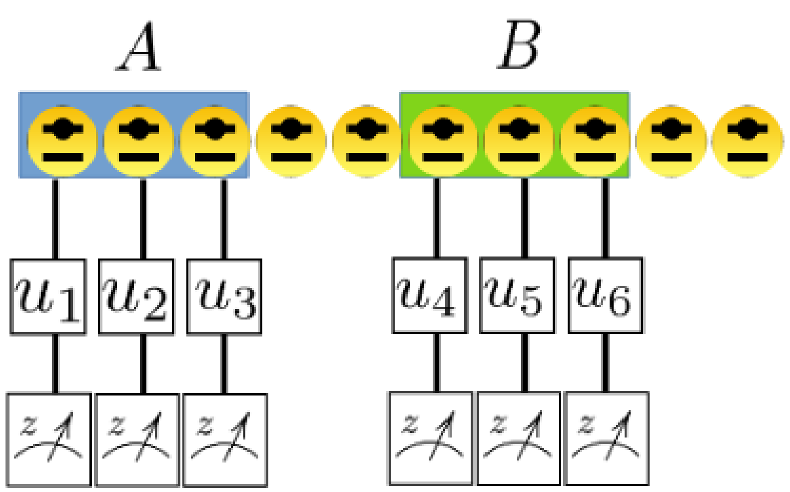
p_3 PPT condition

$$p_3 < p_2^2$$ Implies PPT violation implies entanglement

Measuring PT moments via local randomized measurements

Key ideas:

1) **Randomized measurements are` tomographically complete`** Elben, et al PRA 2019
(see also Ohligher NJP 2013 for Hubbard models)



Measured bit strings

$$\hat{\rho}_{AB}^{(r)} = \bigotimes_{i \in AB} \left[3 (u_i^{(r)})^\dagger \left| k_i^{(r)} \right\rangle \left\langle k_i^{(r)} \right| u_i^{(r)} - \mathbb{I}_2 \right]$$

$$\mathbb{E}[\hat{\rho}_{AB}^{(r)}] = \rho_{AB}$$

2) **Polynomials of the density matrix can be estimated via U-statistics**
(Huang et al, Nature Physics 2020)

$$p_3 = \mathbb{E}\left[\mathrm{Tr}\left((\rho_{AB}^{(r_1)})^{T_A} (\rho_{AB}^{(r_2)})^{T_A} (\rho_{AB}^{(r_3)})^{T_A} \right) \right]$$

→ Multi-linear postprocessing of the data (no tomography)

→ Measurement budget ~$2^{N[AB]}$

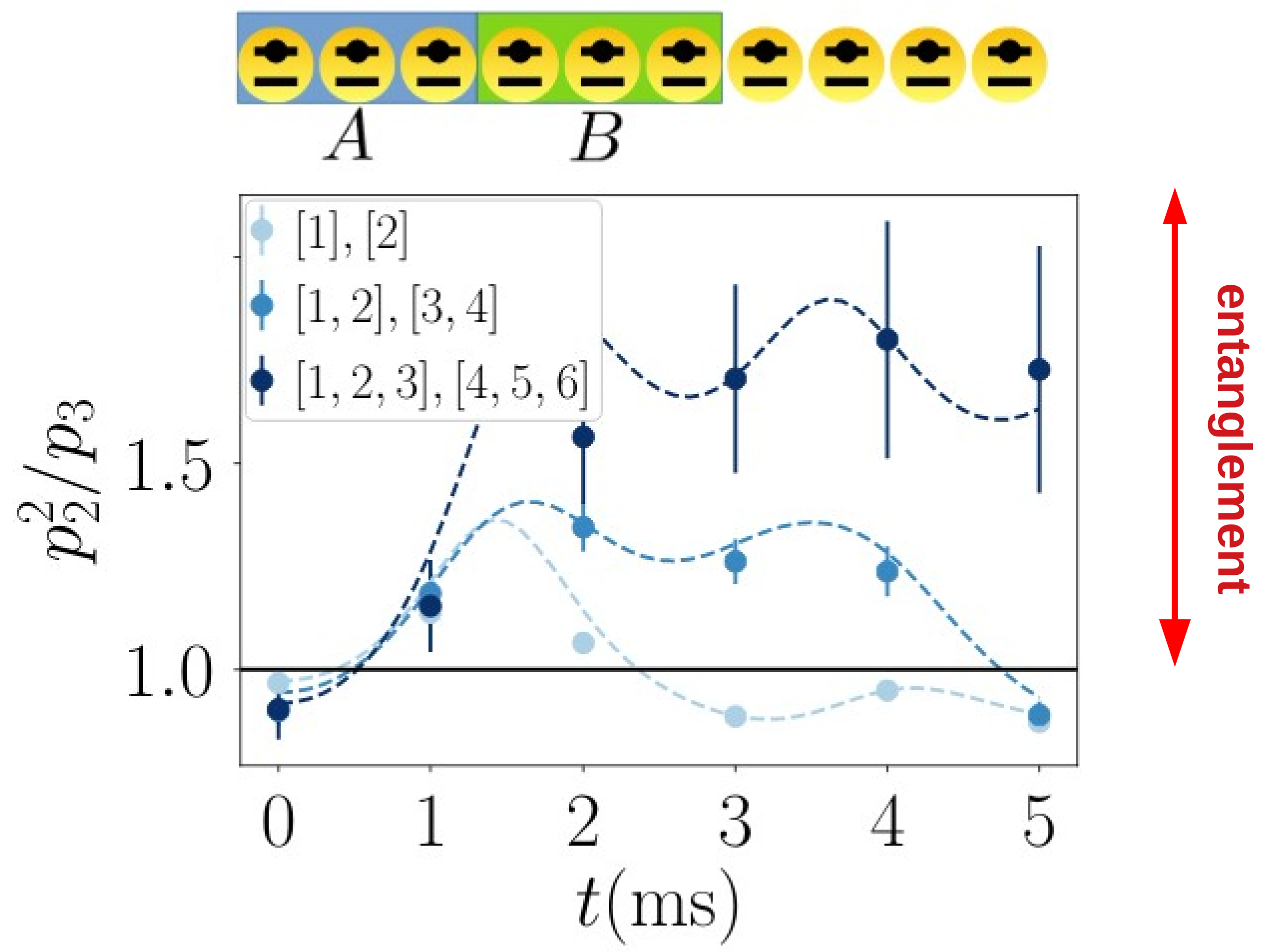
Mixed-state entanglement

First experimental measurements of PT moments

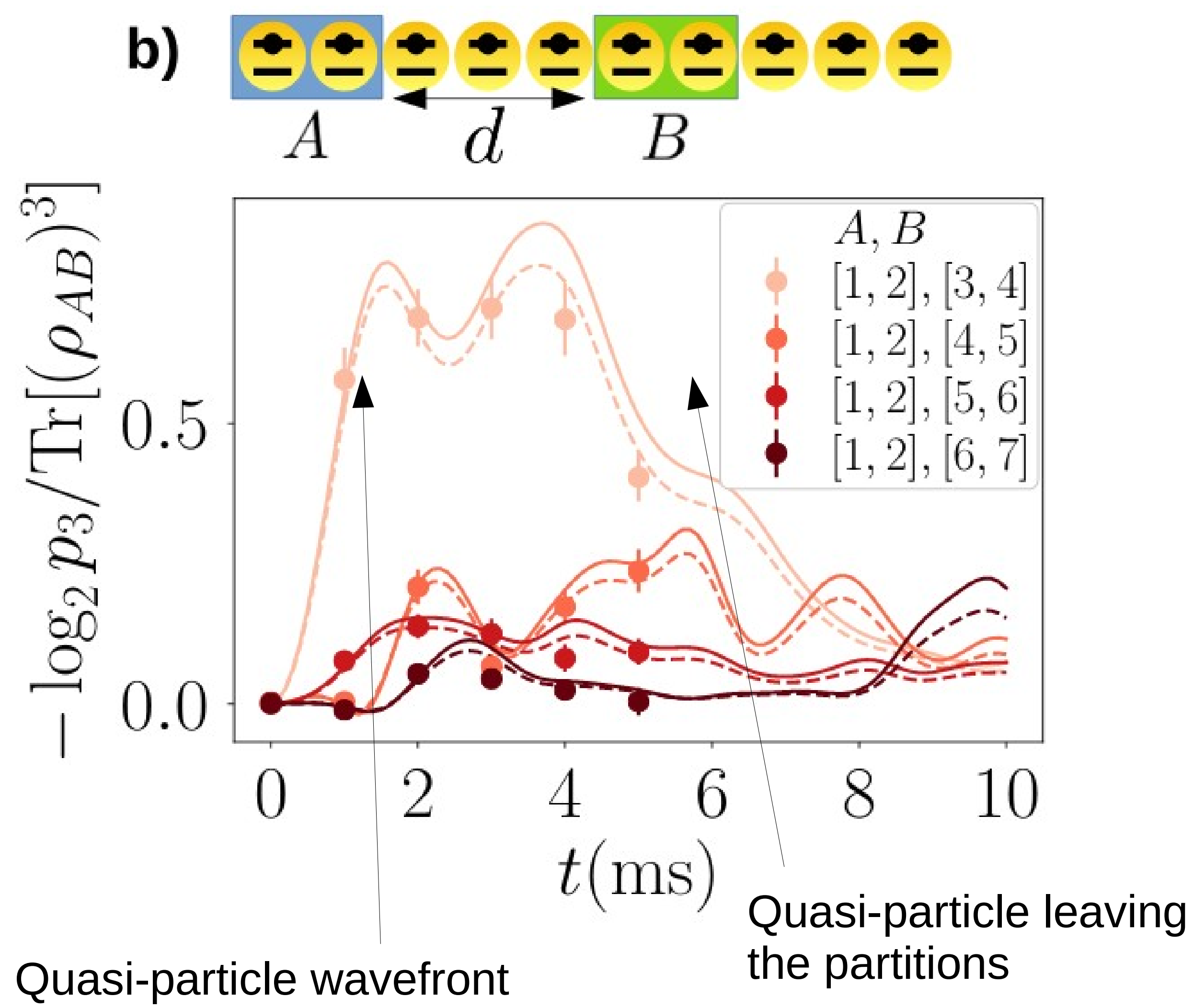
Elben et al, Phys. Rev. Lett. 125, 200501 (2020)

Data: Brydges , Science 2019 (reanalyzed)

Entanglement detection



Entanglement spreading
Quantum-field theory predictions: P. Calabrese et al



Conclusion

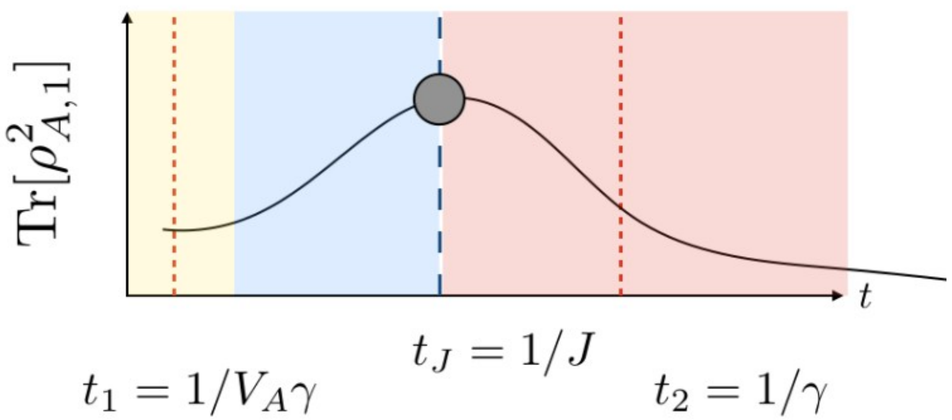
Randomized measurements: a versatile toolbox to probe many-body physics in quantum experiments

Current efforts

Symmetry-resolved entanglement

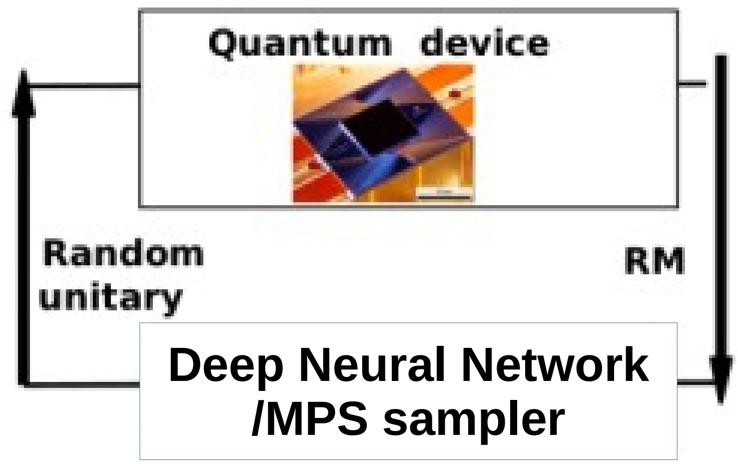
V. Vitale, A. Elben, R. Kueng, A. Neven, J. Carrasco, B. Kraus, P. Zoller, P. Calabrese, BV, M. Dalmonte

https://arxiv.org/abs/2101.07814



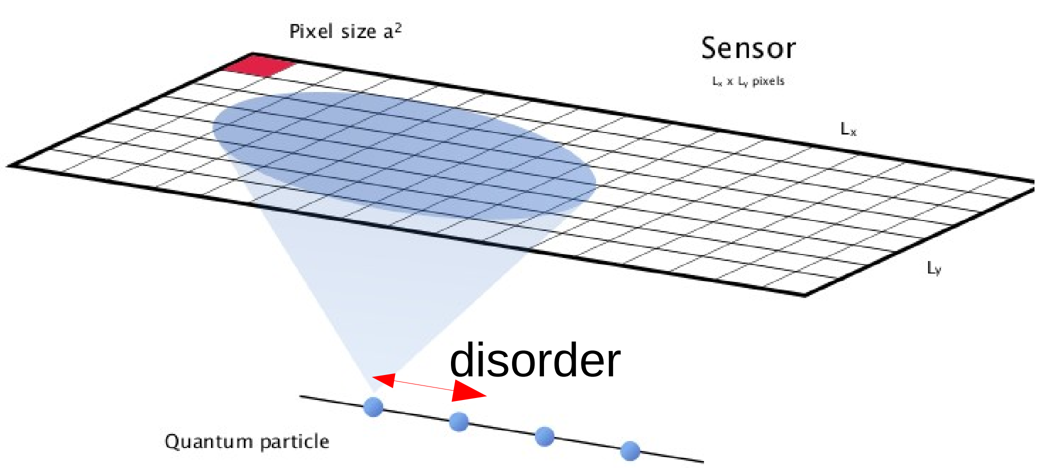
Optimized protocols

A.Rath, A. Elben, R. van Bijnen, P. Zoller, A. Minguzzi



Random Time-of-flight Microscopy

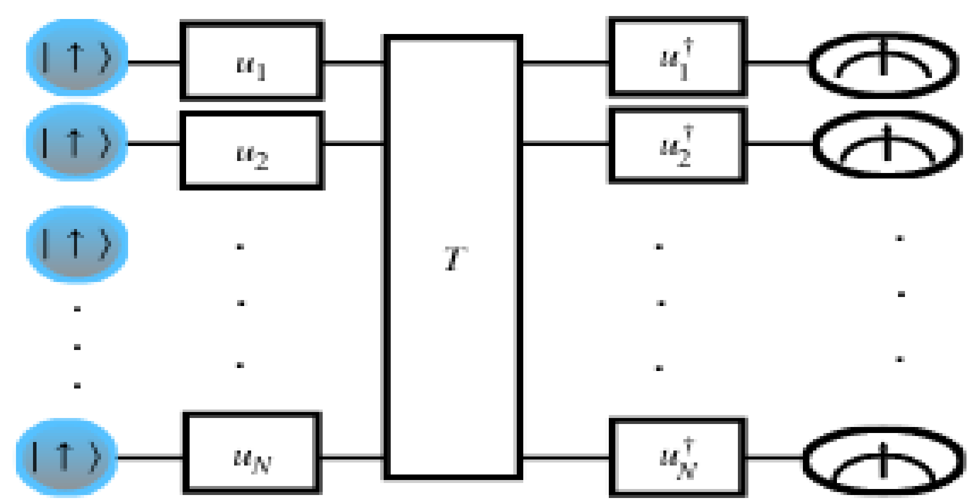
P. Naldesi, A. Elben, P. Zoller, A. Minguzzi



Random Hopping+Time of Flight

Measuring Spectral Form Factors

L. Joshi, A. Elben, P. Zoller



Thank you!

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