Probing entanglement in quantum processors with the randomized measurements toolbox

Heidelberg

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Outline

Entanglement versus quantum computers

The randomized measurement toolbox for measuring entanglement

A recent experiment with a quantum computer

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From classical to quantum computers in the quantum circuit model

Classical computers use classical bits

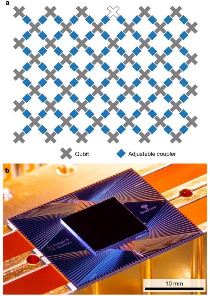
- One classical bit:
  $\rightarrow$ state in 0 or 1
- N classical bits:
  $\rightarrow 2^N$ possibilities for the state
  $(00\ldots00,\ 00\ldots01,\text{etc})$

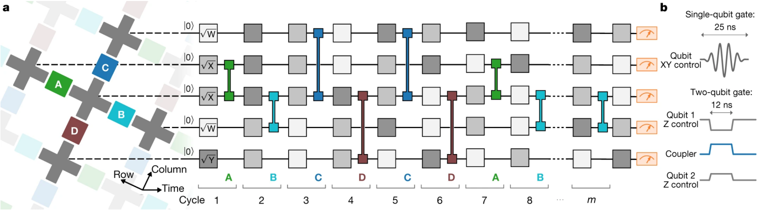
Quantum computers use qubits

- One qubit: 
  $|\psi\rangle = \alpha\,|0\rangle + \beta\,|1\rangle$
- N qubits: 
  $|\psi\rangle = c_{0\ldots0}\,|0\ldots0\rangle + c_{0\ldots1}\,|0\ldots1\rangle + \ldots.$
  The quantum state can be simulatenously in all the 2^N classical states.

What is a quantum computer?

- The Sycamore chip (Image: Google AI)





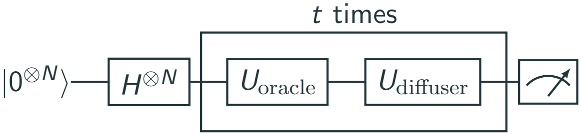
- Any N-qubit state (dim 2^N) can be created (Deutsch 1989).

$$|\psi\rangle = \sum_{s_1,\ldots,s_N} c_{s_1,\ldots,s_N} |s_1\rangle \otimes \cdots \otimes |s_N\rangle \tag{1}$$

- Quantum algorithms are designed to solve classical problems or quantum problems

Why we may expect quantum speedup with quantum parallelism

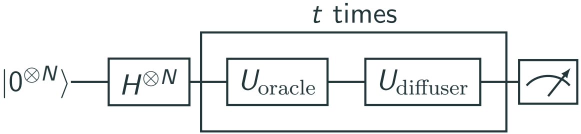
- Unstructured search on a space of 2^N bitstrings: We look for x, such that $f(x = x_1, \ldots, x_N) = 1$.
- Optimal classical algorithm: Random testing, with time complexity $O(2^N)$
- Optimal quantum algorithm: Grover's algorithm



  - The first quantum gate creates a uniform quantum superposition of all 2^N bitstring states
  - Complexity $O(\sqrt{2^N})$: polynomial improvement.
- Note: Quantum parallelism does not guarantee quantum speedup for solving *anything*: One needs to extract relevant classical information from the quantum superposition state.

The measurement

- Every quantum circuits ends up with a measurement



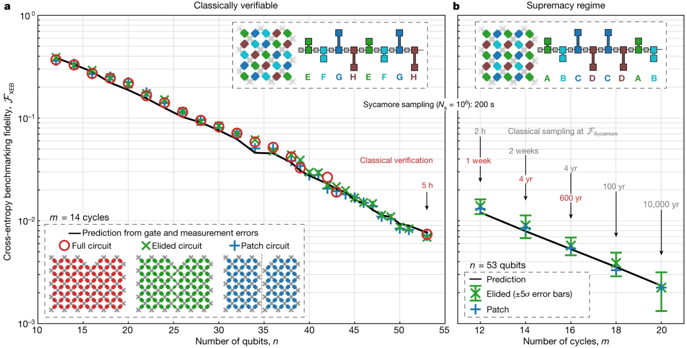
that produces random output $s_1, \ldots, s_N$ ($s_i = 0, 1$) based on Born probabilities.

$$P(s_1, \ldots, s_N) = | \langle s_1, \ldots, s_N | \psi \rangle |^2 \tag{2}$$

- Efficient quantum algorithms are the ones that give the solution to your problem in this *classical* data.
- Measurement precision?: Analytical performance guarantees based on standard tools of statistical estimation (variance, confidence intervals. . .)

Noisy quantum computers

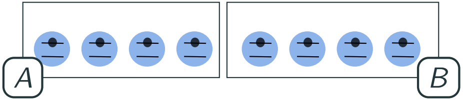
- Error propagation is typically exponential in size and time (Image: Google AI)



- Time is for benchmarks: check the 'quantum aspects' of a quantum computer
- Time is also for quantum simulation: understand a quantum problem using a quantum computer (ground-state physics, non-equilibrium problem, etc)

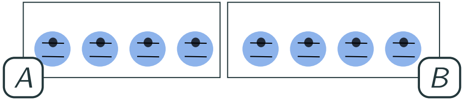
Entanglement

- Take two parts of a quantum system A B (eg sets of qubits)



- A and B are entangled iff $|\psi\rangle \neq |\psi_A\rangle \otimes |\psi_B\rangle$
- Example with two qubits. The Bell state $|\psi\rangle = \frac{1}{\sqrt{2}}(|0\rangle \otimes |0\rangle + |1\rangle \otimes |1\rangle)$ is entangled

How to quantify entanglement?



- The reduced state of a quantum state in region A is defined by a density matrix

$$\rho_A = \mathrm{Tr}_B(|\psi\rangle\langle\psi|) = \sum_{s_B} \langle s_B|\,|\psi\rangle\langle\psi|\,|s_B\rangle \quad (3)$$

- For instance, with the Bell state

$$\rho_A = \frac{1}{2}\sum_{s_B=0,1} \langle s_B|\,|00\rangle\langle 00| + |00\rangle\langle 11| + |11\rangle\langle 00| + |11\rangle\langle 11|\,|s_B\rangle = \frac{1}{2}(|0\rangle\langle 0| + |1\rangle\langle 1|) \quad (4)$$

- Entanglement means: ρ_A has more than one eigenvalue

Quantifying entanglement

- For pure states, entanglement entropies of the reduced state $\rho_A = \mathrm{Tr}_B(|\psi\rangle\langle\psi|)$ are entanglement measures

$$\begin{aligned} S_{\mathrm{vN}} &= -\mathrm{Tr}[\rho_A \log(\rho_A)] \text{ von Neumann Entropies} \\ S_\alpha &= \frac{1}{1-\alpha} \log[\mathrm{Tr}(\rho_A^\alpha)] \text{ Rényi entropies} \end{aligned} \tag{5}$$

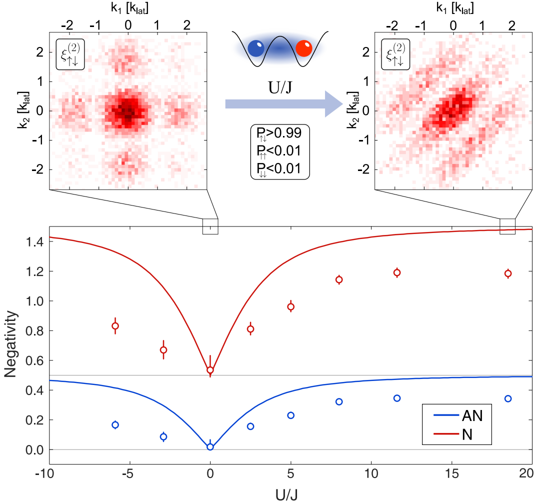
Entanglement in quantum computing

Entanglement is the most important concept in quantum information theory:

- All quantum algorithms involve entangled states.
- Quantum algorithms with a ‘low’ level of entanglement can be efficiently simulated with a classical computer, i.e. they are ‘useless’ (Eisert RMP 2010).
- Decoherence = Presence of entanglement with the environment.
- Distinguish many-body quantum phases in quantum simulation
- Essential for quantum metrology

Digression

- Entanglement in Heidelberg!
- Becher et al, PRL 2020. Fermi-Hubbard model with cold atoms



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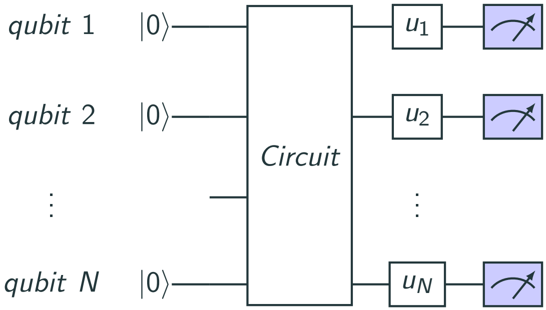
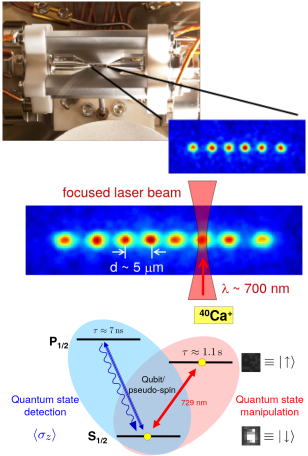
A standard measurement sequence in a quantum computer



- Born rules (density matrix version): Bitstring s is measured with probability $\langle s|u\rho u^{\dagger}|s\rangle$
- High fidelity basis transformations $u = \bigotimes_i u_i$.
- This is repeated $N_u \times N_M$ times to obtain a classical dataset s_{i_u,i_M}.

How can we retrieve quantum information (about ρ) in the classical dataset?

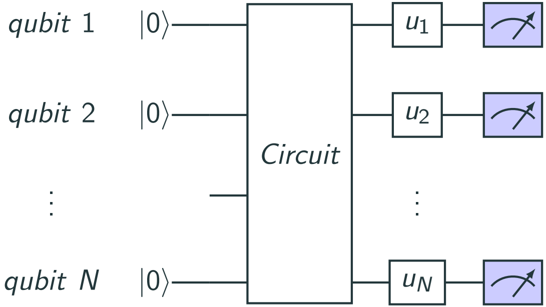
In practice



- Trapped ions 'programmable quantum simulators' (Innsbruck, 2014-)
- from 1 to 100 qubits
- State almost pure $\rho \approx |\psi\rangle\langle\psi|$
- Measurement fidelity $\sim 99.9\%$
- 4 bitstrings per seconds & experiments stable during hours $\rightarrow$ dataset of $N_u N_M \approx 10^5$ bitstrings
- With superconducting qubits: $\sim 10^3$ faster, $N_u N_M \approx 10^7$

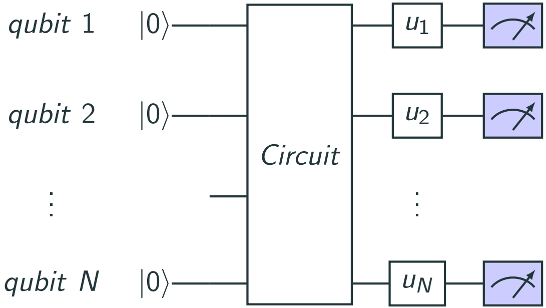
Take-home message: Quantum technologies = a unique opportunity to learn about quantum states!

One approach: Randomized measurements



- u_i chosen independently from the circular unitary ensemble (CUE)
- We extract quantities of interest from the statistics of $P_u(s)$, over random unitary transformations.
- First proposal (van Enk - Beenakker PRL 2012)
- With qubits (Elben et al, PRL 2019)

RM protocol for qubits (A. Elben, BV et al, PRL 2018)



- Protocol:
  (i) apply independent random single qubit rotations
  (ii) measure states $|s\rangle = |s_1, \dots, s_N\rangle$
  (iii) Postprocess data ($D(s, s')$ is the Hamming distance, number of mismatchs between s and s')

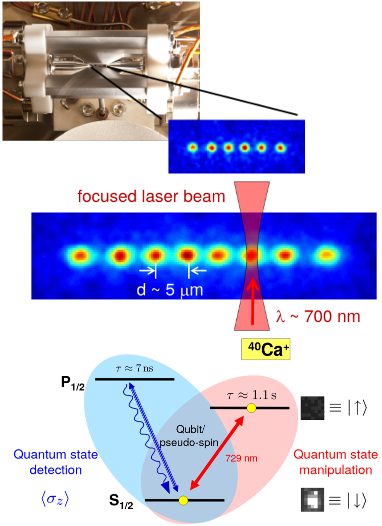
$$\mathrm{Tr}(\rho^2) = 2^N E_u \Big[\sum_{s,s'} (-2)^{-D(s,s')} P_u(s) P_u(s') \Big]$$

RM protocol for qubits (A. Elben, BV et al, PRL 2018)

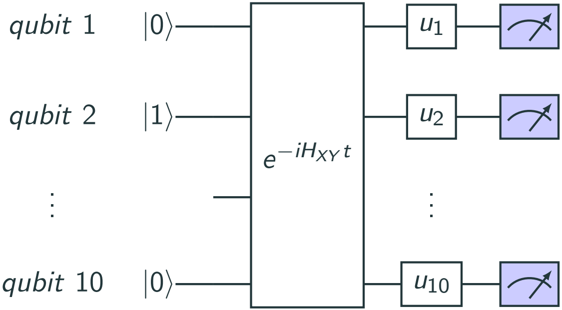
$$\mathrm{Tr}(\rho^2) = 2^N E_u\Big[\sum_{s,s'}(-2)^{-D(s,s')}P_u(s)P_u(s')\Big]$$

- Analytical Proof: Random matrix theory (unitary n-designs).
- 'Cheap' postprocessing of the measurement data.
- Statistical errors $\rightarrow$ 'large' required number of measurements $\sim 2^N$, but 'much smaller' than for quantum state tomography $\sim 4^N - 8^N$.

Demonstration with a trapped ion quantum computer (Brydges et al, Science 2019)



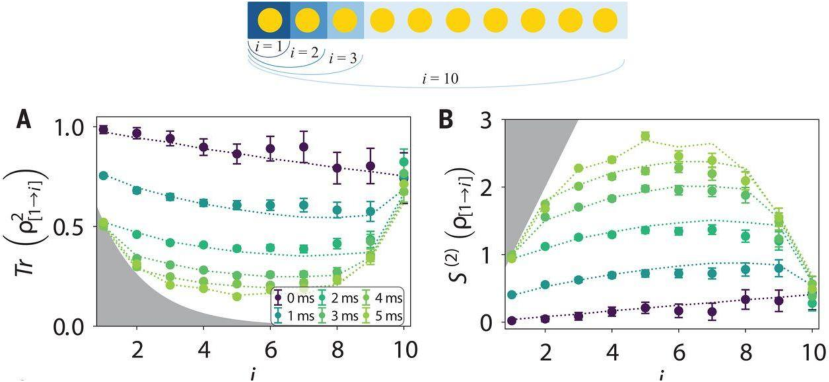
- A programmable quantum simulator



- Goal: Understand entanglement growth in a quantum system

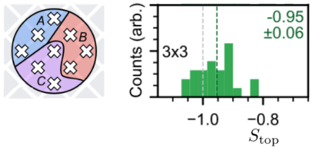
Demonstration with a trapped ion quantum computer (Brydges et al, Science 2019)

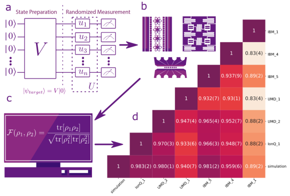
- Demonstration of randomized measurements with the measurement of the purity



RMs are now used routinely in the lab

$$S_{\text{top}} = \sum_{\substack{X= \\ A,B,C}} S_2(\rho_X) - \sum_{\substack{XY= \\ AB,BC,AC}} S_2(\rho_{XY}) + S_2(\rho_{ABC})$$





- Measurement of the topological entanglement entropy (Satzinger et al, Science 2021)
- Cross-Platform verification of devices (A. Elben et al, PRL 2020) and (Zhu et al, 2021 preprint)
- [Theory] Classical shadows formalism (R. Huang et al, Nature Physics 2020)
- Experimental discovery of the p_3-PPT condition (A. Elben, R. Kueng et al, PRL 2020)
- Live measurements of the purity (Stricker et al, PRX quantum 2022)
- Extensions to cold atoms (Naldesi et al, PRL 2023)

Let's illustrate the status of the field with a recent experiment

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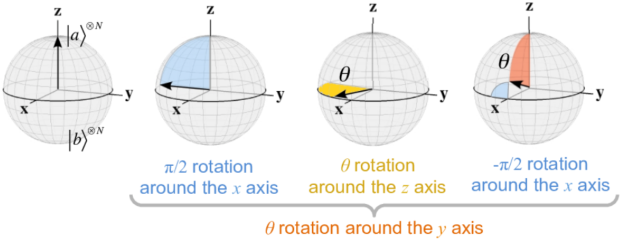
Quantum Fisher information in quantum metrology



Pezze et al, RMP 2018

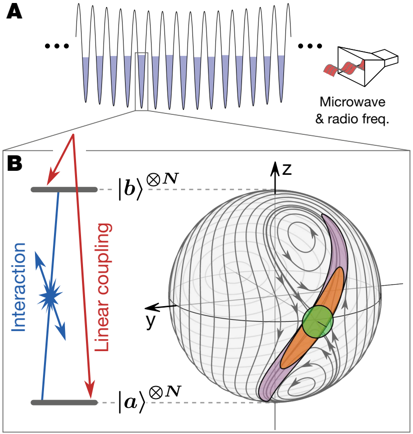
- Quantum Fisher information (QFI) for an Hermitian operator A

$$F_Q = 2 \sum_{(i,j),\lambda_i+\lambda_j>0} \frac{(\lambda_i - \lambda_j)^2}{\lambda_i + \lambda_j} |\langle i| A |j\rangle|^2 \text{ with } \rho = \sum_i \lambda_i |i\rangle\langle i| \tag{6}$$

- Certifies metrological power for parameter estimation
- Access to the entanglement depth in qubit systems

Fisher information in Heidelberg

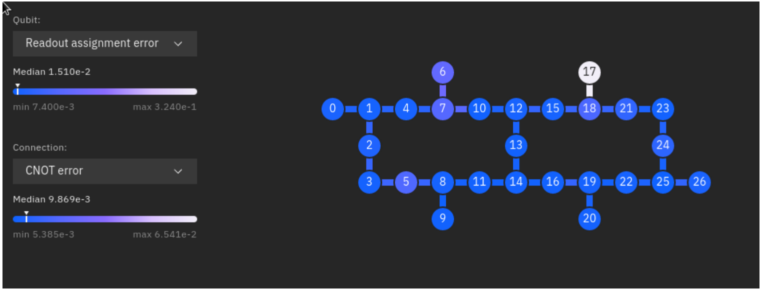
- Strobel et al, Science 2014



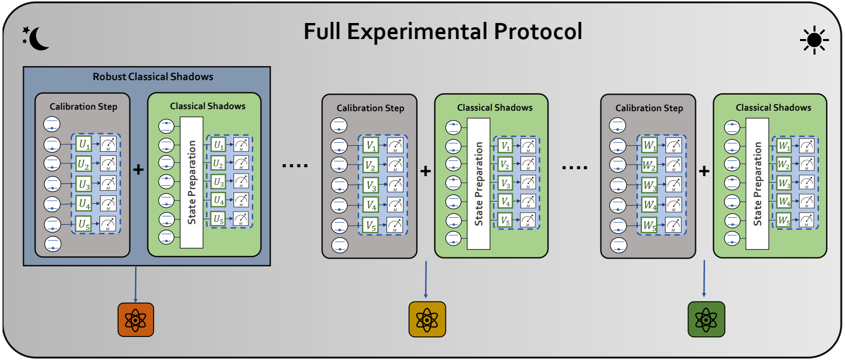
Experimental measurements of the Quantum Fisher information

Vitale, Rath, Jurcevic (IBM), Branciard, Elben (Caltech), BV (arxiv:2307.16882, see also Rath et al PRL 2021)

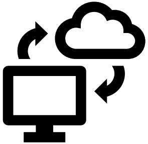
- An experiment performed on the superconducting quantum computer IBMQ Montreal



Our measurement strategy



Post-processing



- Two datasets per batch measurements: calibration on the $|0\rangle^{\otimes N}$, actual measurement
- Several batches to keep track of drifts of measurement errors (it actually matters!)

Postprocessing: Data conversion

- We use 'robust classical shadows' (Chen et al, PRX 2021, improving the seminal Caltech paper: Nature Physics 2020)

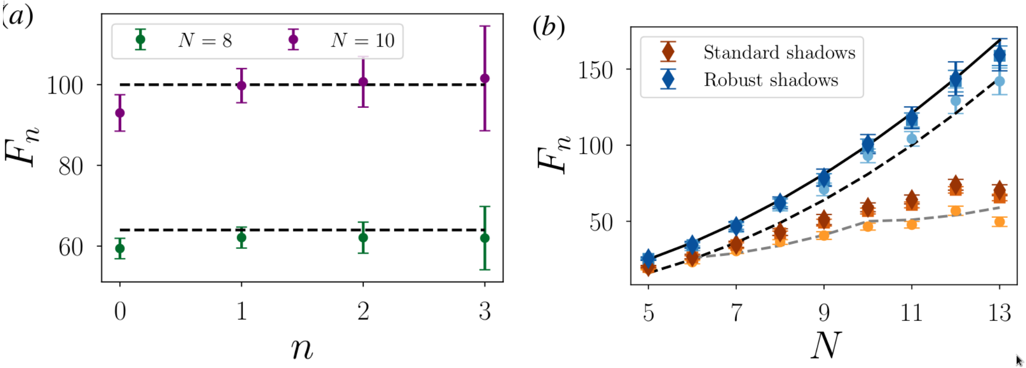
$$\tilde{\rho}^{(r,b)} = \bigotimes_{j=1}^{N} \left(\frac{3}{2F_b[j]-1} U_j^{(r)\dagger} \left|s_j^{(r,b)}\right\rangle \left\langle s_j^{(r,b)}\right| U_j^{(r)} + \frac{F_z[j]-2}{2F_b[j]-1} \mathbf{1} \right), \qquad (7)$$

where the calibration data of each batch b gives access to $F_b[j]$.

- Estimations of functions of ρ are built based on the relation $E[\tilde{\rho}^{(r,b)}] = \rho$

Experimental measurements

- GHZ state $|0\rangle^{\otimes N} + |1\rangle^{\otimes N}$ with expected QFI $F_Q = N^2$, offering optimal performance for quantum metrology, and showing genuine multipartite entanglement.



Conclusion and take-home questions

- Randomized measurement are used routinely to probe entanglement properties of quantum computers, and answer physics questions.
- Physics insights are essential to analyze the data, data analysis provide physics insight. . .
- Randommess is key (deterministic approaches are not possible), BUT there are ways to boost the efficiency using statistical tricks (common random numbers, importance sampling, etc) and physics assumptions (arxiv:2311.08108)

Review: *The randomized measurement toolbox* A. Elben, S. T. Flammia, H.-Y. Huang, R. Kueng, J. Preskill, B. V, P. Zoller, arXiv:2203.11374