Understanding the role of entanglement in quantum computing

Quantum Africa Conference

Benoît Vermersch

September 15 2022

LPMMC Grenoble & IQOQI Innsbruck

Outline

Entanglement versus quantum computers

The randomized measurement toolbox for measuring entanglement

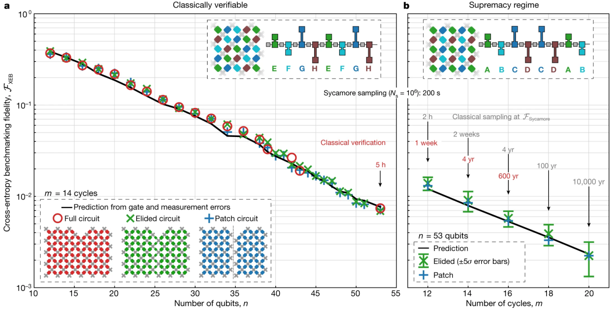
Outline

Entanglement versus quantum computers

The randomized measurement toolbox for measuring entanglement

Noisy quantum computers

- Error propagation is typically exponential in size and time.



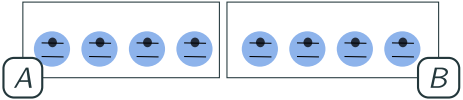
- Time is for benchmarks [Pic: Google AI, Nature 2019]
- Time is also for quantum simulation: study many-body quantum physics on a (noisy) quantum computer

Noisy quantum computers

- One key concept to understand the power and limitations of quantum computers: Entanglement
- One key concept for quantum simulation: Entanglement

Entanglement

- Take two parts of a quantum system A B (eg sets of qubits)



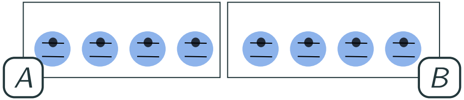
- A and B are entangled iff $|\psi\rangle \neq |\psi_A\rangle \otimes |\psi_B\rangle$
- Example with two qubits. The Bell state $|\psi\rangle = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$ is entangled

Entanglement in quantum computing

Entanglement is the most important concept in quantum information theory:

- All important quantum algorithms involve quantum operations that generate entanglement.
- Quantum algorithms with a low level of entanglement can be efficiently simulated with a classical computer, i.e. they are ‘useless’ (Eisert RMP 2010).
- Universal predictions for large-scale quantum computers (eg Nahum PRX 2017)
- Distinguish many-body quantum phases in quantum simulation

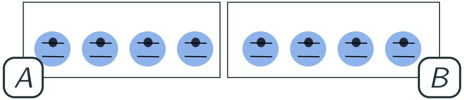
How to quantify entanglement?



For pure states, entanglement entropies of the reduced state $\rho_A = \mathrm{Tr}_B(|\psi\rangle\langle\psi|)$ are entanglement measures

$$\begin{aligned} S_{\mathrm{vN}} &= -\mathrm{Tr}[\rho_A \log(\rho_A)] \text{ von Neumann Entropies} \\ S_\alpha &= \frac{1}{1-\alpha}\log[\mathrm{Tr}(\rho_A^\alpha)] \text{ Rényi entropies} \end{aligned} \quad (1)$$

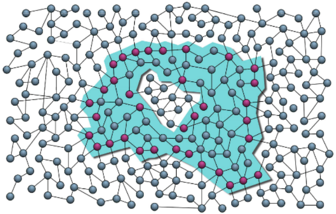
Entanglement entropies for pure states



- Define quantum resources, e.g., number of distillable Bell Pairs
- Define amount of quantum information (e.g Schumacher compression)
- *Excellent notes from J. Preskill on Quantum Shannon Theory*

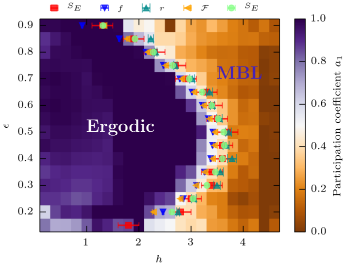
Why should I care about entanglement scaling?

For groundstates of condensed matter systems, we expect an area law $S_{\mathrm{vN}} \propto N_A^{(\mathrm{boundary})}$



Eisert RMP 2010

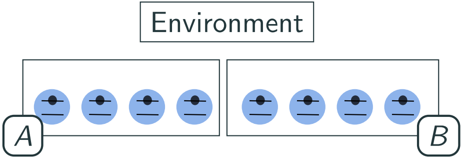
Entanglement scalings indicate phase transitions, a topological phase, etc



Luiz et al, PRB 2015

Entanglement in noisy quantum computers

- Typically the state of a quantum computer is 'mixed' due to an environment



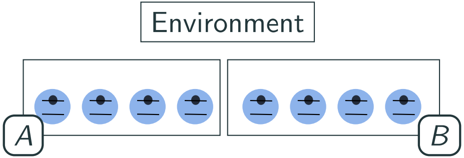
- We define the density matrix of the quantum computer

$$\rho_{AB} = \mathrm{Tr}_E(|\psi\rangle\langle\psi|) \tag{2}$$

- A and B are entangled iff

$$\rho_{AB} \neq \sum_i p_i \rho_A^{(i)} \otimes \rho_B^{(i)} \tag{3}$$

How to detect entanglement?



- Purity $\mathrm{Tr}(\rho^2)$ ($= 1$ iff the state is pure $\rho = |\psi\rangle\langle\psi|$)
- Purity entanglement condition (Horodecki 1996)

$$\mathrm{Tr}(\rho_A^2) < \mathrm{Tr}(\rho_{AB}^2) \implies A \text{ and } B \text{ are entangled} \tag{4}$$

- Example Bell State $|\psi\rangle = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)|0_E\rangle$

$$\begin{aligned} \rho_{AB} &= \frac{1}{2}(|00\rangle + |11\rangle)(\langle 00| + \langle 11|) \implies \mathrm{Tr}(\rho_{AB}^2) = 1 \\ \rho_A &= \frac{1}{2}(|0\rangle\langle 0| + |1\rangle\langle 1|) \implies \mathrm{Tr}(\rho_A^2) = 1/2 \end{aligned} \tag{5}$$

How to measure entanglement?

- The purity $\mathrm{Tr}(\rho^2)$ can be used to detect entanglement.
- The second Rényi entropy $S_2 = -\log_2[\mathrm{Tr}(\rho^2)]$ quantifies entanglement in quantum computers, in particular in the context of quantum simulation.

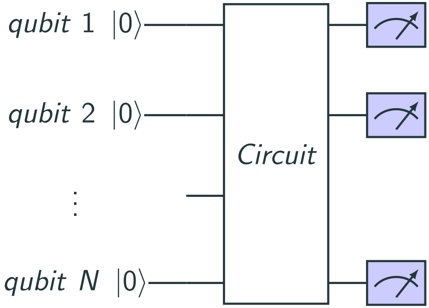
How to measure the purity in an experiment?

Outline

Entanglement versus quantum computers

The randomized measurement toolbox for measuring entanglement

A standard measurement sequence in a quantum computer



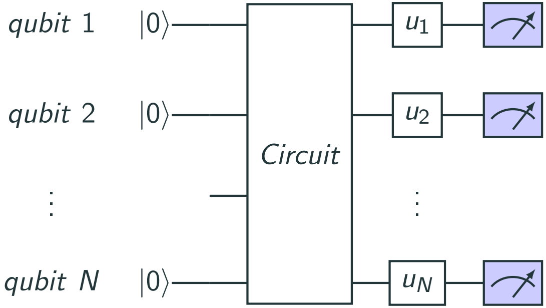
- Quantum measurements: the state $|s\rangle$ is measured with probability $\langle s|\rho|s\rangle$, in a certain measurement basis
- We have access to observables of the type $O\,|s\rangle = O(s)\,|s\rangle$.

⋮

$$\langle O\rangle = \sum_s \langle s|\rho|s\rangle\, O(s) \tag{6}$$

- Measurement in the 3^N combinations of X, Y, Z basis, I can realize state tomography, i.e measure ρ.
- Can I measure the purity $\mathrm{Tr}(\rho^2)$ more directly?

One approach: Randomized measurements



- Randomized measurements: We measure $P_u(s) = \langle s | u \rho u^\dagger | s \rangle$, $u = u_1 \otimes \cdots \otimes u_N$.
- We extract quantities of interest from the statistics of $P_u(s)$, over random unitary transformations.

Original protocol: van Enk-Beenakker (PRL 2012)

- Consider a single qubit

We measure the statistics of $P_u(s) = \langle s|u\rho u^\dagger|s\rangle$.

- Extreme case 1: The state is pure with $\mathrm{Tr}(\rho^2) = 1$, eg $\rho = |0\rangle\langle 0|$, then $P_u(s) = |\langle s|u|0\rangle|^2$ fluctuates in $[0, 1]$.
- Extreme case 2: The state is fully mixed with $\mathrm{Tr}(\rho^2) = 1/2$, $\rho = \mathbf{1}/2$, then $P_u(s) = 1/2$ does not fluctuate.
- Using the properties of the circular unitary ensemble (CUE) $(\mathrm{Tr}(\rho_A^2) - 1/2) \propto \mathrm{Var}_u[P_u(s)^2])$

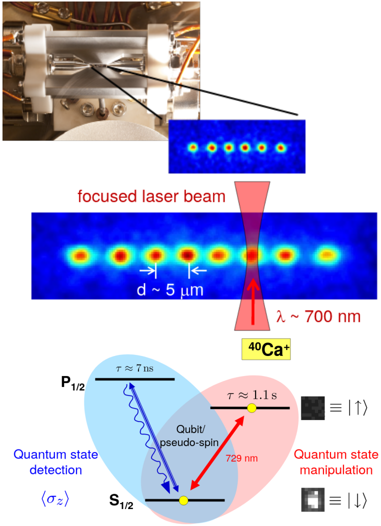
RM protocol for qubits (A. Elben, BV et al, PRL 2018)

- Protocol:
  (i) apply independent random single qubit rotations
  (ii) measure states $|s\rangle = |s_1, \ldots, s_N\rangle$
  (iii) Postprocess data ($D(s,s')$ is the Hamming distance, number of mismatchs between s and s')

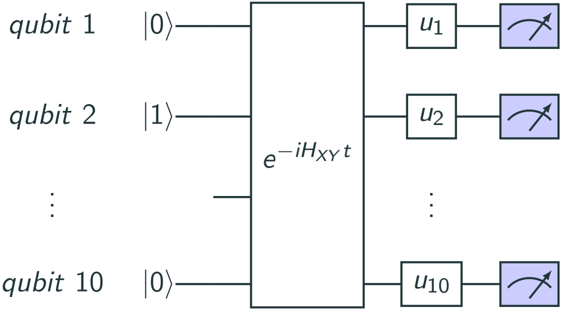
$$\mathrm{Tr}(\rho^2) = 2^N E_u\Big[\sum_{s,s'} (-2)^{-D(s,s')} P_u(s) P_u(s')\Big]$$

- Proof: Random matrix theory and replica tricks.
- State-agnostic estimation without reconstructing the state
- Cheap postprocessing of the measurement data (ie no fitting, etc)
- Info on the unitaries does not appear in the formula $\rightarrow$ estimations are *robust*.
- Statistical errors $\rightarrow$ large required number of measurements $\sim 2^N$, but much smaller than for quantum state tomography $\sim 4^N$.

Demonstration with a trapped ion quantum computer (Brydges et al, Science 2019)



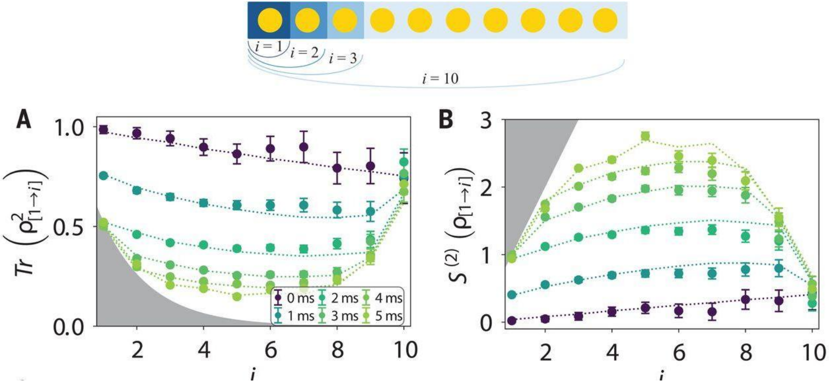
- A programmable quantum simulator



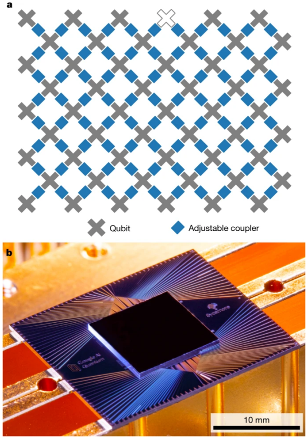
- Goal: Understand entanglement growth in a quantum system

Demonstration with a trapped ion quantum computer (Brydges et al, Science 2019)

- Demonstration of randomized measurements with the measurement of the purity

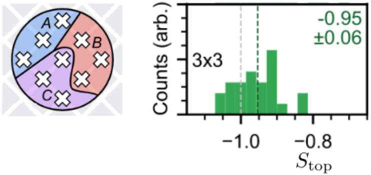


Demonstration with a superconducting quantum computer (Satzinger et al, Science 2021)



- A 2D chip of qubits can be used to implement quantum error correction with a logical qubit
- The toric code model is a surface code that exhibits non-local properties: topological order
- Smoking gun evidence: topological entanglement entropy

$$S_{\text{top}} = \sum_{\substack{X= \\ A,B,C}} S_2(\rho_X) - \sum_{\substack{XY= \\ AB,BC,AC}} S_2(\rho_{XY}) + S_2(\rho_{ABC})$$



Beyond the purity

- Randomized measurements can be generalized to access *many* quantum state properties.
- An illustrative example: Cross-platform comparison of quantum computers

$$\mathrm{F}(\rho_1, \rho_2) = \frac{\mathrm{Tr}(\rho_1 \rho_2)}{\max[\mathrm{Tr}(\rho_1^2), \mathrm{Tr}(\rho_2^2)]}$$

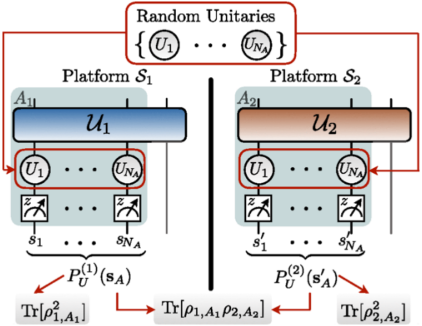
  ρ_1: Quantum computer 1

  ρ_2: Quantum computer 2

- Application: benchmarks of quantum devices, comparisons of different quantum technologies, etc.

Beyond the purity

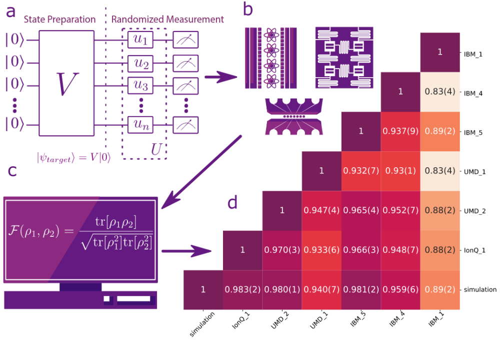
- Protocol (A. Elben, BV et al, PRL 2020)



$$\mathrm{Tr}(\rho_1\rho_2) = 2^N E_u\Big[\sum_{s,s'}(-2)^{-D(s,s')}P_u^{(1)}(s)P_u^{(2)}(s')\Big]$$

- The statistics of randomized measurements (classical information) map to the fidelity (quantum information)!

Demonstration: trapped ions versus superconducting qubits (Zhu et al, 2021 preprint)



- Good news: Fidelity is of order *one* for ‘complicated’ circuits.
- Bad news: Exponential propagation of errors when scaling up the circuits.
- Randomized measurements can be used to benchmark, and build noise models.

Recent developments for randomized measurements

- Randomized measurements are also useful for multiple observable estimations: Classical Shadow Formalism (Huang et al, Nature Physics 2020)
- Many other quantites have been shown to be measurable and some of them have been measured (example partial transpose moments of the density matrix, Elben et al, PRL 2020, but also Symmetry-resolved Rényi entropies, Vitale et al, Sci Post 2022)
- Scaling up the system sizes (Rath,. . . , BV PRL 2021)
- Useful data for machine learning (Huang et al, arxiv:2106.12627)
- Implement randomized measurement in new platforms: ultracold atoms, Rydberg atoms

Review: *The randomized measurement toolbox* A. Elben, S. T. Flammia, H.-Y. Huang, R. Kueng, J. Preskill, B. V, P. Zoller, arXiv:2203.11374

Thank you!