Making randomized measurements a universal measurement toolbox for quantum technologies

Joint Harvard-Innsbruck Seminar

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LPMMC Grenoble & IQOQI Innsbruck

Outline

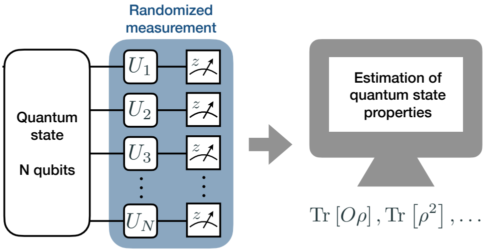
Presentation of Randomized measurements

Access new quantities: Spectral form factors

Extend system sizes: Importance sampling of randomized measurements

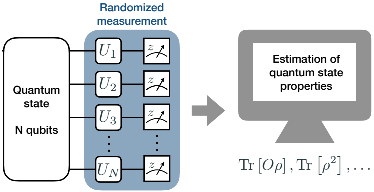
Adapting RM to new platforms: Rydberg atoms

Randomized Measurements (RM)



- Original RM proposal for the purity: van Enk and Beenakker (2012, Phys. Rev. Lett.)
- Protocol for qubits Elben *et al.* (2018, Phys. Rev. Lett.)
- Extended to many quantities: OTOCs, Topological invariants, Symmetry-Resolved Entropies, Entanglement negativites, . . .
- Recent development: Classical shadows formalism to access many local observables Huang *et al.* (2020, Nat. Phys.)

RM protocol for qubits Elben et al. (2018, Phys. Rev. Lett.)



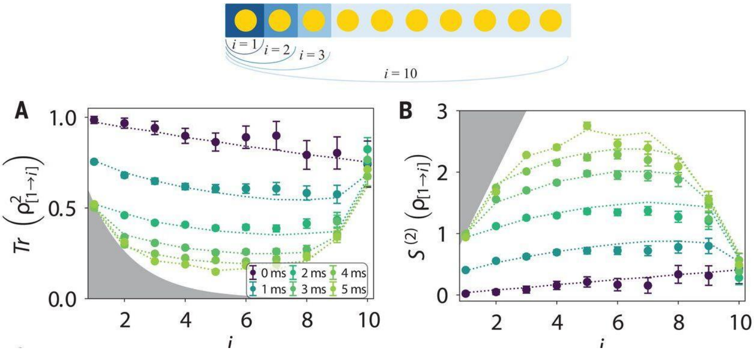
- Protocol: (i) apply random single qubit rotations, (ii) measure bitstrings, (iii) postprocess bitstrings:

$$\mathrm{Tr}(\rho^2) = 2^N E_u \Big[\sum_{s,s'} (-2)^{-D(s,s')} P_U(s) P_U(s') \Big] \quad P_U(s) = \langle s | u \rho u^\dagger | s \rangle$$

- State-agnostic estimation without reconstructing the state (i.e no tomography, no machine learning or matrix-product-state (MPS) ansatz)
- Cheap postprocessing of the measurement data (ie no fitting, etc)
- Info on the unitaries does not appear in the formula $\rightarrow$ estimations are *robust*.

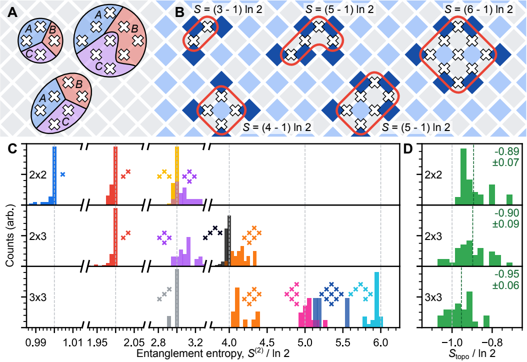
Entanglement probing with the purity

- Purity detects entanglement: $\mathrm{Tr}(\rho_A^2) < \mathrm{Tr}(\rho_{AB}^2) \to AB$ entangled.
- Purity quantifies entanglement for pure states via the second Rényi entropy $S_2(\rho_A) = -\log_2[\mathrm{Tr}(\rho_A^2)]$
- Demonstration with trapped ions Brydges *et al.* (2019, Science)



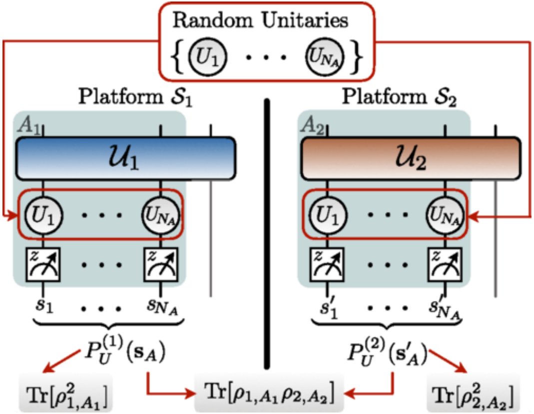
Entanglement probing with the purity (2)

- Purity detects universal features of quantum matter in and out-of equilibirum: quantum phase transitions, topology, many-body localization, etc
- Demonstration of intrinsic topological order in the toric code with Google's Sycamore: Satzinger *et al.* (2021, arxiv)

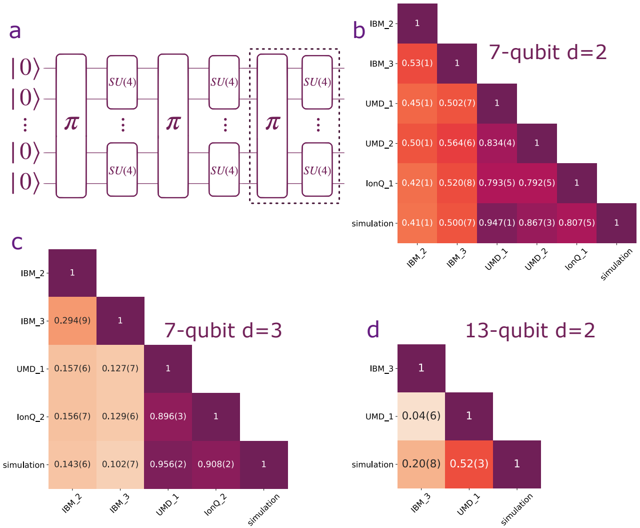


Cross-platform verification of quantum computers

- Proposals theory-experiment: Flammia and Liu (2011, Phys. Rev. Lett.)
- Proposal experiment-experiment: Elben *et al.* (2020, Phys. Rev. Lett.)



Demonstration: trapped ions versus superconducting qubits Zhu et al. (2021, arxiv)



Today's talk: making RM a universal measurement toolbox

- Three aspects
  - Try to access any *physical quantity* of interest w.r.t quantum computing and quantum simulation.
  - *Extend* the range of system sizes where RM can be applied: From 13 qubits to $25 - 30$ qubits (without fitting models).
  - *Adapt* the protocols, sampling strategies and error mitigation, to the constraints of each physical platform.
- Practical interest for experiments but also important conceptual questions: what limits our abilities to probe quantum systems?

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Spectral form factors and RM

- Innsbruck-Maryland Collaboration: L. Joshi, A. Elben ($\rightarrow$ Caltech), A. Vikram, BV, V. Galitski, P. Zoller (Joshi *et al.*, 2021, arxiv:2106.15530)
- Motivation 1: we have a good experience with probing quantum states: what about time-evolution operators? $|\psi(t)\rangle = T(t)\,|\psi(0)\rangle$.
- Motivation 2: the spectral form factor is the central quantity in many-body quantum chaos

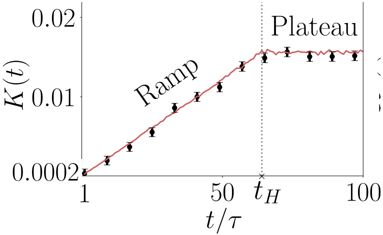
$$K(t) = \mathrm{Tr}[T(t)]\mathrm{Tr}[T^{\dagger}(t)]$$

Spectral form factors and quantum chaos

- The SFF probes level repulsion in quantum chaos (see eg. F. Haake's book), e.g with $T(t) = \sum_m e^{-i\epsilon_m t} |\epsilon_m\rangle \langle\epsilon_m|$:

$$K(t) = \sum_{m,n} e^{-i(\epsilon_m - \epsilon_n)t}$$

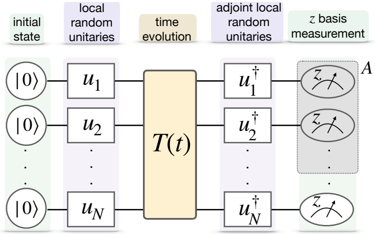
- For a chaotic evolution, universal ramp-plateau structure



- Probe also aspects of *many-body* quantum chaos: Eigenstate thermalization Hypothesis, Many-body localization, etc

Randomized measurements of the SFF

- Approach 1: Quantum Process Tomography: Reconstruct the unitary $T(t)$ matrix from information complete measurements with ϵ accuracy. Cost: $\sim 32^{N_A}\ldots$
- Approach 2: Ancilla-based (D. Vasilyev et al, PRX Quantum 2021)
- Approach 3: RM
  - Insight: We want to study how 'various' initial states evolved via $T(t)$.



  - Chaotic evolution: the RM should be 'featureless'— Non-Chaotic: the RM should be correlated around $|0\rangle^{\otimes N}$.
  - Note: such measurements are not information-complete

Connection between the RM and the SFF

- Statistical correlations between unitaries can be expressed analytically as

$$\mathbb{E}_U\left[(U^*\otimes U)(O^T\otimes\rho_0)(U^T\otimes U^\dagger)\right]=2^{-N}\left|\Phi_N^+\right\rangle\left\langle\Phi_N^+\right|,\quad \left|\Phi_N^+\right\rangle=2^{-N/2}\sum_s|\mathbf{s}\rangle\otimes|\mathbf{s}\rangle \tag{1}$$

with $O=(|0\rangle\langle 0|-\frac{1}{2}|1\rangle\langle 1|)^{\otimes N}$

- 'Surgery' on the quantity of interest

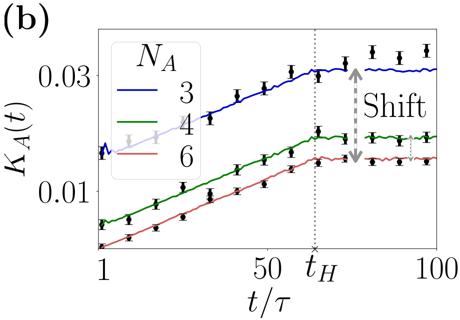
$$K(t)=\langle\Phi_N^+|\mathbf{1}\otimes T(t)|\Phi_N^+\rangle\langle\Phi_N^+|\mathbf{1}\otimes T^\dagger(t)|\Phi_N^+\rangle\,. \tag{2}$$

- Finally,

$$K(t)=\mathbb{E}_U\left[\mathrm{Tr}\,[O\,\underbrace{U^\dagger T(t)U\rho_0U^\dagger T^\dagger(t)U}_{\rho_f(t)}]\right] \tag{3}$$

Illustration

- Measurement cost: $\sim 10^N$
- The characteristic ramp can be seen with $N = 6$, $M = 10^5$ measurements in current devices!
- Useful information from reduced systems measurements: partial SFF $K_A(t)$.
- Error mitigation can be applied based on purity measurements, if needed



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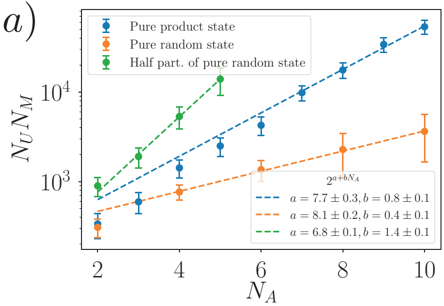
Statistical errors in RM protocols

- Main challenge for RM: statistical errors, with two crucial parameters who scale exponentially with system size
  - N_u number of applied unitaries
  - N_M number of measurement for each unitary



- Access to ~ 15 qubits, assumption free, cost in postprocessing: few seconds.

Importance sampling for probing entanglement

- Work with A. Rath, A. Elben, R. van Bijnen, and P. Zoller (arXiv:2102.13524 , PRL in press)
- Our goal: reduce the required $N_U N_M$ exponentially (in particular N_u) $\rightarrow$ access to $30 - 35$ qubits.
- Why? Access universal regimes for entanglement: scaling laws, central charge, topological entropy, etc - Assess fundamental limits about measurements
- Our idea:
    - Importance Sampling: we use/learn information about the state *before* we measure
    - We *still* have unbiased estimators, i.e assumption-free, and cheap data postprocesing). We simply boost the convergence w.r.t statistical errors.

Importance sampling for probing entanglement: The basic idea

- Interpet RM as the evaluation of an integral

$$\mathrm{Tr}(\rho^2) = 2^N \int dU \left[\sum_{s,s'} (-2)^{-D(s,s')} P_U(s) P_U(s') \right]$$

- Consider a 'well-chosen' probability distribution, instead of the uniform one

$$\mathrm{Tr}(\rho^2) = 2^N \int p_{\mathrm{IS}}(U) dU \left[\frac{\sum_{s,s'} (-2)^{-D(s,s')} P_U(s) P_U(s')}{p_{\mathrm{IS}}(U)} \right]$$

- The unitaries u are sampled according to $p_{\mathrm{IS}}(U)$, this will change the convergence properties of the integral evaluation with finite number of samples, i.e measurements.
- The experimental and the postprocessing task remain unchanged.

Importance sampling for probing entanglement: A concrete example

- Instead of the ideal pure N-qubit GHZ state $|\psi\rangle = (|0\rangle^{\otimes N} + |1\rangle^{\otimes N})/\sqrt{2}$, we realize a mixed-state version ρ of $|\psi\rangle$.
- To measure the purity of ρ, define the importance sampler

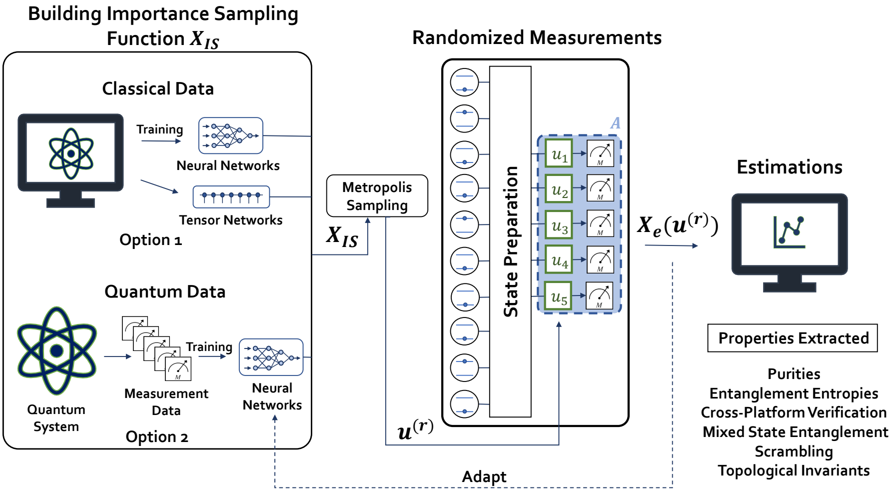
$$p_{\mathrm{IS}}(U) = 2^N \left[\sum_{s,s'} (-2)^{-D(s,s')} P_U^{(\psi)}(s) P_U^{(\psi)}(s') \right] \tag{4}$$

- Sample N_u unitaries according to $p_{\mathrm{IS}}(U)$ and estimate

$$[\mathrm{Tr}(\rho^2)]_e = \sum_U \left[\frac{\sum_{s,s'} (-2)^{-D(s,s')} P_U(s) P_U(s')}{p_{\mathrm{IS}}(U)} \right] \tag{5}$$

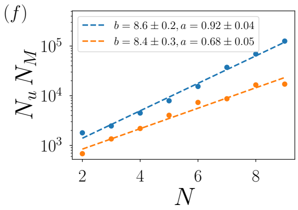
- As $\rho \approx |\psi\rangle \langle\psi|$, the integrand has been 'flattened'
  - Exponential reduction of the required number of unitaries N_u.
  - The effect of 'shot noise' exponentially reduced for well-conditioned states.

The full protocol Rath et al. (2021, arxiv:2102.13524)



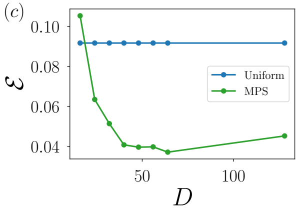
Performances Rath et al. (2021, arxiv:2102.13524)

Example: GHZ states



→ Exponential reduction of the number of measurements, with $N_u = O(1)$.

Example: 20 *qubit* many-body entangled states, 10 qubits sampled via MPS



→ Better approximations lead to better performances

- Also tested on topological entropy of 2D topological ground states.
- Tutorial and python scripts: `https://github.com/bvermersch/RandomMeas`

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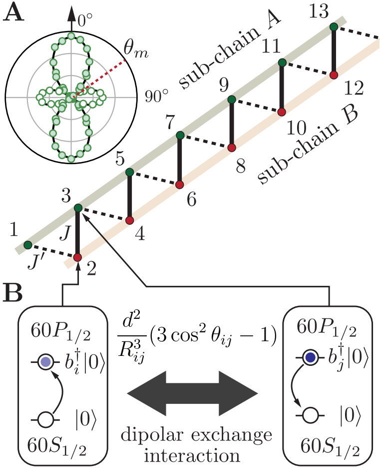
RM with Rydberg atoms

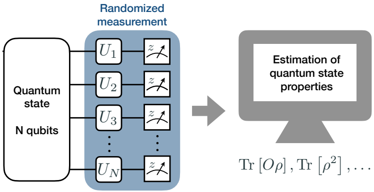
- Rydberg atoms:
  - A scalable platform with strongly interacting qubits
  - Dipolar & van der Waals interactions → quantum magnetism, spin liquids, lattice gauge theories, topology, classical/quantum optimization.
- Being able to apply RM would allow:
  - benchmarking, cross-platform comparison with superconducting qubits and trapped ions.
  - measure non-local order parameters (entanglement entropies, topological invariants, etc)
  - Hamiltonian $\langle H \rangle$ cost functions in quantum optimization

de Léséleuc et al, Science 2019



RM with Rydberg atoms

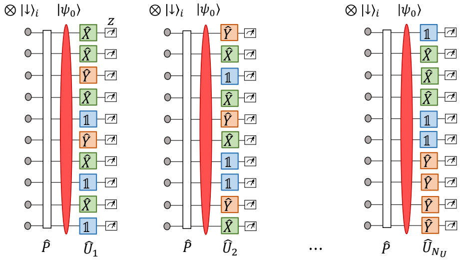
- Problem for RM: the Rydberg interations are always ‘on’. How to apply local unitaries?



  - Approach 1: Map to hyperfine ground state→possible interacting induced errors during the mapping.
  - Approach 2: Realize quasi-local unitaries via quantum quenches Elben *et al.* (2018); Hu *et al.* (2021)→ generating procedure is system-dependent.
  - Approach 3: ‘Fast’ generation of local unitaries in presence of residual interactions.

The Rydberg RM protocol

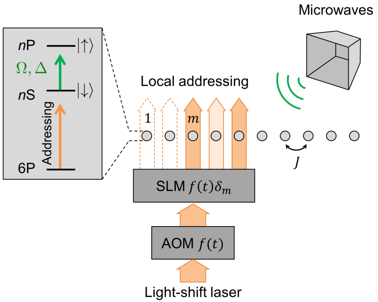
- Work with S. Notarnicola, A. Elben, S. Montangero, T. Lahaye, A. Browaeys, and P. Zoller (to appear soon on arxiv).



- Idea: Generate with optical and MW controls X, Y and Z measurements
- Single qubit Clifford measurements are sufficient to apply randomized measuremnents Huang *et al.* (2020), with similar required N_u.

The Rydberg RM protocol

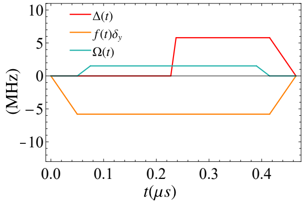
- How to realize three measurement settings as fast as possible, i.e faster w.r.t the interaction timescales?
- Each qubit is subject to X, Y, Z measurement
  - A fixed disorder pattern from a SLM induces a light shift $\delta_{X,Y,Z}$
  - MW drives put sequentially the group X, Y, Z on resonance and realizes the required rotation.
- The number of controls does not increase with qubit number.



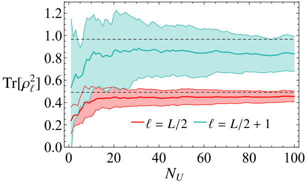
The Rydberg RM protocol

- Case study: a XY model with Rydberg s and p levels
- The required time scales match the experimental possibilities
- Protocol tested on the purity and Hamiltonian variance $\langle H^2 \rangle$ estimations.
- If needed, measurement error mitigation can be applied in RM protocols.

Realizing a Y measurement



Purity estimated – SSH model



Conclusion

- RM become more well suited to probe large synthetic quantum systems.
  - Remarkable ability to treat large datasets in a state-agnostic way and to extract key quantities (e.g SFF, quantum Fisher information Rath et al, arxiv:2105.13164)
  - Can be coupled to other measurement protocols: eg. Entanglement Hamiltonian Tomography (Kokail et al, Nature Physics 2021), Machine-Learning methods for importance sampling.
  - Copes well with the specificities of each experimental platform

- Thank you for your attention!

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