

# Randomized measurements for large-scale quantum experiments

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Quobly, and LPMMC Université Grenoble Alpes (on leave)

UTC Quantum Workshop at Chattanooga, Tennessee

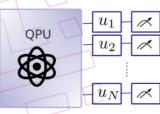
## **Quobly was launched in November 2022**



## Quantum Information Team @ Quobly

Measurements & Benchmarking







**Tensor-Network Simulations** & quantum algorithms R&D (open-source)



#### Postprocessing

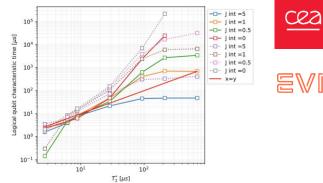


Observables, Entanglement, Fidelity, XEB Fidelity, Tomography, etc



Large scale compilation, Tensor quantum programming, Qubitization 111111111111

#### Architectures and quantum error correction Surface-17 code



**Internal Simulator** 

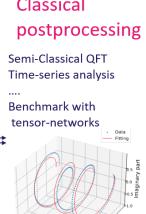
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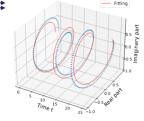
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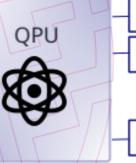
## Classical

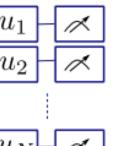




## Randomized measurements for large-scale experiments

Randomized measurements







P. Zoller (UIBK) A. Elben (Psi)





J. Preskill, R. Huang (Caltech) R. Kueng (Linz) J.I.C Cirac (MPQ B. Kraus (TUM)

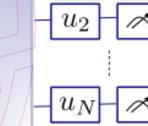


M. Serbyn. M. Ljubotina (ISTA)

M. Votto, W. Lam (UGA)

V. Vitale (Pasqal)

A. Rath (IQM)







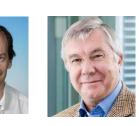


Xiao Mi (Google)





T. Brydges, M. Joshi,



C. Roos, R. Blatt (UIBK) And many more...









P.Jurcevic (IBM)

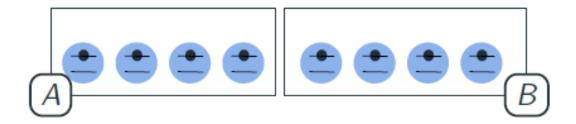






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## Motivation: Entanglement, a central quantity in quantum computing & quantum simulation



Purity

 $\operatorname{Tr}(\rho_{\Delta}^2)$ 

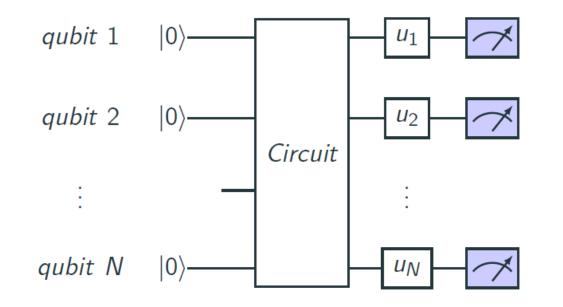
Entanglement is quantified via Entropies

$$S_{\rm vN} = -\text{Tr}[\rho_A \log(\rho_A)]$$
 von Neumann Entropies  
 $S_\alpha = \frac{1}{1-\alpha} \log[\text{Tr}(\rho_A^\alpha)]$  Rényi entropies

Entropies (related quantities) central to describe quantum computational complexity, and for understanding quantum simulation experiments (Eisert RMP 2010)



## Randomized measurements: A single data acquisition procedure



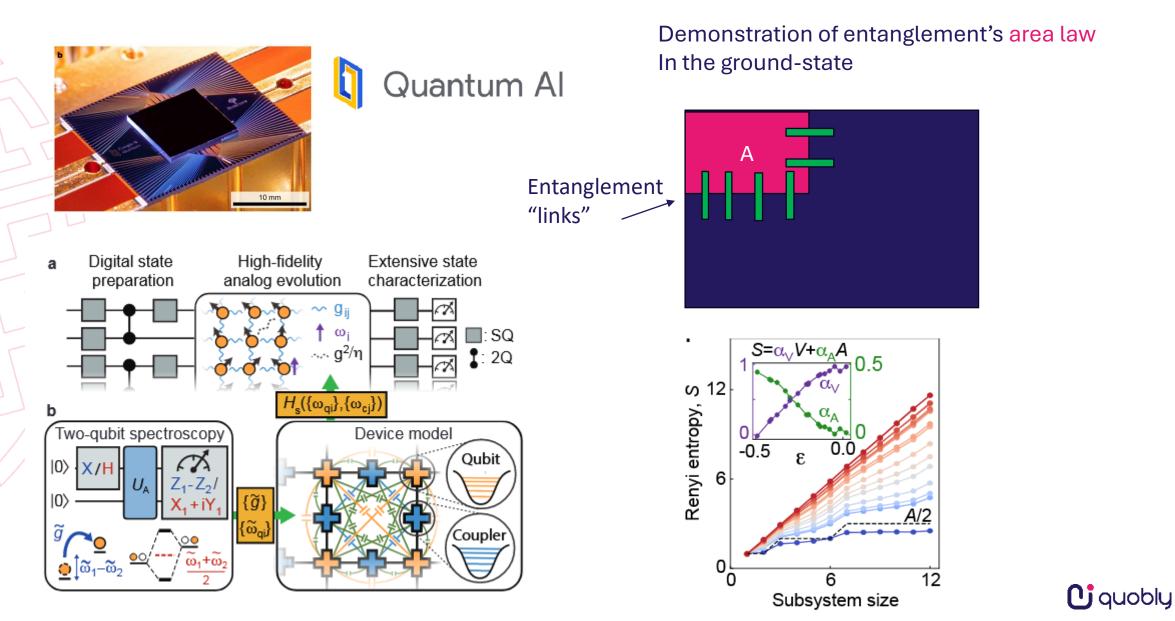
- Randomized measurements: We measure  $P_u(s) = \langle s | u \rho u^{\dagger} | s \rangle$ ,  $u = u_1 \otimes \cdots \otimes u_N$ .
- *u<sub>i</sub>* chosen independently from the circular unitary ensemble (CUE)
- We extract quantities of interest from the statistics of P<sub>u</sub>(s), over random unitary transformations.

For example, the purity formula (Elben, BV, et al, PRL 2018)

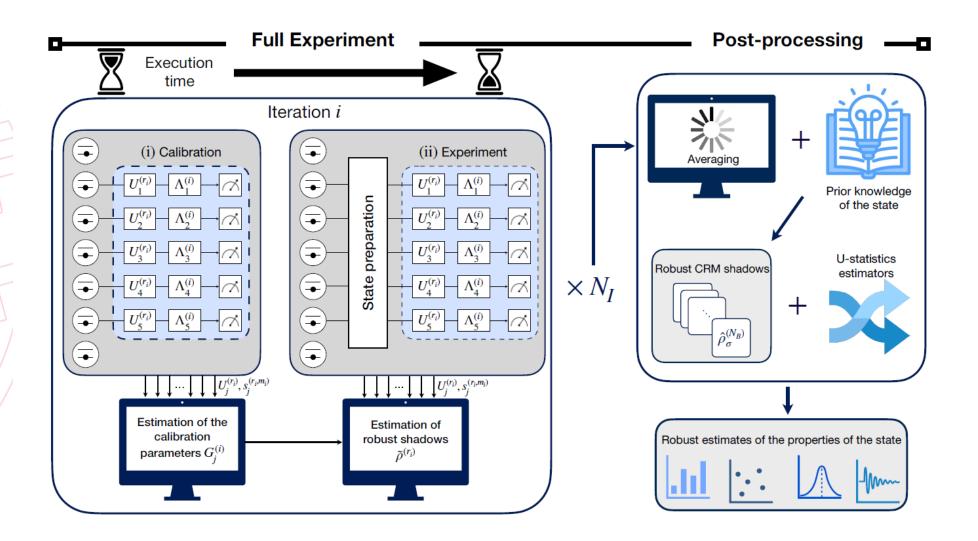
$$\mathrm{Tr}(\rho^2) = 2^N E_u \Big[ \sum_{s,s'} (-2)^{-D(s,s')} P_u(s) P_u(s') \Big]$$



## Recent use of RM: Andersen et al, Nature 2025



## Experimental Robust Shadow Estimation (Vitale et al, PRX Q 2024)





## Beyond the purity: Classical Shadows as the modern framework to postprocess randomized measurements

Data are processed 'robust classical shadows' (Chen et al, PRX 2021, improving the seminal Caltech paper: Nature Physics 2020)

$$\tilde{\rho}^{(r,b)} = \bigotimes_{j=1}^{N} \left( \frac{3}{2F_{b}[j] - 1} U_{j}^{(r)^{\dagger}} |s_{j}\rangle \langle s_{j}| U_{j}^{(r)} + \frac{F_{z}[j] - 2}{2F_{b}[j] - 1} \mathbf{1} \right),$$

where the calibration data of each batch *b* gives access to  $F_b[j]$ . Estimations of functions of  $\rho$  are built based on the relation  $E[\tilde{\rho}^{(r,b)}] = \rho$ 

$$\operatorname{tr}[\rho O] \simeq \frac{1}{N_u} \sum_r \operatorname{tr}[\hat{\rho}^{(r)}O], \ \operatorname{tr}[\rho^2] \simeq \frac{1}{N_u(N_u-1)} \sum_{r_1 \neq r_2} \operatorname{tr}[\hat{\rho}^{(r_1)}\hat{\rho}^{(r_2)}]$$



(2)

#### Some recent uses of randomized measurements to access entanglement

Reference

Brydges et al Science 2019

Joshi et al, PRL 2020

Dong et al, PRL 2025

Elben et al, PRL 2020

Elben et al, PRL 2020

Zhu et al, Nature Comm 2022

Satzinger al, Science 2021

Vitale et al, Sci Post 2022

Stricker et al, PRX Q 2022

Rath et al, PRX Q 2023

Joshi et al, PRL 2024

16, 2943 (2025)

Vitale et al, PRX Quantum 2024

Hu et al, Nature Communications

Andersen et al, Nature 2025

Votto et al, in preparation

Quantities/Concepts	Platform
Entropies	lons
OTOCs	lons
Spectral Form Factors	Superconducting qubits
Mixed-state entanglement*	lons
Cross-Platform verification*	Ions & superconducting qubits
Topological entropies	Superconducting qubits
SR Entropies*	lons
Entropies (live)	lons
Operator entropies*	lons
Quantum Mpemba effect	lons
Quantum Fisher information (Robust):	Superconducting qubits
Entropies (2D)	Superconducting qubits
Entropies (Robust)	Superconducting qubits
Entropies(large-scale) & tomography	Superconducting qubits

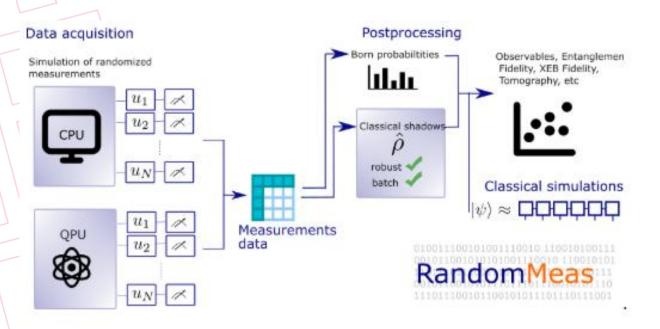
10 \* : Data from Brydges et al

## RandomMeas.jl: Open-Source Package

RandomMeas: The randomized measurement toolbox in Julia

#### docs dev 💭 CI passing License Apache 2.0

This package provides efficient routines for sampling, simulating, and post-processing randomized measurements, including classical shadows, to extract properties of many-body quantum states and processes. RandomMeas relies heavily on <u>ITensors.jl</u>.



https://github.com/bvermersch/RandomMeas.jl & Julia's General Registry

#### 

#### **Classical shadows**

- 1. Energy/Energy variance measurements with classical shadows
- 2. Robust Shadow tomography
- 3. Process Shadow tomography
- 4. Classical shadows with shallow circuits
- 5. Virtual distillation

A. Elben (Psi)

#### Quantum benchmark

- 6. Cross-Entropy/Self-Cross entropy benchmarking
- 7. Fidelities from common randomized measurements
- 8. Cross-Platform verification

#### Entanglement

- 9. Entanglement entropy of pure states"
- 10. Analyzing the experimental data of Brydges et al, Science 2019
- 11. Surface code and the measurement of the topological entanglement entropy
- 12. Mixed-state entanglement: The p3-PPT condition and batch shadows

#### Miscellanous

13. Noisy circuit simulations with tensor networks



## Summary first part & transition

Many physical quantities have been recently measured with RM: Experimentally friendly acquisition, and numerically friendly "universal" postprocessing.

Robustness aspects well understood and experimentally demonstrated

For the purity and related quantities, number of measurements scales as  $2^{aN}$ , a  $\approx 1$ , implies typically N <14

More info in our review **"The randomized measurement toolbox"** Elben, Flammia, Huang, Kueng, Preskill, BV, Zoller, Nat Rev Phys 2023

Let's expand this toolbox to large-scale systems reconstructions!



## Large-scale entropies and tomography via randomized measurements

Probing many-body quantum states in large-scale experiments with randomized measurements, BV et al, Phys. Rev. X 2025

Learning mixed quantums in large-scale experiments (to appear on arxiv)





P. Zoller (UIBK) L. Piroli (Unibo)



M. Serbyn. M. Ljubotina (ISTA)



J.I.C Cirac (MPQ

#### **Connections to MPS/MPO Gibbs State Tomography** Litterature

Baumgratz et al, NJP 2013 Torlai et al, Nature Comm 2023

Anshu et al, Nature Physics 2021 (Review) Joshi et al, Nature 2023

...



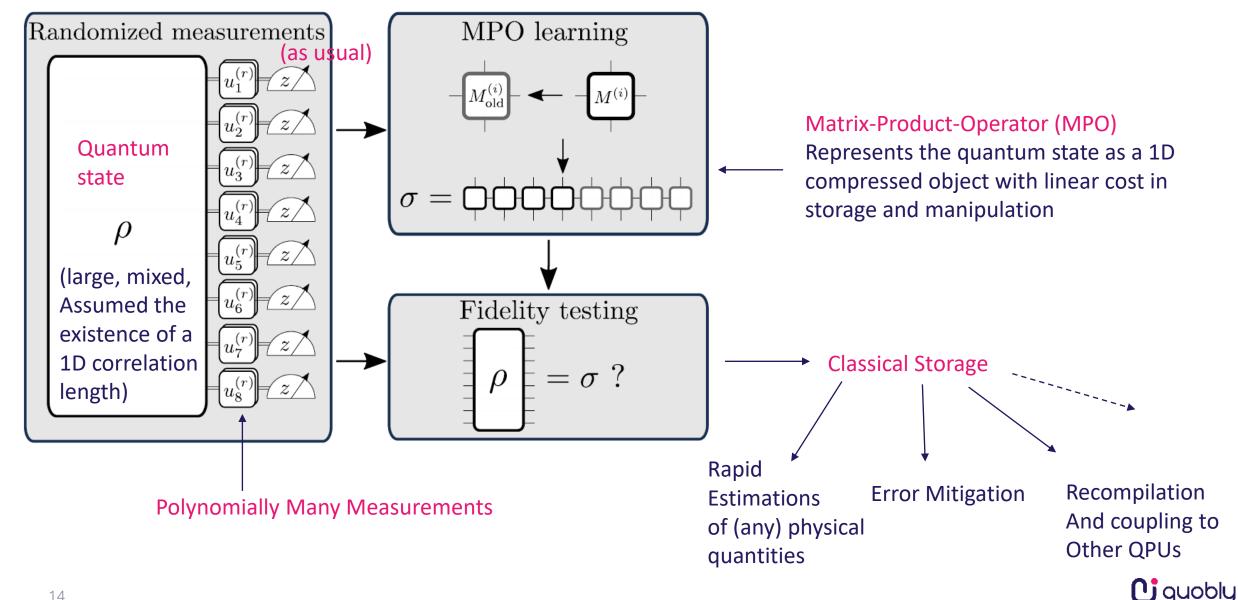
M. Votto (UGA)



C. Lancien (UGA)

**Pi** quoblu

### Learning mixed quantums in large-scale experiments: General picture & usage



## Learning mixed quantums in large-scale experiments: Technical statements (boring but needed...)

We assume the existence of a finite correlation length in the MPO framework. This implies the approximate factorization condition (Vermersch al Phys. Rev. X 2024)

$$\left| \operatorname{tr}[\rho_{ABC}^2]^{-1} \frac{\operatorname{tr}[\rho_{AB}^2] \operatorname{tr}[\rho_{BC}^2]}{\operatorname{tr}[\rho_B^2]} - 1 \right| \le \alpha e^{-|B|/\xi_{\rho}^{(2)}}$$

This assumption is satisfied in 1D quantum circuits, quantum Gibbs states (Capel et al, arxiv: 2024 in particular.

Votto et al (arxiv:..) : In this case, one can learn the state using the inner product as cost function

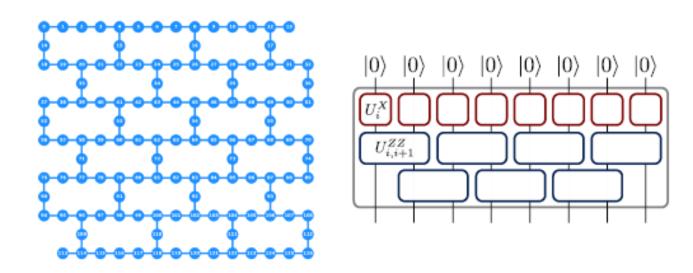
$$\mathcal{F}_{\rm GM}(\rho,\sigma) = \frac{\mathrm{tr}[\rho\sigma]}{\sqrt{\mathrm{tr}[\rho^2\sigma^2]}} \quad \text{With the MPO} \quad \sigma = \sum_{\{s_j\},\{s'_j\}} M^{(1)}_{s_1,s'_1} M^{(2)}_{s_2,s'_2} \dots M^{(N)}_{s_N,s'_N} \left|\{s_j\}\right\rangle \left\langle\{s'_j\}\right\rangle$$

With arbitrary small total error and polynomially many measurements

Proof idea: Local gradient updates can be estimated faithfully from reduced classical shadows



Learning mixed quantums in large-scale experiments: Demonstration



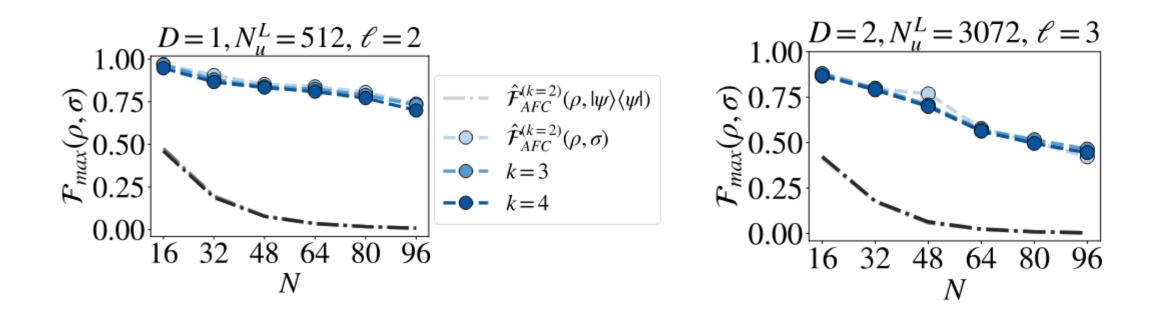
 Experimental setup: superconducting quantum processor (IBM Brisbane QPU), N<sub>tot</sub> = 96 qubits, depth D = 1,2 kicked Ising model

$$U_i^X = e^{-i\pi X_i/8}, \ U_i^{ZZ} = e^{i\pi Z_i Z_{i+1}/4}$$

• We perform  $M = N_u \times N_M = D \cdot 3072 \times 1024$  measurements



### Learning mixed quantums in large-scale experiments: Demonstration

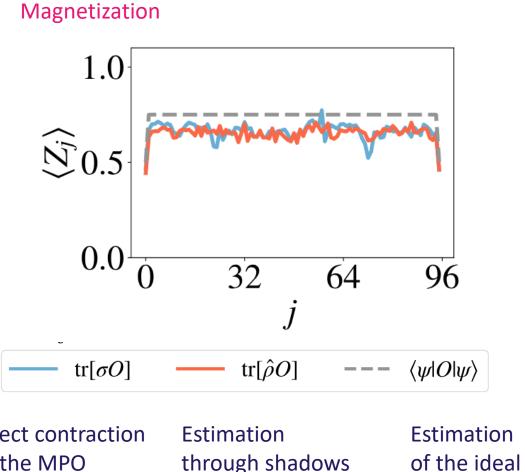


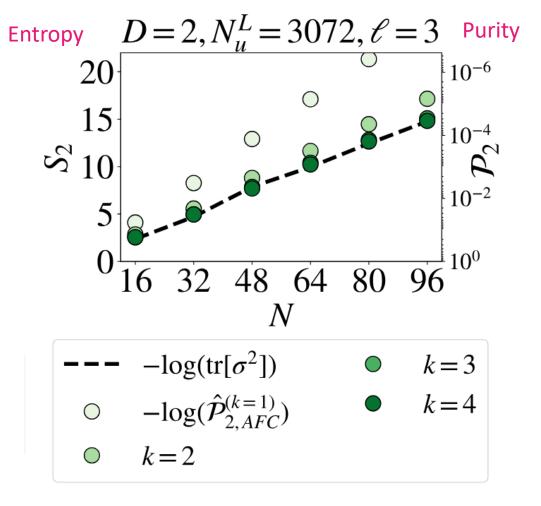
Experimental tomography of a N = 96 mixed qubit state (previous results: N = 20 qubits, MPS (Kurmapu et al PRXQ 2023) or Gibbs states (Joshi et al, Nature 2023)

<sup>17</sup> The MPO  $\sigma$  captures the noisy features of the experiment.



### Learning mixed quantums in large-scale experiments: Extracting physical quantities





**C** auobly

Direct contraction Of the MPO

through shadows (lenghty calculation) Pure state

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### Learning mixed quantums in large-scale experiments: Applications

#### Quantum Error Mitigation:

Based on a noisy experimental device, estimation of noiseless observable estimation values based on noise extrapolation, noise models & extra sampling, virtual distillation, etc (Cai et al, RMP 2023)

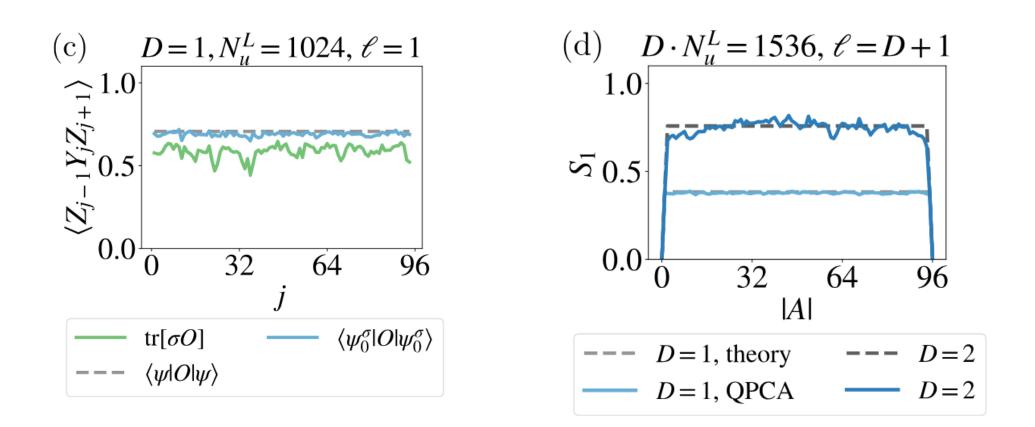
$$|\psi\rangle = U |\psi_0\rangle \longrightarrow \rho = \mathcal{U}(\gamma)\rho_0 \longrightarrow \langle O \rangle = \langle \psi | O | \psi \rangle$$

Here: Quantum Principal Component analysis (stronger version of virtual distillation) via the DMRG algorithm of tensor-networks (Review: Schollwoeck, Annals of Physics, 2011)

$$\rho = \sum_{a} \Lambda_{a} |\psi_{a}^{\rho}\rangle \langle \psi_{a}^{\rho}|, \text{ where } \Lambda_{a} > \Lambda_{a+1}$$

We search classically for the ground state of  $H = -\sigma$ 

Learning mixed quantums in large-scale experiments: Applications



Large-scale resconstruction of noiseless expectation values, and von Neumann entanglement entropies (Despite the original exponentially small original mixed state)



### Thank you

#### The Quantum Information Team at Quobly



**Benoît Vermersch** 



**Thibaud Louvet** 

Nathan Miscopein

#### **Carlos Ramos-Marimón**



**Dimitri Lanier** 

Amara Keita

Contact me if you want to know more, or join!











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L. Piroli (Unibo)

Learning mixed quantums in large-scale experiments (to appear on arxiv)



M. Votto (U



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ancien (UGA)

## Quobly was born from the combined expertise of CEA and CNRS at Grenoble

