Probing entanglement in quantum processors with the randomized measurements toolbox

GDR Meso Aussois

Benoît Vermersch

Nov 30 2022

LPMMC Grenoble & IQOQI Innsbruck

Outline

Entanglement versus quantum computers

The randomized measurement toolbox for measuring entanglement

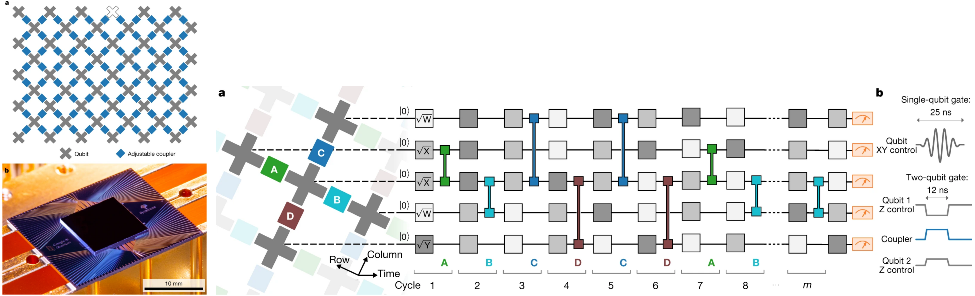
Outline

Entanglement versus quantum computers

The randomized measurement toolbox for measuring entanglement

What is a quantum computer?

- The Sycamore chip (Image: Google AI)



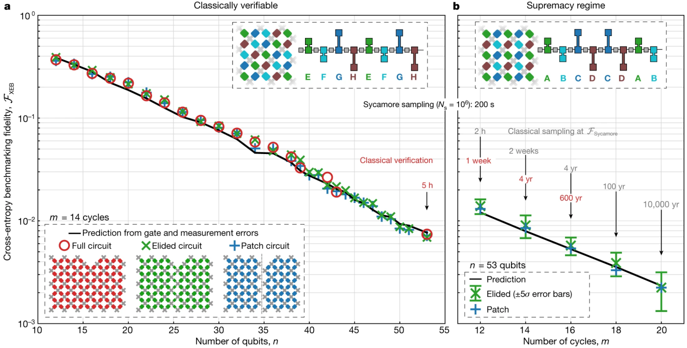
- Any N-qubit state (dim 2^N) can be created (Deutsch 1989).

$$|\psi\rangle = \sum_{s_1,\ldots,s_N} c_{s_1,\ldots,s_N} |s_1\rangle \otimes \cdots \otimes |s_N\rangle \tag{1}$$

- Quantum algorithms are designed to solve classical problems or quantum problems

Noisy quantum computers

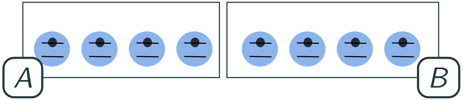
- Error propagation is typically exponential in size and time (Image: Google AI)



- Time is for benchmarks: check the 'quantum aspects' of a quantum computer
- Time is also for quantum simulation: understand a quantum problem using a quantum computer (ground-state physics, non-equilibrium problem, etc)

Entanglement

- Take two parts of a quantum system A B (eg sets of qubits)



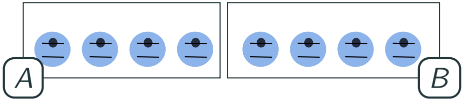
- A and B are entangled iff $|\psi\rangle \neq |\psi_A\rangle \otimes |\psi_B\rangle$
- Example with two qubits. The Bell state $|\psi\rangle = \frac{1}{\sqrt{2}}(|0\rangle \otimes |0\rangle + |1\rangle \otimes |1\rangle)$ is entangled

Entanglement in quantum computing

Entanglement is the most important concept in quantum information theory:

- All quantum algorithms involve entangled states.
- Quantum algorithms with a ‘low’ level of entanglement can be efficiently simulated with a classical computer, i.e. they are ‘useless’ (Eisert RMP 2010).

How to quantify entanglement?



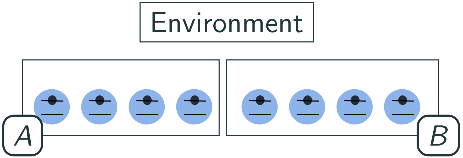
- For pure states, entanglement entropies of the reduced state $\rho_A = \mathrm{Tr}_B(|\psi\rangle\langle\psi|)$ are entanglement measures

$$\begin{aligned} S_{\mathrm{vN}} &= -\mathrm{Tr}[\rho_A \log(\rho_A)] \text{ von Neumann Entropies} \\ S_\alpha &= \frac{1}{1-\alpha} \log[\mathrm{Tr}(\rho_A^\alpha)] \text{ Rényi entropies} \end{aligned} \tag{2}$$

- Entanglement entropies measure quantum resources & quantum information (excellent notes from J. Preskill)
- Entanglement entropies distinguish quantum phases in quantum simulation (ex: RMP by J. Eisert)

Entanglement in noisy quantum computers

- Typically the state of a quantum computer is 'mixed' due to an environment



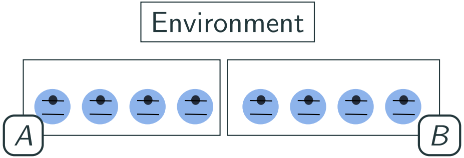
- We define the density matrix of the quantum computer

$$\rho_{AB} = \mathrm{Tr}_E(|\psi\rangle\langle\psi|) \tag{3}$$

- A and B are entangled iff

$$\rho_{AB} \neq \sum_i p_i \rho_A^{(i)} \otimes \rho_B^{(i)} \tag{4}$$

How to detect entanglement?



- Purity $\mathrm{Tr}(\rho^2)$ ($= 1$ iff the state is pure $\rho = |\psi\rangle \langle\psi|$)
- Purity entanglement condition (Horodecki 1996)

$$\mathrm{Tr}(\rho_A^2) < \mathrm{Tr}(\rho_{AB}^2) \implies A \text{ and } B \text{ are entangled} \tag{5}$$

- Example Bell State $|\psi\rangle = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)\,|0_E\rangle$

$$\begin{aligned} \rho_{AB} &= \frac{1}{2}(|00\rangle + |11\rangle)(\langle 00| + \langle 11|) \implies \mathrm{Tr}(\rho_{AB}^2) = 1 \\ \rho_A &= \frac{1}{2}(|0\rangle \langle 0| + |1\rangle \langle 1|) \implies \mathrm{Tr}(\rho_A^2) = 1/2 \end{aligned} \tag{6}$$

How to measure entanglement?

- The purity $\mathrm{Tr}(\rho^2)$ can be used to detect entanglement.
- The second Rényi entropy $S_2 = -\log_2[\mathrm{Tr}(\rho^2)]$ quantifies entanglement

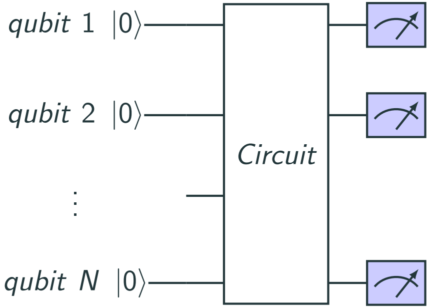
How to measure the purity in an experiment?

Outline

Entanglement versus quantum computers

The randomized measurement toolbox for measuring entanglement

A standard measurement sequence in a quantum computer

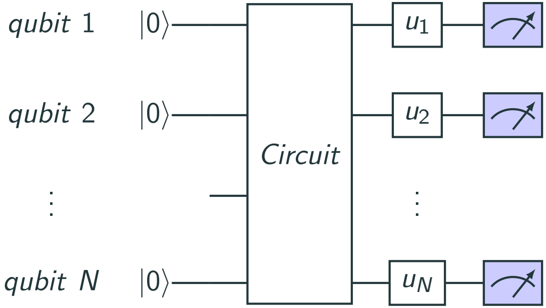


- Quantum measurements: the state $|s\rangle$ is measured with probability $\langle s|\rho|s\rangle$, in a certain measurement basis
- We have access to observables of the type $O|s\rangle = O(s)|s\rangle$.

$$\langle O\rangle = \sum_s \langle s|\rho|s\rangle\, O(s) \tag{7}$$

- Measurement of 4^N observables: I can realize state tomography, i.e measure ρ.
- Can I measure the purity $\mathrm{Tr}(\rho^2)$ more directly?

One approach: Randomized measurements



- Randomized measurements: We measure $P_u(s) = \langle s|u\rho u^\dagger|s\rangle$, $u = u_1 \otimes \cdots \otimes u_N$.
- u_i chosen independently from the circular unitary ensemble (CUE)
- We extract quantities of interest from the statistics of $P_u(s)$, over random unitary transformations.

Original protocol: van Enk-Beenakker (PRL 2012)

- Consider a single qubit

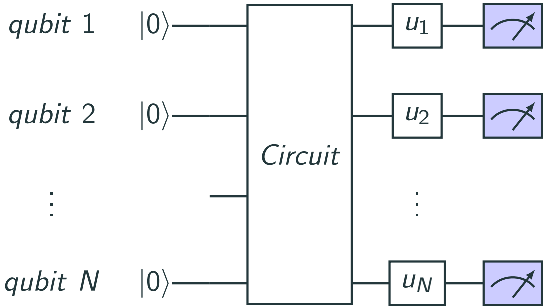
- We evaluate the statistics of $P_u(s) = \langle s|u\rho u^\dagger|s\rangle = \sum_{m,n} u_{s,m}\rho_{m,n}u^*_{s,n}$

$$E[[P_u(s)]^2] = \sum_{m,n,m',n'} \rho_{m,n}\rho_{m',n'}E[u_{s,m}u^*_{s,n}u_{s,m'}u^*_{s,n'}] \quad (8)$$

- Using Random Matrix Theory (2-design identities)

$$\overline{[P_u(s)]^2} = \frac{1}{6}\sum_{m,n,m',n'} \rho_{m,n}\rho_{m',n'}\left(\delta_{m,n}\delta_{m',n'} + \delta_{m,n'}\delta_{m',n}\right) = \frac{1+\mathrm{Tr}(\rho^2)}{6} \quad (9)$$

RM protocol for qubits (A. Elben, BV et al, PRL 2018)



- Protocol:
  (i) apply independent random single qubit rotations
  (ii) measure states $|s\rangle = |s_1, \ldots, s_N\rangle$
  (iii) Postprocess data ($D(s, s')$ is the Hamming distance, number of mismatchs between s and s')

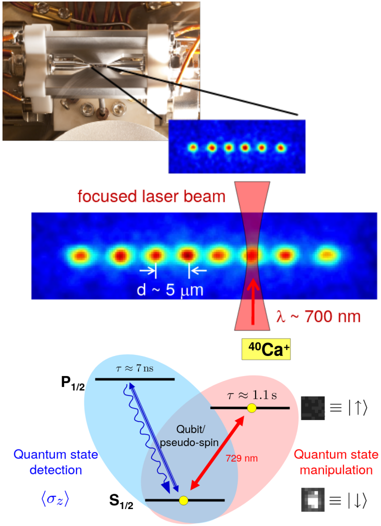
$$\mathrm{Tr}(\rho^2) = 2^N E_u \Big[\sum_{s,s'} (-2)^{-D(s,s')} P_u(s) P_u(s') \Big]$$

RM protocol for qubits (A. Elben, BV et al, PRL 2018)

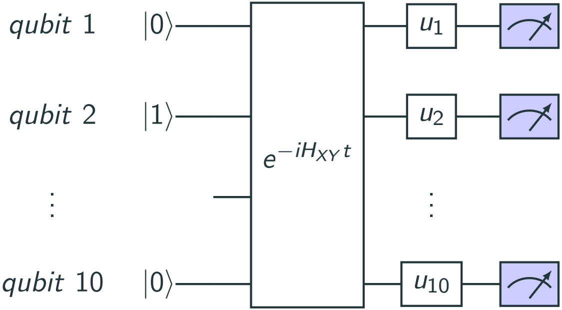
$$\mathrm{Tr}(\rho^2) = 2^N E_u \left[\sum_{s,s'} (-2)^{-D(s,s')} P_u(s) P_u(s') \right]$$

- Analytical Proof: Random matrix theory (twirling channels or tensor networks diagrams)
- State-agnostic estimation without reconstructing the state.
- Cheap postprocessing of the measurement data.
- Statistical errors $\rightarrow$ 'large' required number of measurements $\sim 2^N$, but 'much smaller' than for quantum state tomography $\sim 4^N - 8^N$.

Demonstration with a trapped ion quantum computer (Brydges et al, Science 2019)



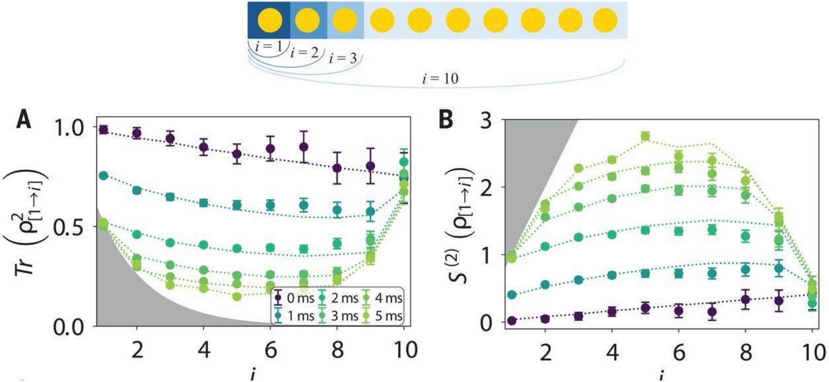
- A programmable quantum simulator



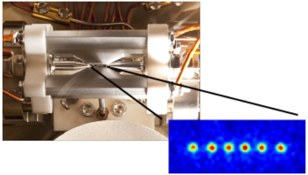
- Goal: Understand entanglement growth in a quantum system

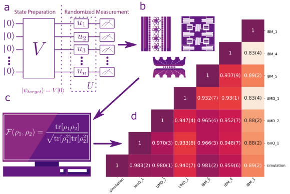
Demonstration with a trapped ion quantum computer (Brydges et al, Science 2019)

- Demonstration of randomized measurements with the measurement of the purity



RMs are now used routinely in the lab





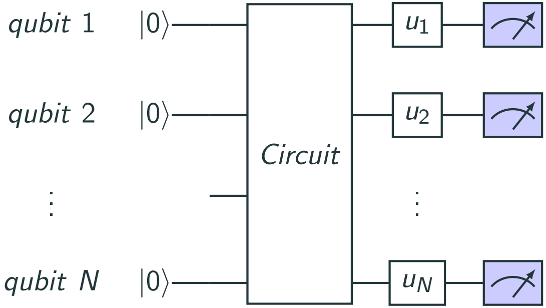
- Measurement of the topological entanglement entropy (Satzinger et al, Science 2021)
- Cross-Platform verification of devices (A. Elben et al, PRL 2020) and (Zhu et al, 2021 preprint)
- Experimental discovery of the p_3-PPT condition (A. Elben, R. Kueng et al, PRL 2020)
- Live measurements of the purity (Stricker et al, PRX quantum 2022)

How can we improve the randomized measurement toolbox?

Outline

- Measurement of the Quantum Fisher information
  - Theory: Rath, Branciard, Minguzzi, BV, Phys. Rev. Lett. 127, 260501
  - Experiment: Rath, Vitale, Elben, Branciard, BV, IBM
- Importance sampling of randomized measurements
  - For the purity: Rath, Elben, van Bijnen, Zoller, BV, Phys. Rev. Lett. 127, 200503 2021
  - For any polynomial: Rath, Elben, BV, in preparation

Measuring new quantities



- Important upgrade to the toolbox: Classical shadows (Huang et al, Nature Physics 2020)

$$\rho^{(r)} = \bigotimes_{i \in AB} \left(3 u_i \left| k_i^{(r)} \right\rangle \left\langle k_i^{(r)} \right| u_i^\dagger - \mathbf{1}_i \right) \tag{10}$$

- For a single measurement, unbiased estimations of the density matrix

$$E[\rho^{(r)}] = \rho \tag{11}$$

Measuring new quantities

- Classical shadows are very promising for observable estimations (ex energy estimation in quantum simulation)

$$\mathrm{Tr}(O\rho) = E[\mathrm{Tr}(\rho^{(r)}O)] \tag{12}$$

- Classical shadows provide access to 'multi-copy observables' (MCO)

$$\mathrm{Tr}(O\rho^{\otimes n}) = E[\mathrm{Tr}(O\rho^{(r_1)} \otimes \cdots \otimes \rho^{(r_n)})] \tag{13}$$

- Examples: Rényi entropies, Partial-Transpose moments (Elben et al, PRL 2020), Symmetry-resolved entropies (Vitale et al, Sci Post 2021), etc
- Note: The acquisition tasks is the same for all observables, i.e. Measure first, ask questions later

Quantum Fisher information

- Quantum Fisher information (QFI) for an operator A

$$F_Q = 2 \sum_{(i,j),\lambda_i+\lambda_j>0} \frac{(\lambda_i - \lambda_j)^2}{\lambda_i + \lambda_j} |\langle i| A |j\rangle|^2 \text{ with } \rho = \sum_i \lambda_i |i\rangle \langle i| \qquad (14)$$

- Certifies metrological power (Quantum Cramer Rao bound)
- For $A = \sum_i \sigma_i^z$, the entanglement depth (how many particles are entangled)
- Diverges for quantum phase transitions.
- Rath et al (PRL 2021): QFI is the limit of a series of MCO, i.e can be measured

$$F_n = 2\,\mathrm{Tr}\left(\sum_{\ell=0}^{n} (\rho\otimes\mathbf{1}-\mathbf{1}\otimes\rho)^2(\mathbf{1}\otimes\mathbf{1}-\rho\otimes\mathbf{1}-\mathbf{1}\otimes\rho)^\ell \mathbf{S}(A\otimes A)\right) = \mathrm{Tr}(O\rho^{\otimes n}). \qquad (15)$$

Quantum Fisher information and MCO: Statistical Errors

- The series of MCO F_n converges to QFI, and is estimated via randomized measurements

$$F_n = \mathrm{Tr}(O\rho^{\otimes n}) = E[\mathrm{Tr}(O\rho^{(r_1)} \otimes \cdots \otimes \rho^{(r_n)})] \tag{16}$$

- We provide analytical variance estimation ('error bars')

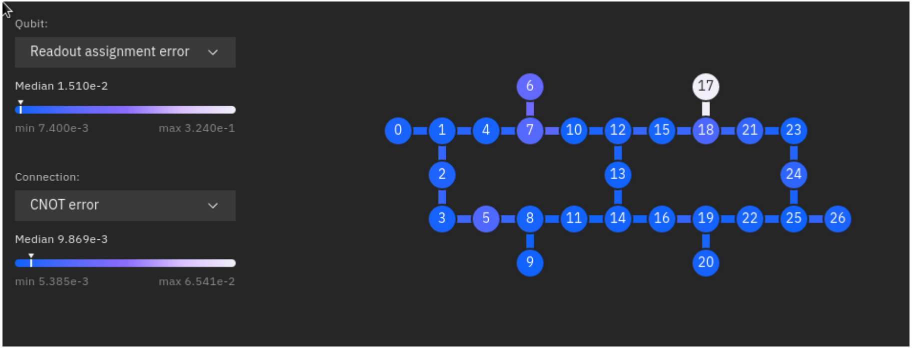
$$\mathrm{var}[\hat{F}_n] = \binom{M}{n}^{-2} \sum_{k=0}^{n} \binom{M}{k} \binom{M-k}{n-k} \binom{M-n}{n-k} \mathrm{var}\left[\mathrm{Tr}\left(O_k \otimes_{i=1}^{k} \hat{\rho}^{(i)}\right)\right] \tag{17}$$

$$O_k = \frac{1}{n!} \sum_{\pi} \mathrm{Tr}_{\{k+1\ldots n\}}(\pi^\dagger O \pi[\mathbf{1}^{\otimes k} \otimes \rho^{\otimes(n-k)}]) \tag{18}$$

- Person behind this: Aniket Rath

Experimental measurements of the Quantum Fisher information

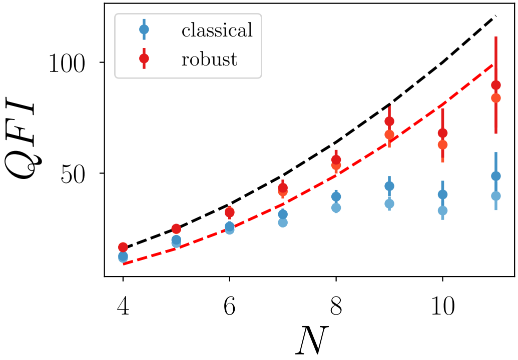
- Device: IBMQ Montreal



- Two case studies: GHZ states $|\phi\rangle = |0\rangle^{\otimes N} + |1\rangle^{\otimes N}$, and critical Ising groundstate
- System sizes up to 14 qubits, $(10^7 - 10^8$ measurements)

Experimental measurements of the Quantum Fisher information

- Calibration steps on the $|0\rangle^{\otimes N}$ provides a robust 'map' to build random measurements estimations for any density matrix ρ (Assumption: local gate independent noise)
- Fast postprocessing algorithms using 'batch shadows' (Rath et al, https://arxiv.org/abs/2209.04393)
- Observation of Heisenberg scaling: $F \propto N^2$, and genuine multipartite entanglement, on a noisy GHZ state via the two bounds F_0, F_1



Outline

- Measurement of the Quantum Fisher information
  - Theory: Rath, Branciard, Minguzzi, BV, Phys. Rev. Lett. 127, 260501
  - Experiment: Rath, Vitale, Elben, Branciard, BV, IBM
- Importance sampling of randomized measurements
  - For the purity: Rath, Elben, van Bijnen, Zoller, BV, Phys. Rev. Lett. 127, 200503 2021
  - For any polynomial: Rath, Elben, BV, in preparation

Importance sampling of randomized mesurements

- IBM experiments uses $N_u \sim 10^5$ measurement circuits ($N_M = 100$ shots)
- For classical shadows $\rho(u)$, such that $\int \rho(u)du = \rho$
- We can define 'importance sampling shadows' by defining $\int [\rho(u)/p(u)]p(u)du = \rho$ for any distribution $p(u)$
- We can choose $p(u)$ to minimize the variance of a multi-copy observable estimation O

$$\mathrm{Var}(\hat{O}) = \frac{1}{N_u}\mathrm{Var}_p(\mathrm{Tr}(O_1\rho(u)/p(u)) + \mathcal{O}\left(\frac{1}{N_u^2}\right) \tag{19}$$

Importance sampling of randomized mesurements

- Rath,Elben,BV (in preparation): If $p(u) \propto \mathrm{Tr}(O_1 \rho(u))$, and $N_M \propto N_u$

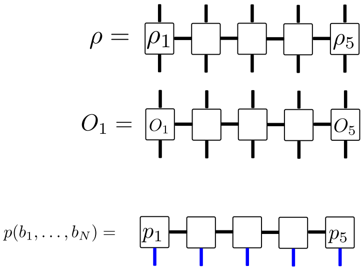
$$\mathrm{Var}(\hat{O}) = \mathcal{O}(\frac{1}{N_u^2}) \tag{20}$$

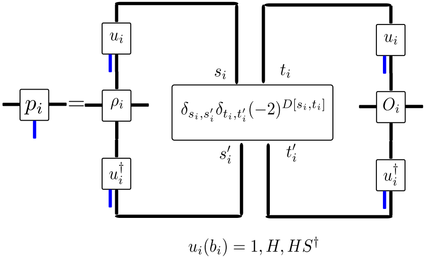
- Algorithm 1: I use a state approximation $\rho^{(0)} \approx \rho$, $p(u)$ can be efficiently constructed.
- Algorithm 2: $p(u)$ can be adaptively learnt from an experiment, because it is a multi-copy observable (the dependence on u appears only in postprocessing).

Importance sampling of randomized mesurements

- Algorithm 1 with tensor networks
  - Requirement 1: a classical approximate representation of the state $\rho^{(0)}$
  - Requirement 2: discretized unitaries (local Clifford transformations $u_i = 1, H, HS^\dagger$)
- The sampling function is a 'local contraction of the state'

$$p(u) \propto \mathrm{tr}(\rho^{(0)}(u) O_1), \quad \rho^{(0)}(u) = \sum_{s,s'} (-2)^{D[s,s']} P_u(s) u^\dagger |s'\rangle \langle s'| u$$





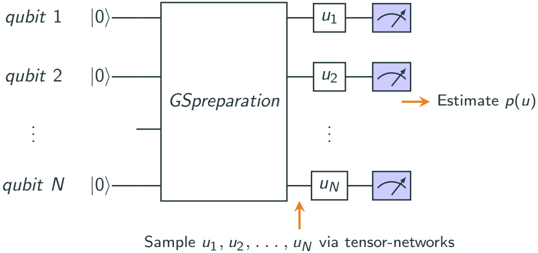
- Review on tensor-networks: U. Schollwock.

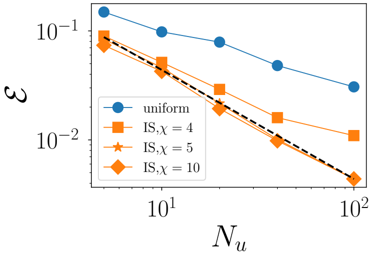
Importance sampling of randomized mesurements

- Example of algorithm 1: Measurement of the second moment $\langle H^2 \rangle$ of the ground state energy of the critical Ising model (64 qubits)

$$H = \sum_i \sigma_i^z \sigma_{i+1}^z + \sum_i \sigma_i^x \tag{21}$$

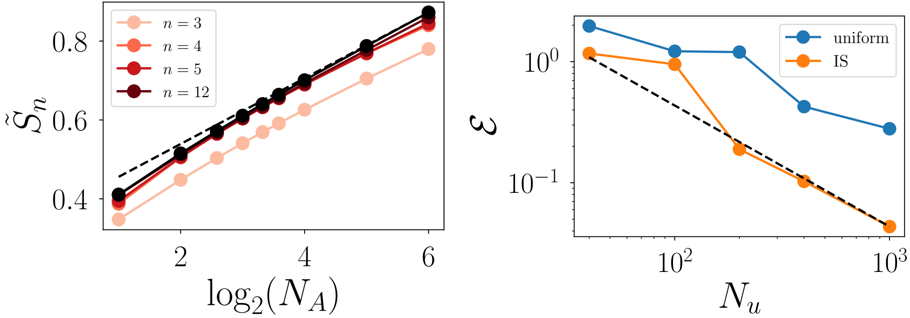
- Importance sampling performs well, even with 'poor' classical approximations (small bond dimensions χ).
- TBD: compare with other methods for this specific example





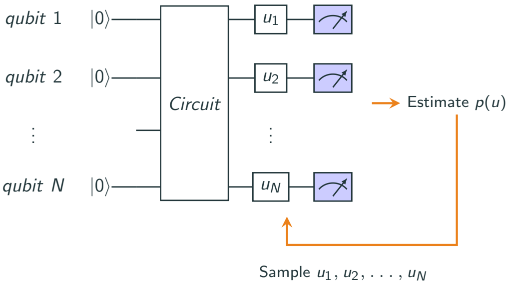
von Neumann entropy

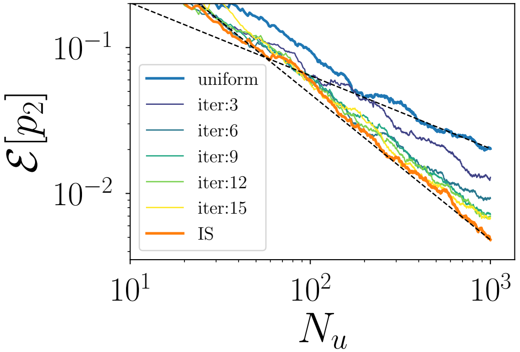
- The critical point of the Ising model is characterized a logarithmic dependence of the von Neumann entropy with subsystem size.
- With algorithm 1, one can measure any polynomial interpolation of the von Neumann entropy $S_{\mathrm{vN}}(\rho) \approx \sum_n c_n \mathrm{Tr}(\mathrm{O}\rho^{\otimes \mathrm{n}})$.
- Right panel: Average statistical error for a reduced state of 8 qubits.



Adaptive purity measurement

- Example of algorithm 2:
  - Adaptive measurement of the puriry of 7 qubit reduced state of a 14 qubit random circuit (depth 5).
  - Randomized measurement of experimental cycles $1 \rightarrow n$, is used to sample unitaries for cycle $n+1$.





Conclusion

- Randomized measurement are used routinely to probe entanglement properties of quantum computers, and answer physics questions.
- Generation of huge 'quantum science' datasets motivates the development of improved preprocessing and postprocessing methods (classical algorithms assisting quantum experiments)

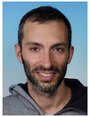
Review: *The randomized measurement toolbox* A. Elben, S. T. Flammia, H.-Y. Huang, R. Kueng, J. Preskill, B. V, P. Zoller, arXiv:2203.11374

Team:


A. Rath


M. Votto


V. Vitale (Trieste→Grenoble)


P. Naldesi (Innsbruck)


L. Joshi (Innsbruck)