Randomized measurements protocols for quantum technologies

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Machine Learning for Quantum
06/07/21

Outline

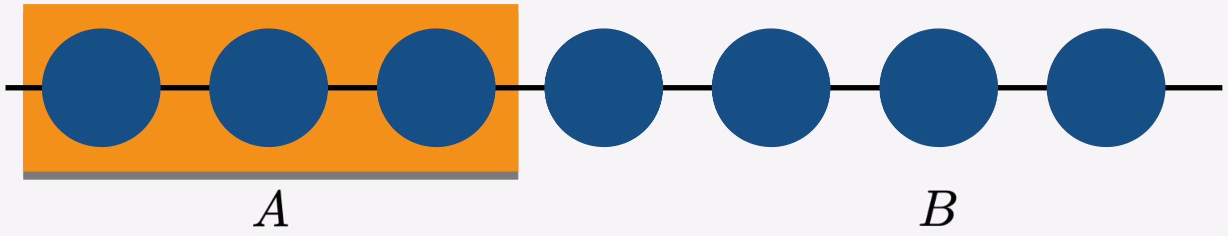
- Context: Measuring entanglement

- Our method with randomized measurements

- Optimized protocols using importance sampling

A. Rath, R. van Bijnen, A. Elben, P. Zoller, B. Vermersch   arXiv:2102.13524

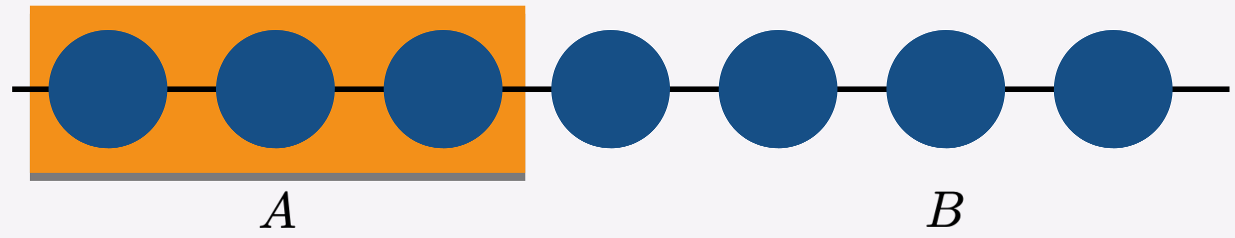
Context entanglement



Two subsystems A and B are **bipartite entangled** iff

$$|\Psi\rangle \neq |\Psi_A\rangle \otimes |\Psi_B\rangle \quad \rho \neq \sum_j p_j \rho_j^{(A)} \otimes \rho_j^{(B)}$$

Beyond low-order correlation functions: entanglement



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Detecting and quantifying entanglement

- Reduced density matrix $\rho_A = \mathrm{Tr}_B(\rho)$
- Entanglement condition (Horodecki 1996) $\mathrm{Tr}\left[\rho_A^2\right], \mathrm{Tr}\left[\rho_B^2\right] < \mathrm{Tr}\left[\rho^2\right]$ (**purity**: $\mathrm{Tr}\left[\rho_A^2\right]$)

Example: Bell state $|\Psi\rangle = \frac{1}{\sqrt{2}}\left(|0\rangle \otimes |0\rangle + |1\rangle \otimes |1\rangle\right) \qquad \rho = |\psi\rangle\langle\psi|$

$$\rho_A = \frac{1}{2}\left(|0\rangle\langle 0| + |1\rangle\langle 1|\right) \qquad \mathrm{Tr}(\rho_A^2) = \frac{1}{2} < 1$$

- Entanglement measures (pure states)

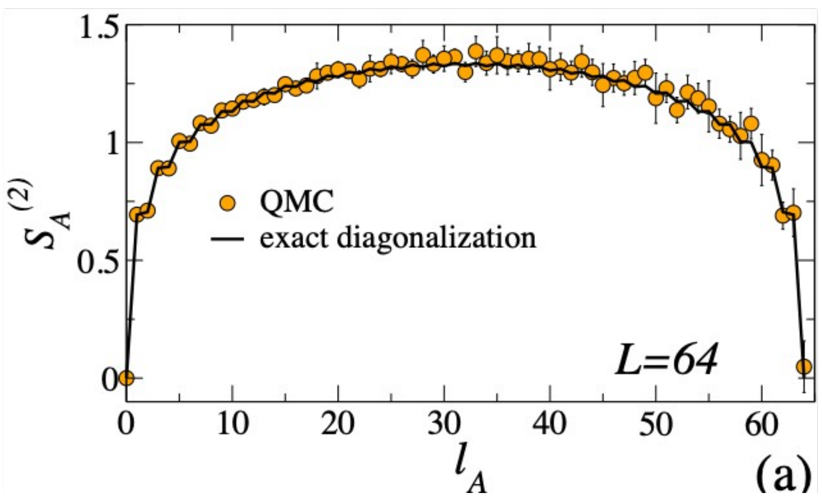
von-Neumann entropy $S_A = -\mathrm{Tr}_A\left[\rho_A \log \rho_A\right]$

Rényi entropy $S_A^{(n)} = \frac{1}{1-n} \log \mathrm{Tr}_A\left[\rho_A^n\right] \quad \leq S_A$

Measuring entanglement entropies: so what?

Measuring entanglement entropies in quantum simulation

Quantum Phase transitions

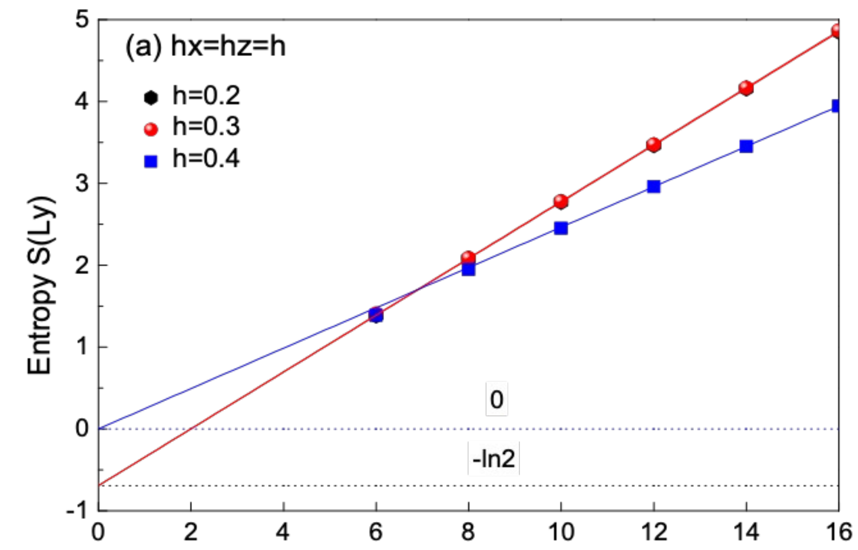


P. Calabrese and J. Cardy, J. Stat. Mech (2004).
Humeniuk, Roscilde PRB (2012)

$$S_A^{(2)} \approx (c/4)\log(L_A)$$

central charge

Topological order



Kitaev, Preskill, PRL 2006
Levin, Wen, PRL 2006
Jian et al, NP 2012

$$S_A^{(n)} \approx \alpha_n L_A - \gamma$$

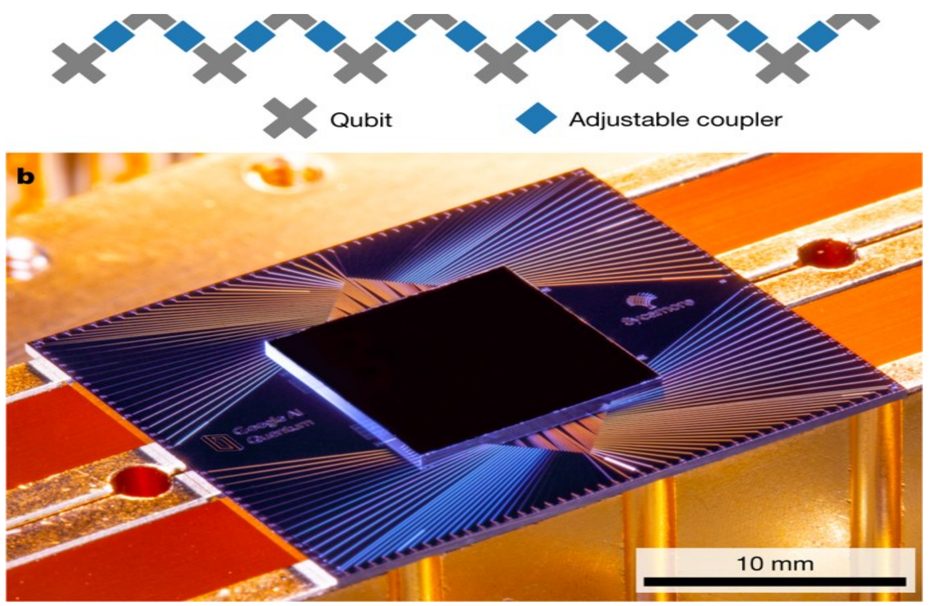
Topological entanglement Entropy

Measuring entanglement entropies: so what?

Measuring the entanglement in quantum computers

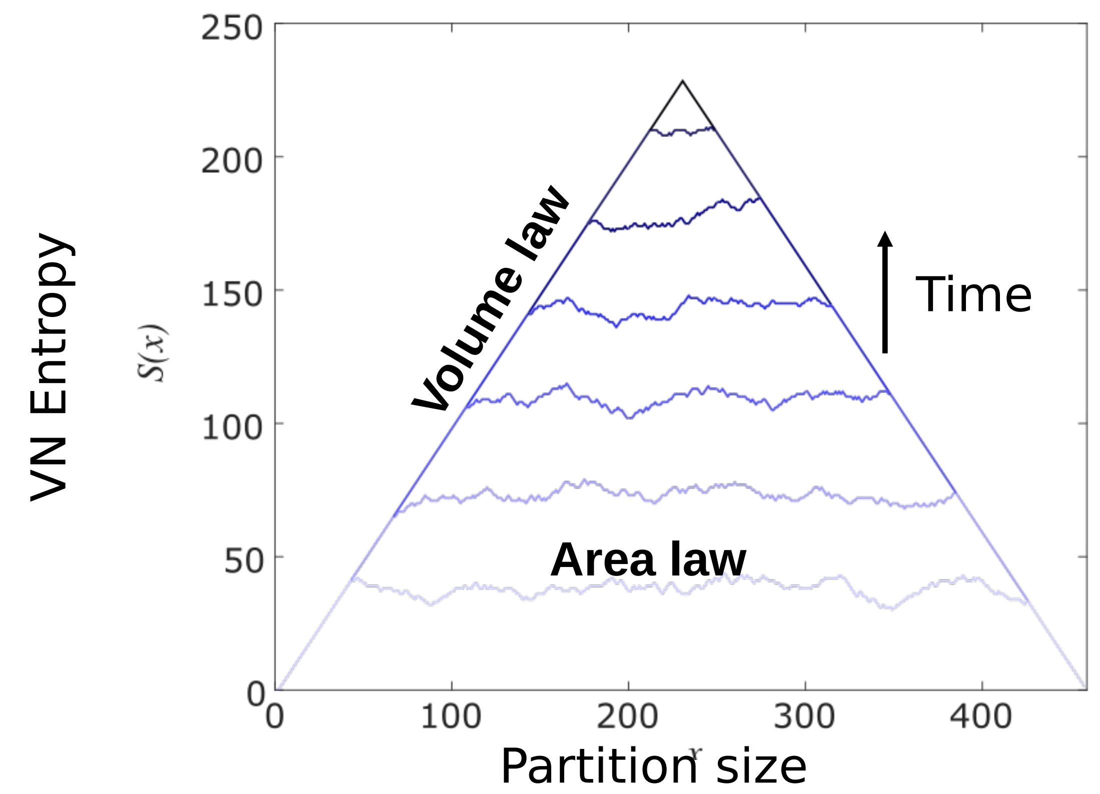
Checks

- Purity checks
- Entanglement checks



Google Sycamore chip

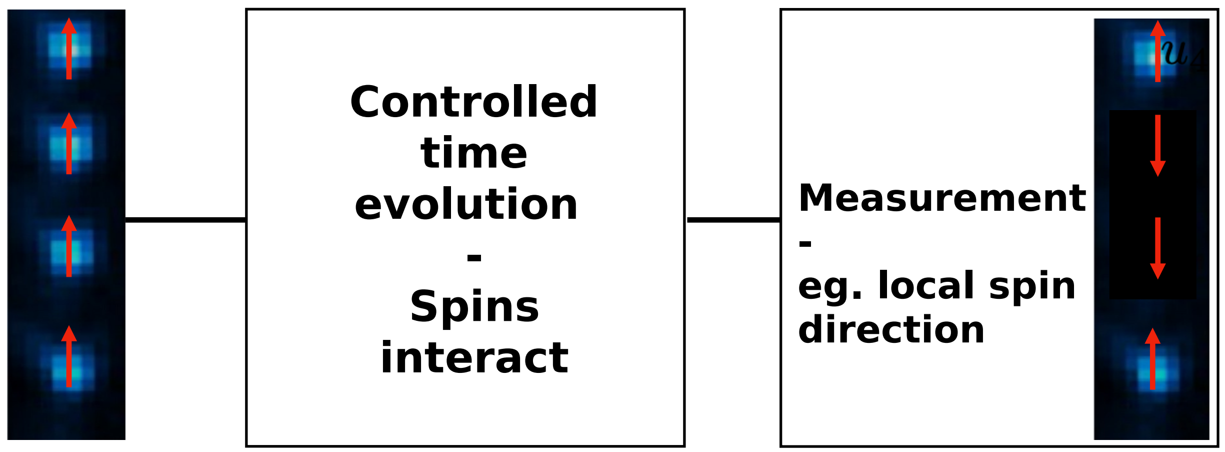
Universal behaviors



Nahum et al, Phys. Rev. X 7, 031016 (2017)

How to measure the purity/entanglement entropies in such many-body quantum systems?

A standard measurement protocol



Simple 'classical' initial state

Quantum simulation/computation

Quantum state ρ

Repeat a finite number of times N_M

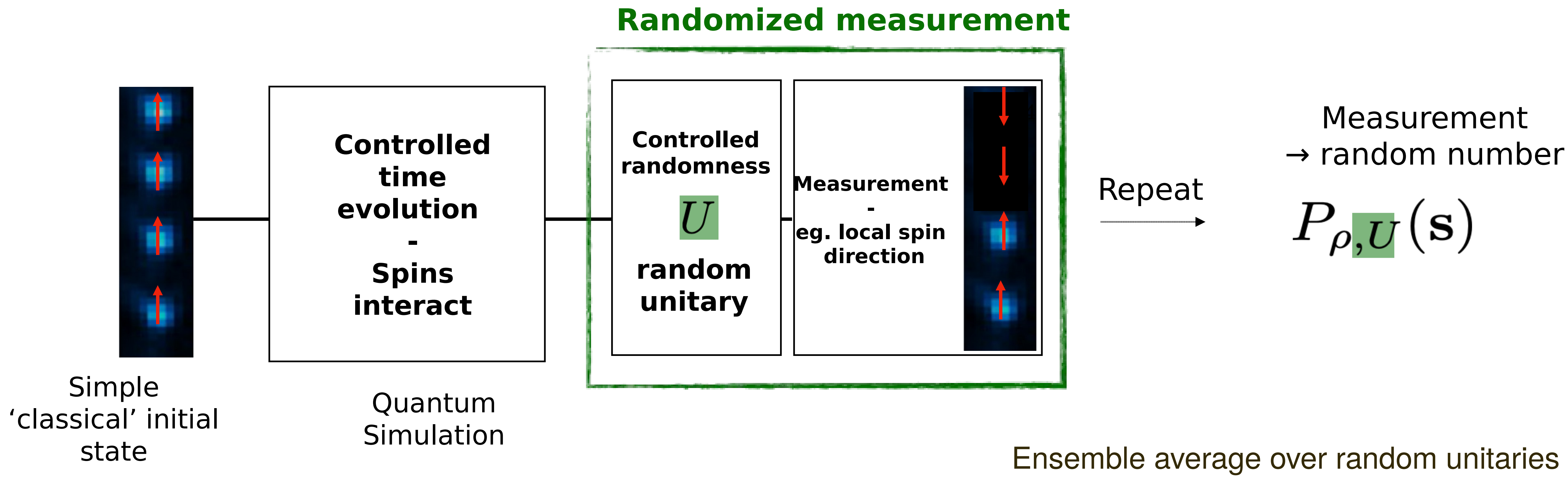
Probability to see a specific configuration

$$P_\rho(\mathbf{s}) = \langle s|\,\rho\,|s\rangle + \frac{\epsilon}{\sqrt{N_M}}$$

'Projection' noise

- **Limited to `observables',** correlation functions, etc $\mathrm{tr}(\rho^2)$
- Not directly applicable to nonlinear functions w.r.t the density matrix
- Possible approach using auxiliary systems by D. Jaksche, H. Pichler, A. Daley, P. Zoller,.

A new tool: randomized measurement protocols



Ensemble average over random unitaries

Correlations of probabilities $\overline{P_{\rho,U}(\mathbf{s}_1)P_{\rho,U}(\mathbf{s}_2)}$

Mixed-state entanglement

Zhou et al, PRL 2020
Elben, .., BV PRL 2020 **[Exp** (re-analized)**]**
Neven, .., BV, , arXiv:2103.07443 **[Exp** (re-analized)**]**

See also works by Knips, Ketterer

Purity-Entanglement entropies

van Enk, Beenakker, PRL 2012
Elben, BV, et al. PRL 2018, PRA 2018, PRA 2019
Brydges,..,BV,.., Science 2019 **[Exp]**
Huang, Kueng, Preskill, Nature Physics 2020
Vovrosh, arXiv:2101.01690 **[Exp]**
Elben,.BV,.. arXiv:2101.07814 **[Exp** (re-analized)**]**
Satzinger, arXiv:2104.01180 **[Exp]**
Rath, .., BV arXiv:2102.13524

Scrambling

BV et al., PRX 2019
Nie, arxiv:1903.12237 **[Exp]**
Qi, arXiv:1906.00524
Joshi, BV, et al PRL 2020 **[Exp]**
Zhou et al, arXiv:2102.01008

State overlaps

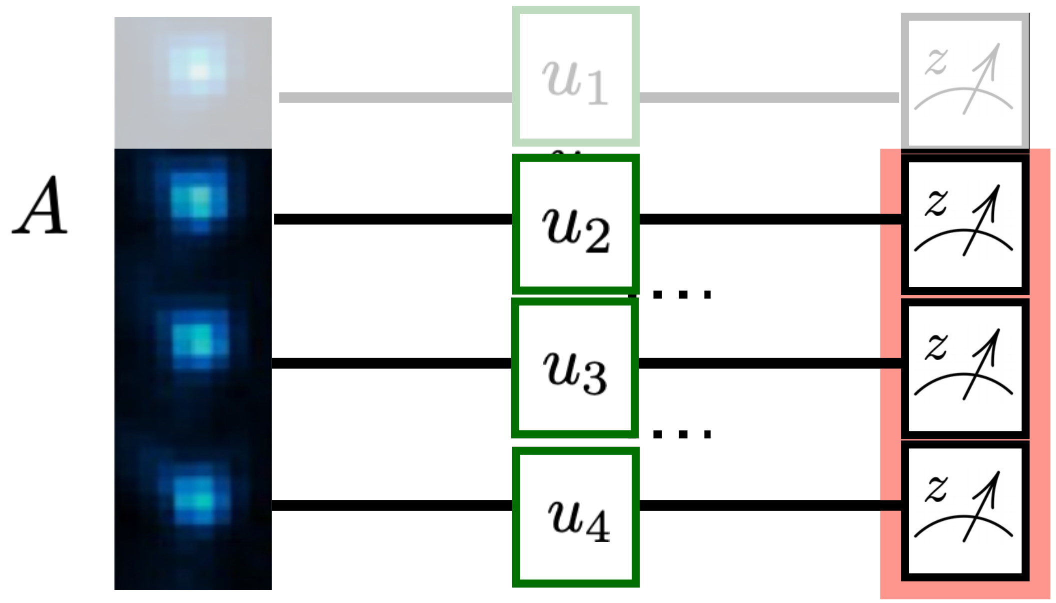
Elben, BV, et al., PRL 2020 **[Exp** (re-analized)**]**
Yu,.., arXiv:2104.00519 **[Exp]**

Topological invariants

Elben,..,BV Sci. Adv. 6 (2020)
ZP Cian,.. BV, et al PRL (2021)

Randomized measurements protocol with local unitaries

Protocol for qubit systems with local random unitaries *Elben, BV et al. (PRL 2018, PRA 2019)*



$u_i \in \mathrm{CUE}(d)$

Local random unitaries (CUE)

$U_A = \bigotimes u_i$

Hamming distance

$$\mathrm{Tr}\left[\rho_A^2\right] = \overline{X_U} \quad \text{with} \quad X_U = 2^{N_A} \sum_{s_A, s'_A} (-2)^{-D[s_A, s'_A]} P_U(s_A) P_U(s'_A)$$

Cross correlations

Proof: average over local unitary 2-designs

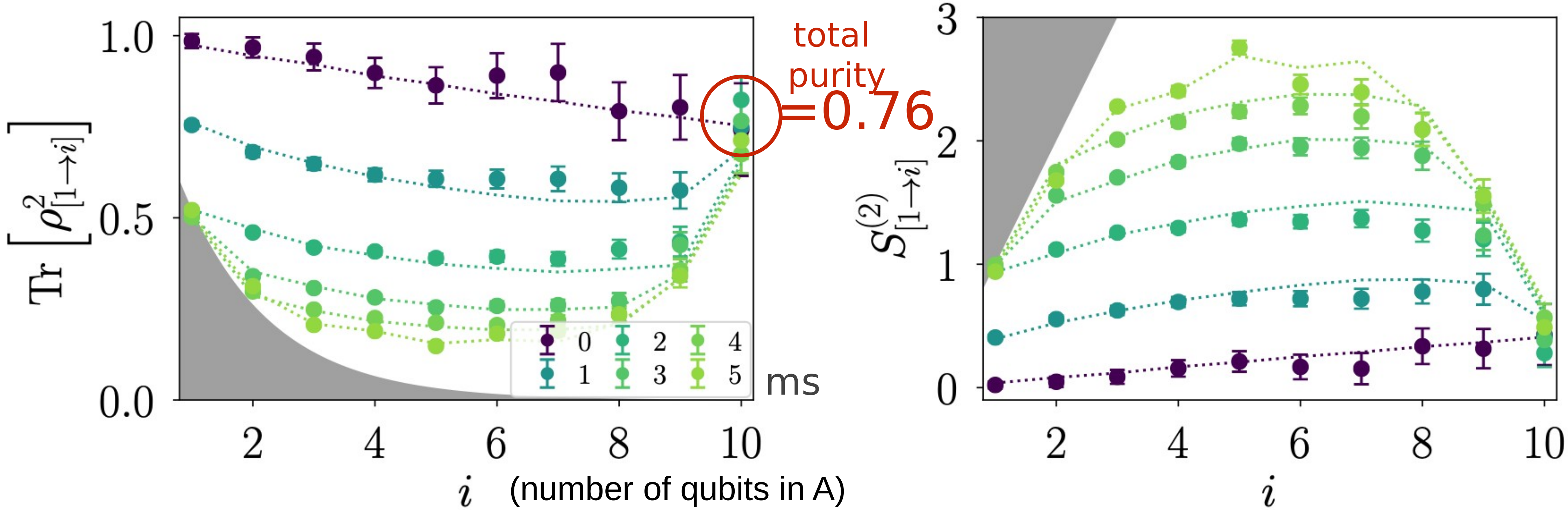
Number of measurements to overcome stat. errors : ~ $2^{N[A]}$ (Compared to tomography: ~ $4^{N[A]}$)

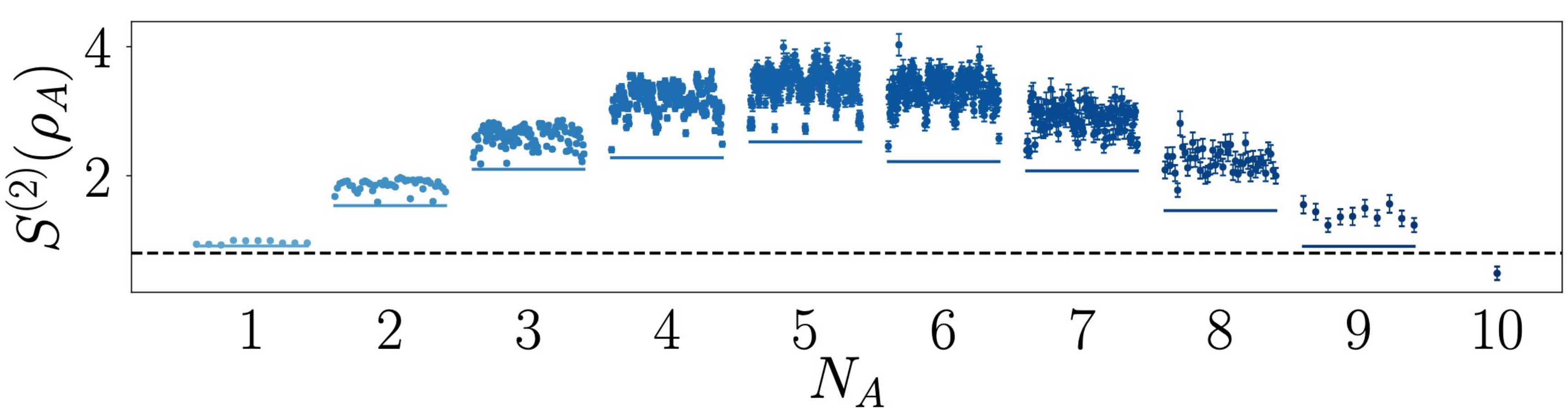
Experimental demonstration with trapped ions

Following the growth of entanglement as a function of time

Brydges et al, Science 2019

Purity   Renyi entropy



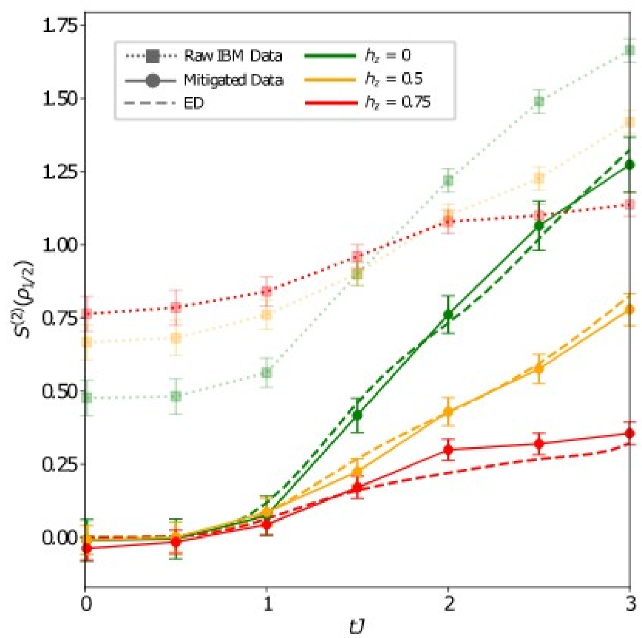


all sub partitions …
entangled

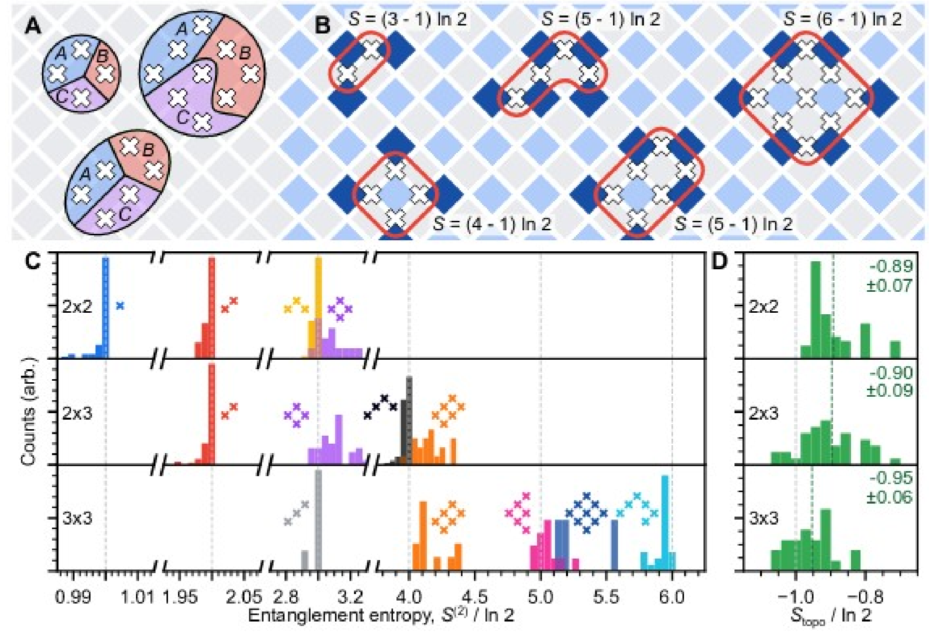
Recent works

Purity Measurement with superconducting qubits

arXiv:2101.01690

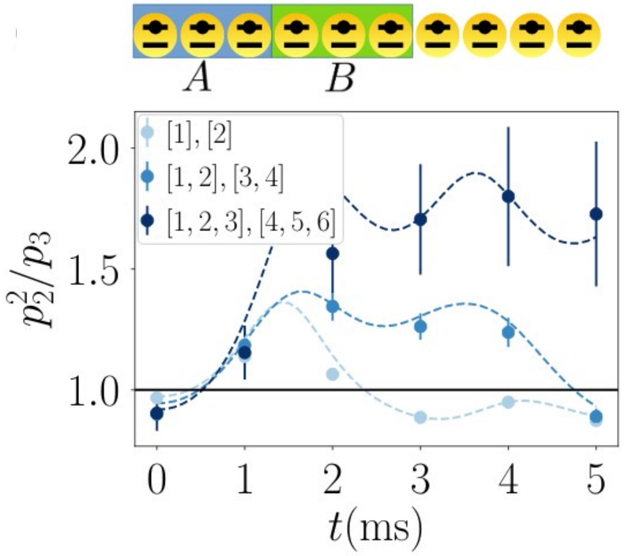


arXiv:2104.01180



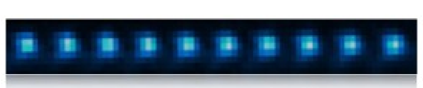
Mixed-state entanglement

Elben et al, Phys. Rev. Lett. 125, 200501 (2020)
arXiv:2101.07814
arXiv:2103.07443 - arXiv:2103.06897 (O. Guehne's group)



Cross-Platform Measurement

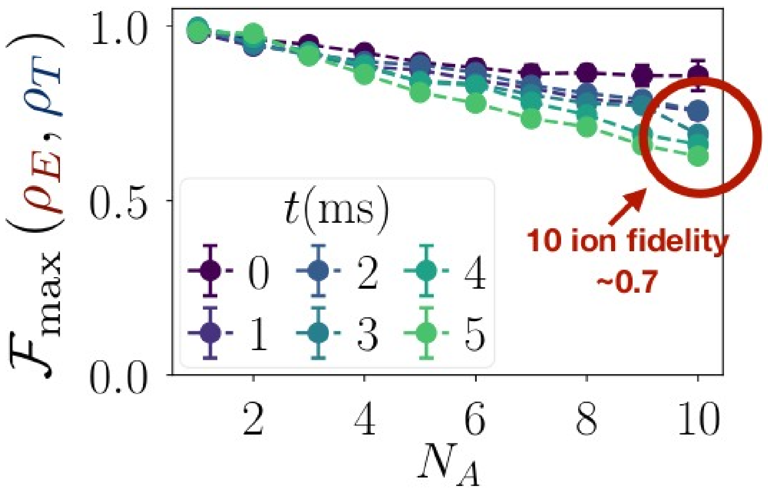
Experiment ρ_E



Many-body Quench dynamics within the long-range XY-model

Theory ρ_T

Simulation of the theoretical model



Number of spins

Data: Brydges, AE, et al., Science 2019

Number of unitaries: 500

Number of measurements per unitary: 150

Proof of principle

-

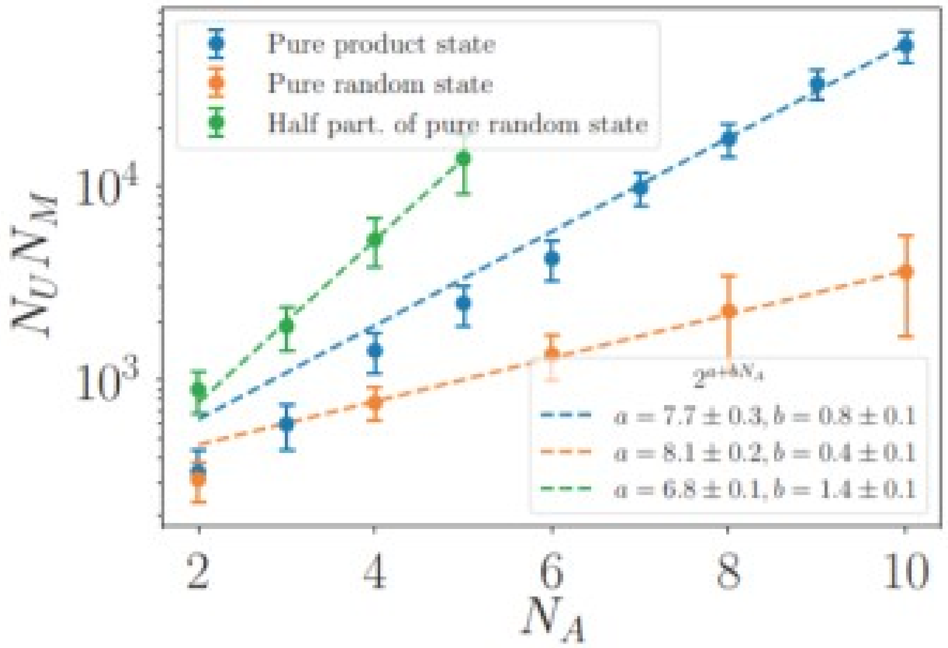
Expected to be applicable in a regime of 10 to 20 qubits

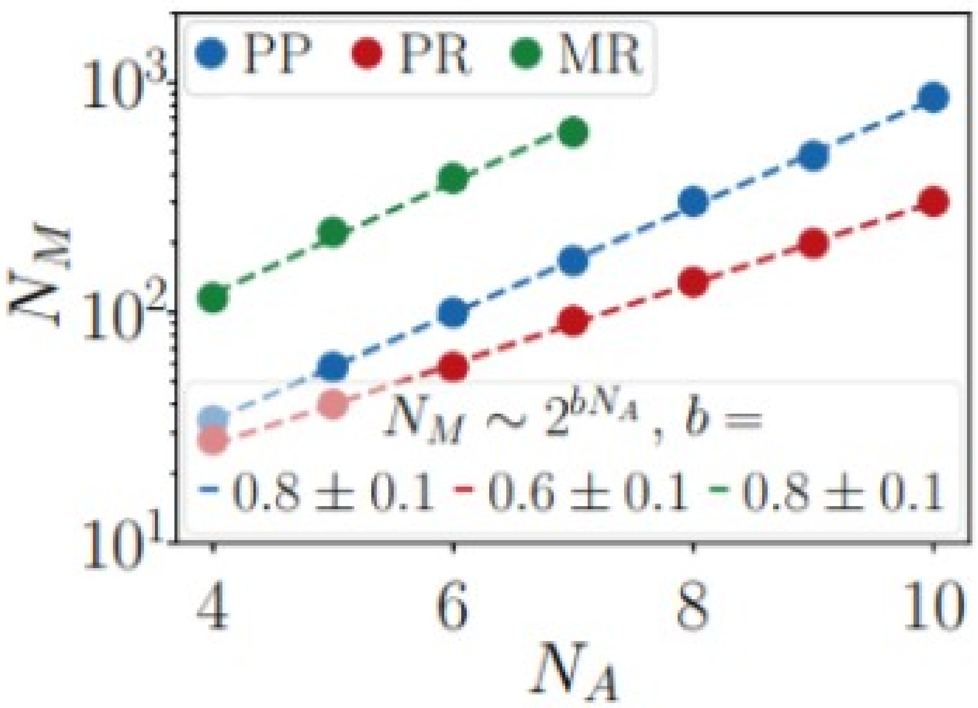
Statistical errors

N number of qubits

N_M number of measurements per unitary

N_U number of unitaries used





Quantum state tomography scales as: $\sim \mathbf{2^{2N}}$

Current protocol's measurement budget $(N_M N_U)$ to overcome statistical errors: $\sim \mathbf{2^{1N}}$

Can we estimate the purity more efficiently ?

Brydges et al Science 2019, Elben et al 2020

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- Context: Measuring entanglement

- Our method with randomized measurements

- **Optimized protocols using importance sampling**

A. Rath, R. van Bijnen, A. Elben, P. Zoller, B. Vermersch arXiv:2102.13524

Averaging over the CUE with uniform sampling (pdf = 1):

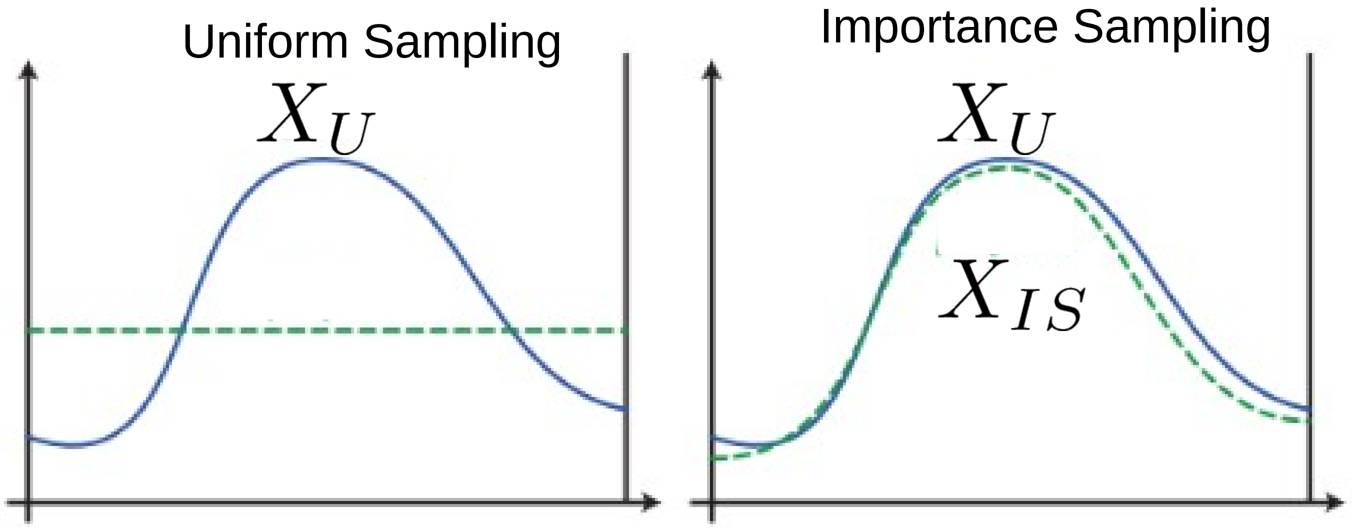
$$\mathrm{Tr}[\rho^2] = \overline{X_U} \equiv \int X_U\, P(\theta,\varphi) d\theta\, d\varphi \approx \mathbb{E}[X_U] = \frac{1}{N_U}\sum_{r=1}^{N_U} X^{(r)}$$

Randomized measurements

$2N$ angles for N qubits

Performing Importance sampling (pdf = $X_{IS} \propto X_U$) :

$$\mathrm{Tr}[\rho^2] = \int \frac{X_U}{X_{IS}} \times X_{IS}\, P(\theta,\varphi)\, d\theta\, d\varphi \approx \frac{1}{N_U} \times \mathbb{E}[X_{IS}] \times \sum_{r=1}^{N_U} \frac{X^{(r)}}{X_{IS}^{(r)}}$$

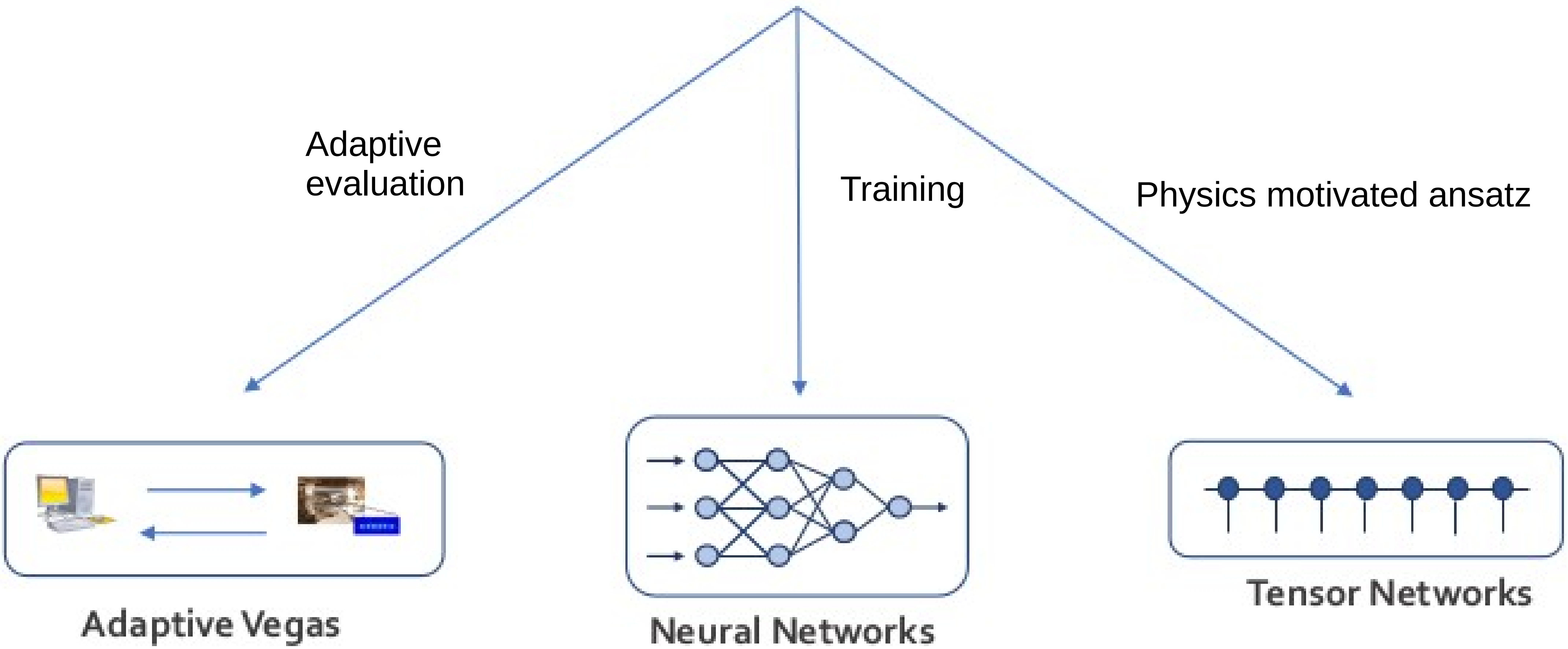


Standard deviation for the estimation of the purity (no shot noise)

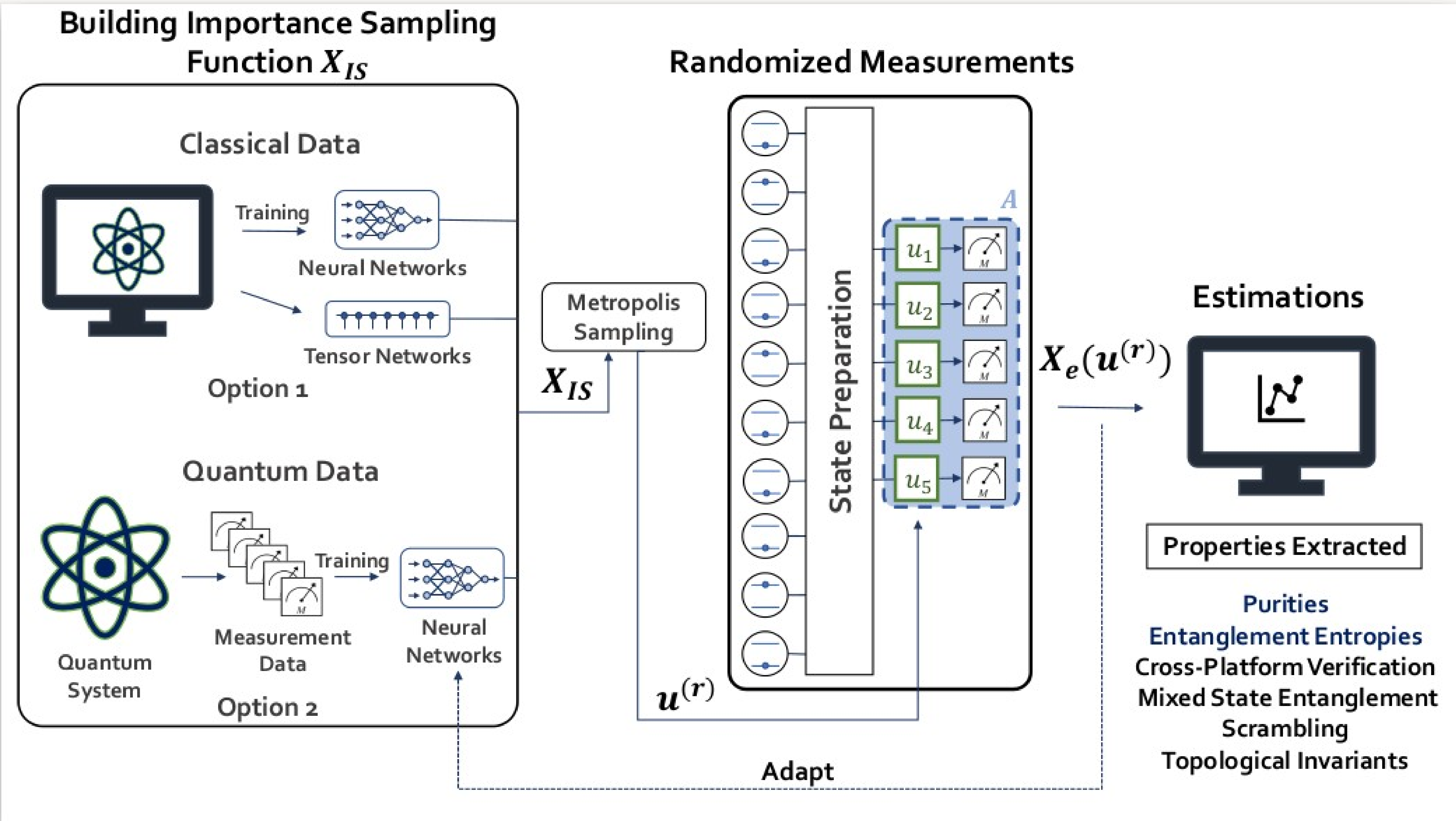
$$\mathrm{std}_{\mathrm{IS}}(X_U/X_{IS})/\sqrt{N_u}$$

How to build the Importance sampling probability distribution X_{IS} ?

Prior knowledge of the state (prior measurements or a Theory Model)



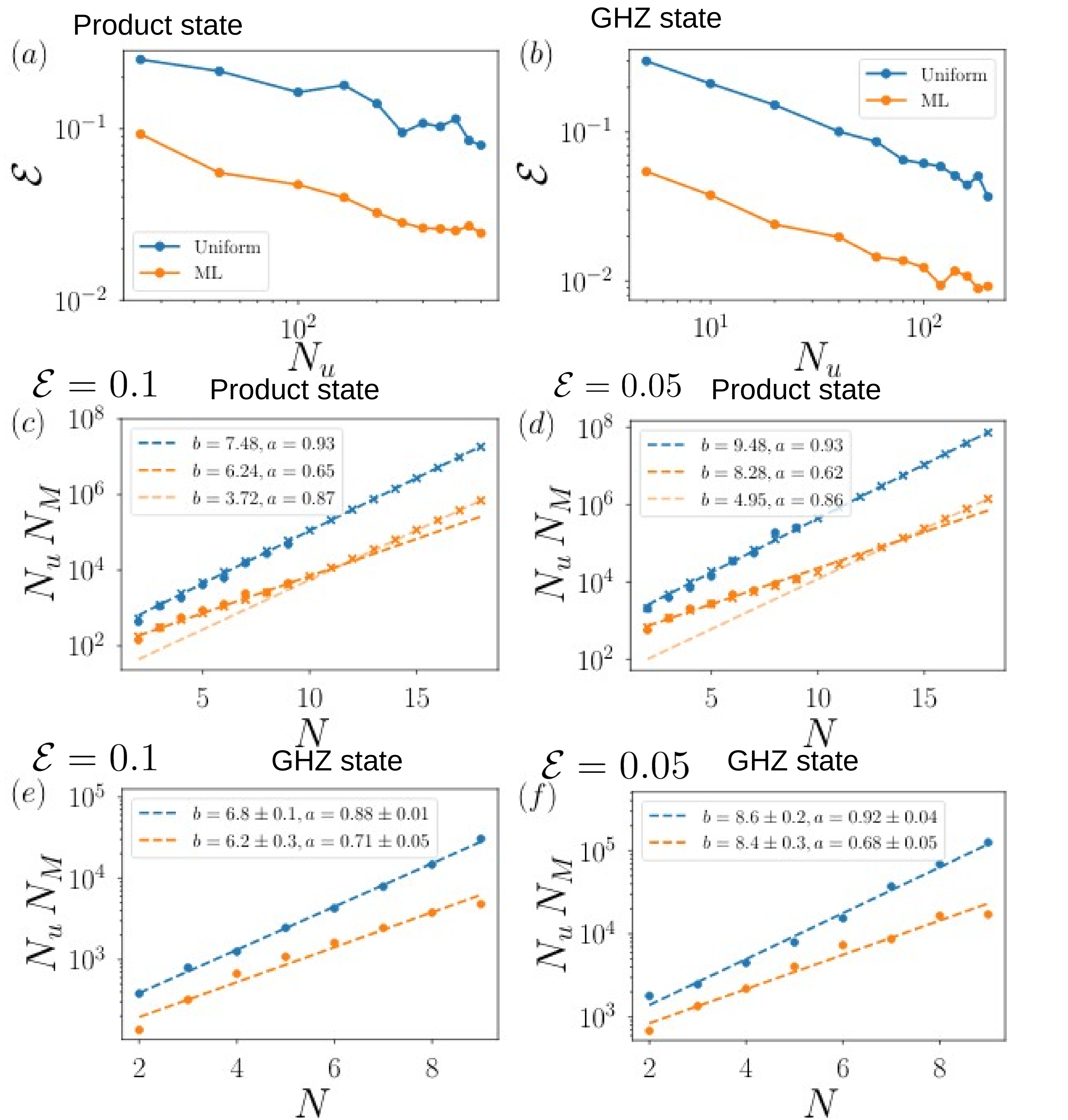
$$X_{IS}(u) = X_{DNN}(u) \qquad X_{IS}(u) = X_{\psi_{MPS}}(u)$$



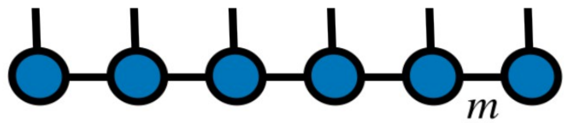
See also works for observables by R. Huang, R. Kueng, J. Preskill

Large NM (no shot noise)- Machine Learning Sampler

With shot noise - Perfect sampler



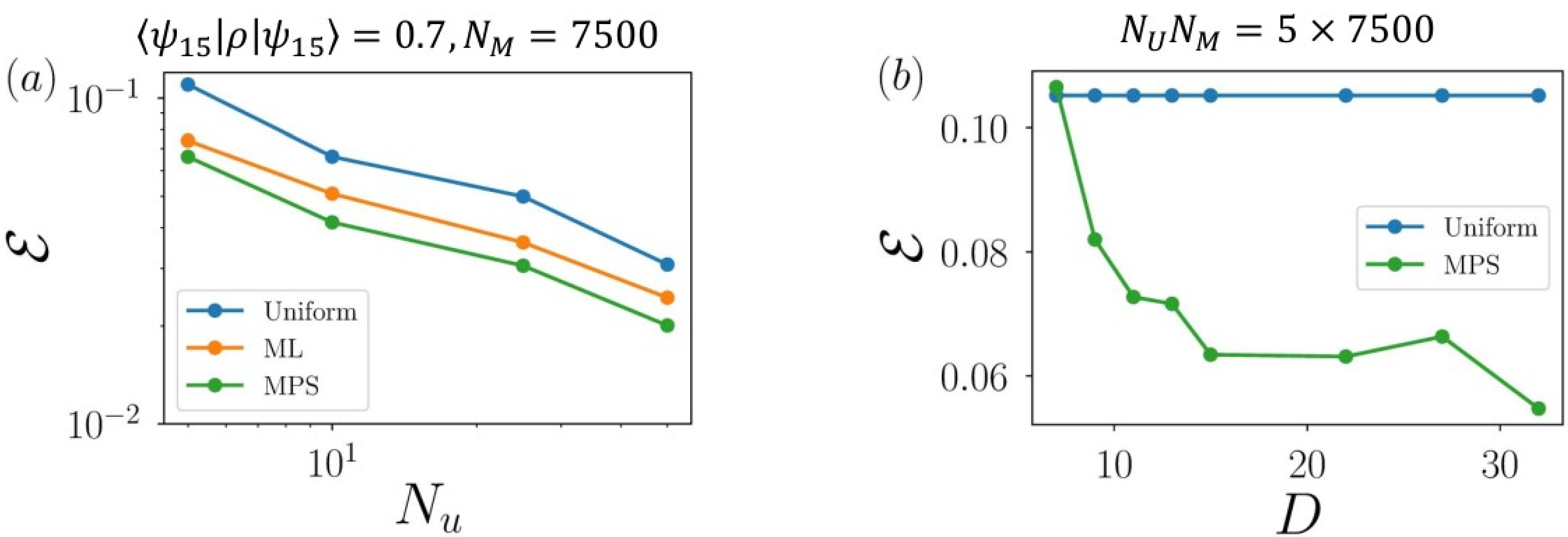
Sampling from a tensor-network approximation of the quantum state



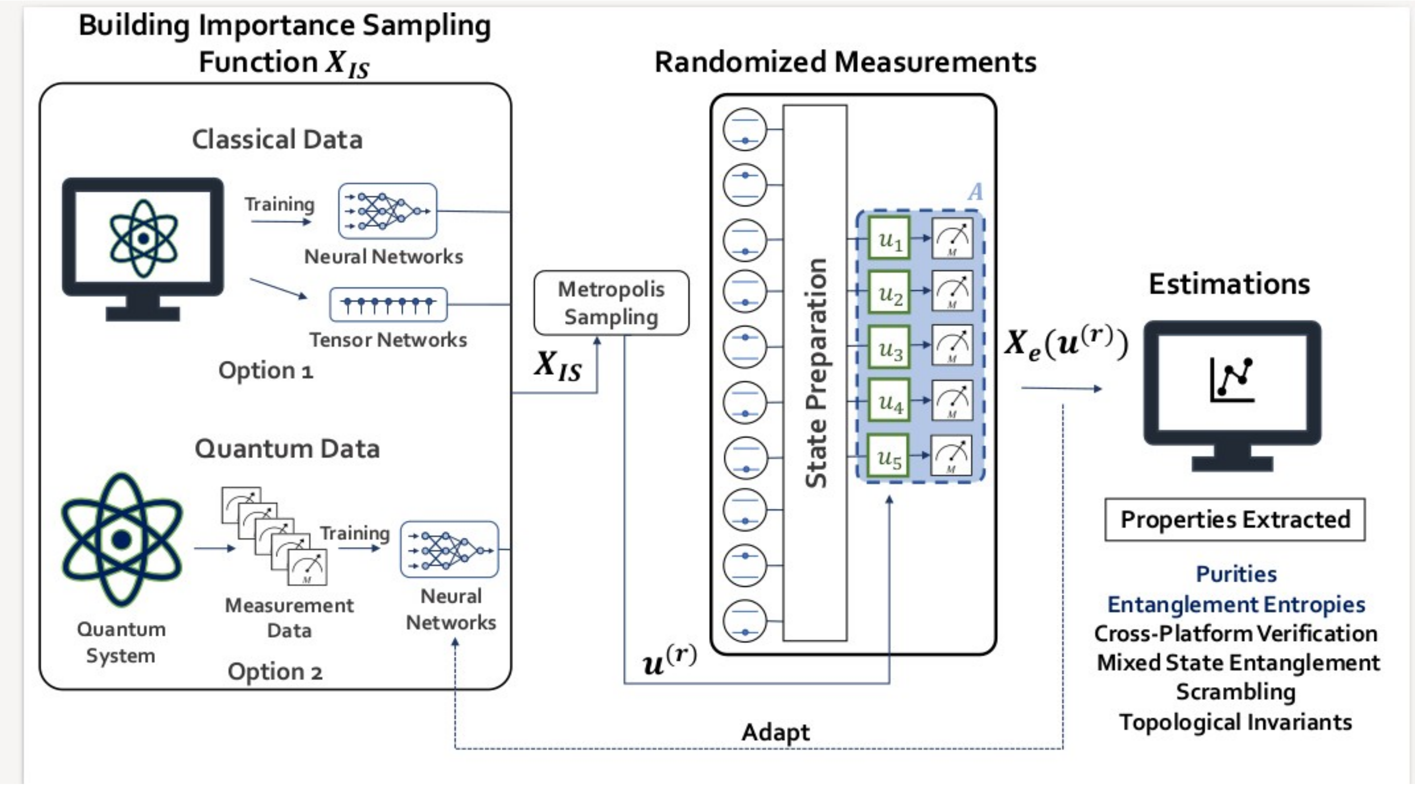
10- Qubit highly entangled state produced in a Quantum Simulation with $\mathbf{Tr}(\boldsymbol{\rho}^2) = \mathbf{0.62}$

Brydges et al, Science 2019

Importance sampling done from a pure MPS representation $|\boldsymbol{\psi_D}\rangle$ of $\boldsymbol{\rho}$



Accuracy of the MPS



- A new protocol using importance sampling of local random unitaries with improved performances for all states.
- Measurement budget $(N_M N_U)$ to overcome statistical errors for product and GHZ states: $\sim \mathbf{2^{0.6N}}$
- Investment of classical resources paying off in terms of reducing statistical errors in experiments.
- Applicable in most randomized measurement protocols giving access to different properties.

- Scaling up to 30-40 qubits possible by sampling from large scale MPS.

A. Rath, R. van Bijnen, A. Elben, P. Zoller, B. Vermersch   arXiv:2102.13524

The randomized measurement crew

M. Dalmonte, I. Cirac, R. Kueng, R. Huang, J. Preskill, B. Kraus. C. Kokail, R, van Bijnen, L. Sieberer, J. Carrasco, A. Neven, F. Pollmann, Z. P Cian, M. Hafezi, G. Zhu, J. Yu, H. Dehghani, M. Barkeshli,N. Yao, M. Joshi, T. Brydges, C.Maier, B. Lanyon, P. Jurcevic, C. Roos, R. Blatt, P. Calabrese, V. Vitale, C. Branciard, A. Minguzzi

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