Probing entanglement in quantum processors with the randomized measurements toolbox

NEQM2 - Sankt Anton

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Outline

Entanglement and randomized measurements

Measurement of the quantum Fisher information in a quantum processor

Enhanced estimations of quantum state properties via common randomized measurements

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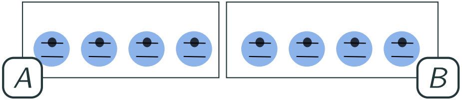
Entanglement

- Take two parts of a quantum system A B (eg sets of qubits)



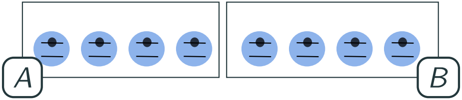
- A and B are entangled iff $|\psi\rangle \neq |\psi_A\rangle \otimes |\psi_B\rangle$
- Example with two qubits. The Bell state $|\psi\rangle = \frac{1}{\sqrt{2}}(|0\rangle \otimes |0\rangle + |1\rangle \otimes |1\rangle)$ is entangled

Entanglement in quantum computing

Entanglement is the most important concept in quantum information theory:

- All quantum algorithms involve entangled states.
- Quantum algorithms with a ‘low’ level of entanglement can be efficiently simulated with a classical computer.
- Universal predictions for large-scale quantum computers (eg Nahum PRX 2017)

How to quantify entanglement?



- For pure states, entanglement entropies of the reduced state $\rho_A = \mathrm{Tr}_B(|\psi\rangle\langle\psi|)$ are entanglement measures

$$\begin{aligned} S_{\mathrm{vN}} &= -\mathrm{Tr}[\rho_A \log(\rho_A)] \text{ von Neumann Entropies} \\ S_\alpha &= \frac{1}{1-\alpha} \log[\mathrm{Tr}(\rho_A^\alpha)] \text{ Rényi entropies} \end{aligned} \tag{1}$$

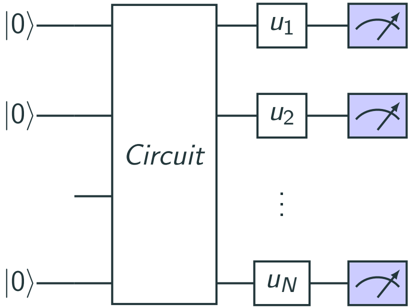
- Entanglement entropies measure quantum resources & quantum information (excellent notes from J. Preskill)
- Entanglement entropies distinguish quantum phases/dynamics in quantum simulation (ex: RMP by J. Eisert)

How to measure entanglement?

- The purity $\mathrm{Tr}(\rho^2)$ can be used to detect entanglement.
- The second Rényi entropy $S_2 = -\log_2[\mathrm{Tr}(\rho^2)]$ quantifies entanglement

How to measure the purity in an experiment? (if you cannot afford Bell-state measurements ;))

One approach: Randomized measurements



- Randomized measurements: We measure $P_u(s) = \langle s|u\rho u^\dagger|s\rangle$, $u = u_1 \otimes \cdots \otimes u_N$.
- u_i chosen independently from the circular unitary ensemble (CUE)
- We extract quantities of interest from the statistics of $P_u(s)$, over random unitary transformations.

Original protocol: van Enk-Beenakker (PRL 2012)

- Consider a single qubit

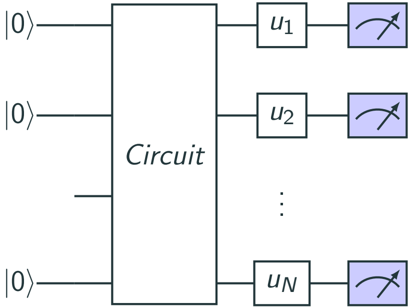
- We evaluate the statistics of
  $P_u(s) = \langle s|u\rho u^\dagger|s\rangle = \sum_{m,n} u_{s,m}\rho_{m,n}u^*_{s,n}$

$$E[[P_u(s)]^2] = \sum_{m,n,m',n'} \rho_{m,n}\rho_{m',n'}E[u_{s,m}u^*_{s,n}u_{s,m'}u^*_{s,n'}] \qquad (2)$$

- Using Random Matrix Theory (2-design identities)

$$\begin{aligned}\overline{[P_u(s)]^2} &= \frac{1}{6}\sum_{m,n,m',n'} \rho_{m,n}\rho_{m',n'}\left(\delta_{m,n}\delta_{m',n'} + \delta_{m,n'}\delta_{m',n}\right) \\ &= \frac{1+\mathrm{Tr}(\rho^2)}{6} \qquad (3)\end{aligned}$$

RM protocol for qubits (A. Elben, BV et al, PRL 2018)

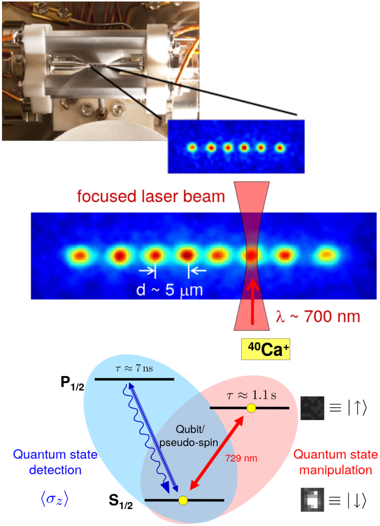


- Protocol:
  (i) apply independent random single qubit rotations
  (ii) measure states $|s\rangle = |s_1, \ldots, s_N\rangle$
  (iii) Postprocess data ($D(s,s')$ is the Hamming distance, number of mismatchs between s and s')

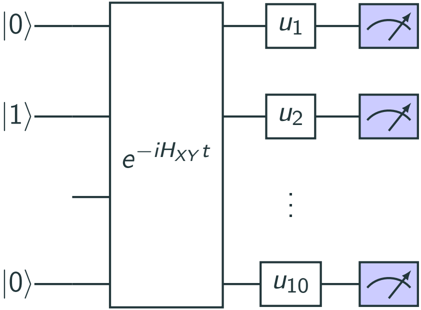
$$\mathrm{Tr}(\rho^2) = 2^N E_u \big[\sum_{s,s'} (-2)^{-D(s,s')} P_u(s) P_u(s') \big]$$

Rough measurement budget: 2^N

Demonstration with a trapped ion quantum computer (Brydges et al, Science 2019)



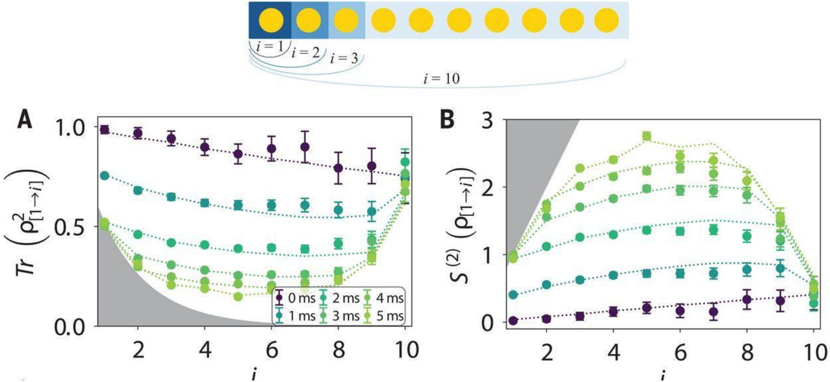
- A programmable quantum simulator



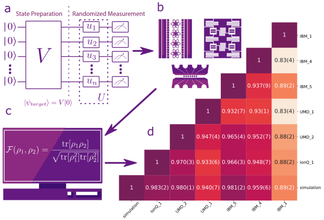
- Goal: Understand entanglement growth in a quantum system

Demonstration with a trapped ion quantum computer (Brydges et al, Science 2019)

- Demonstration of randomized measurements with the measurement of the purity



RMs are now used routinely in the lab



- Topological entanglement entropy (Satzinger et al, Science 2021)
- Cross-Platform verification (A. Elben et al, PRL 2020) and (Zhu et al, Nature Comm. 2023)
- The classical shadow formalism (H. Huang et al, Nature Physics 2020)
- Experimental discovery of the p_3-PPT condition (A. Elben, R. Kueng et al, PRL 2020), new entropies are measured (Vitale, Rath, et al 2021 2022)
- Live measurements of the purity (Stricker et al, PRXQ 2022)

Some challenges for randomized measurements

- Review: Elben, Flammia, Kueng, Preskill, BV, Zoller, Nature Review Physics 2022, is it the end?
- I dont think so, we still need
  - A Methodology to access a given physical quantity, with a more complicated form than the purity? Performance guarantees? Systematic errors versus statistical errors?
  - Practical feasibility of the RM toolbox in the many-body qubit scenario: measurement errors accumulate, postprocessing time explodes.
  - Can we reduce the measurement effort by adding prior knowledge?
  - Entangling measurements? Symmetries? Fermions? (not this talk, but check the review)
  - Learning tasks based on RM data (this was Richard's great talk!)

Outline

- Measurement of the Quantum Fisher information in a quantum processor
    - Theory: Rath, Branciard, Minguzzi, BV, Phys. Rev. Lett. 127, 260501
    - Experiment: Rath, Vitale, Elben, Branciard, BV, IBM, *in preparation*
- Boosting Randomized measurements via common random numbers
    - BV, Rath, Branciard, Sundar, Preskill, Elben, arxiv:2304.12292

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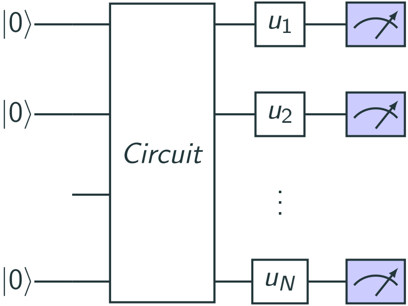


- Classical shadows (Huang et al, Nature Physics 2020)

$$\rho^{(r)} = \bigotimes_{i \in AB} \left(3 u_i \left| k_i^{(r)} \right\rangle \left\langle k_i^{(r)} \right| u_i^{\dagger} - \mathbf{1}_i \right) \tag{4}$$

- For a single measurement, unbiased estimations of the density matrix

$$E[\rho^{(r)}] = \rho \tag{5}$$

Shadows

- Classical shadows are very promising for observable estimations (ex energy estimation in quantum simulation)

$$\mathrm{Tr}(O\rho) = E[\mathrm{Tr}(\rho^{(r)}O)] \tag{6}$$

- Provide access to 'multi-copy observables' (MCO)

$$\mathrm{Tr}(O\rho^{\otimes n}) = E[\mathrm{Tr}(O\rho^{(r_1)} \otimes \cdots \otimes \rho^{(r_n)})] \tag{7}$$

Quantum Fisher information

- Quantum Fisher information (QFI) for an operator A

$$F_Q = 2 \sum_{(i,j),\lambda_i+\lambda_j>0} \frac{(\lambda_i - \lambda_j)^2}{\lambda_i + \lambda_j} |\langle i| A |j\rangle|^2 \text{ with } \rho = \sum_i \lambda_i |i\rangle \langle i| \tag{8}$$

- Certifies metrological power (Quantum Cramer Rao bound)
- Access to the entanglement depth

$$F_Q > \left\lfloor \frac{N}{k} \right\rfloor k^2 + \left(N - \left\lfloor \frac{N}{k} \right\rfloor k\right)^2 \rightarrow \text{ depth } \geq k+1 \tag{9}$$

- Grows at quantum phase transitions (see eg Hauke et al, Nature Physics 2016)

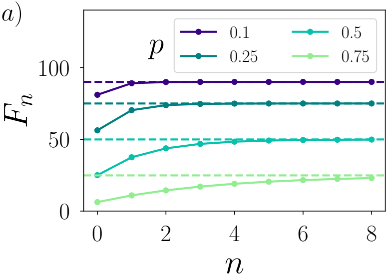
Quantum Fisher information

- Rath et al (PRL 2021): QFI is the limit of a series of MCO, i.e can be measured

$$F_n = 2\,\mathrm{Tr}\left(\sum_{\ell=0}^{n} (\rho\otimes\mathbf{1}-\mathbf{1}\otimes\rho)^2(\mathbf{1}\otimes\mathbf{1}-\rho\otimes\mathbf{1}-\mathbf{1}\otimes\rho)^{\ell}\mathbf{S}(A\otimes A)\right) \tag{10}$$

- The series convergences exponentially faster to the QFI: Example for noisy GHZ states ($N = 10$)

(a)



Quantum Fisher information and MCO: Statistical Errors

- The series of MCO F_n is estimated via randomized measurements

$$F_n = \mathrm{Tr}(O\rho^{\otimes n}) = E[\mathrm{Tr}(O\rho^{(r_1)} \otimes \cdots \otimes \rho^{(r_n)})] \qquad (11)$$

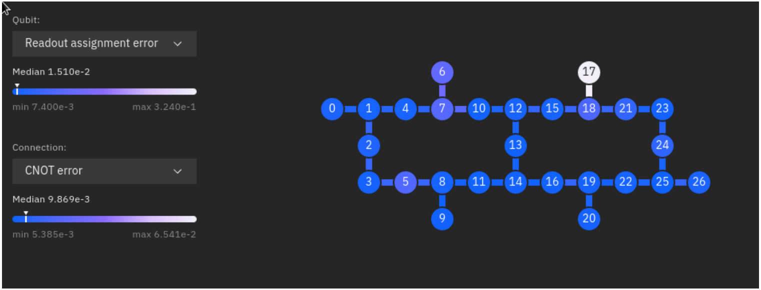
- We provide analytical variance bounds ('error bars')

$$\mathbb{V}[\hat{F}_n] \leq \sum_{k=1}^{n} \frac{n!^2 2^{kN}}{k!(n-k)!^2(M-k+1)^k} \mathrm{Tr}\left([O_k]^2\right)$$
$$O_k = \frac{1}{n!} \sum_{\pi} \mathrm{Tr}_{\{k+1 \dots n\}}(\pi^\dagger O \pi [\mathbf{1}^{\otimes k} \otimes \rho^{\otimes(n-k)}]) \qquad (12)$$

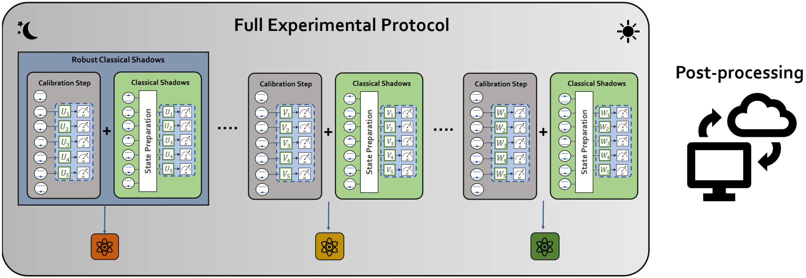
- Expressions valid for any O: $\rightarrow$ can be used for variance reductions techniques (see second part, with an example which is the von Neumann entropy)

Experimental measurements of the Quantum Fisher information

- Device: IBMQ Montreal



Our measurement strategy



- Two datasets per batch measurements: calibration on the $|0\rangle^{\otimes N}$, actual measurement
- Several batches to keep track of drifts of measurement errors (it actually matters!)

Postprocessing

- We use robust classical shadows (Chen et al, PRX 2021) under the assumption of local gate independent noise

$$\tilde{\rho}^{(r,b)} = \bigotimes_{j=1}^{N} \left(\frac{3}{2F_b[j]-1} U_j^{(r)\dagger} |s_j\rangle \langle s_j| U_j^{(r)} + \frac{F_z[j]-2}{2F_b[j]-1} \mathbf{1} \right), \tag{13}$$

  where the calibration data of each batch b gives access to $F_b[j] = \frac{1}{2}\sum_{s_j} \langle s_j| \Lambda_{j,b}(|s_j\rangle \langle s_j|) |s_j\rangle$. via direct postprocessing of the qubit marginals probabilities.
- The calibration data can be also use to verity the assumption of local noise.

Postprocessing

- Standard shadow estimation:

$$F_n = \mathrm{Tr}(O\rho^{\otimes n}) = E[\mathrm{Tr}(O\rho^{(r_1)} \otimes \cdots \otimes \rho^{(r_n)})] \tag{14}$$

averaged over M^n outcomes, $M \sim 10^{7-8}$ for $N \sim 12 - 13 \rightarrow$ impossible.

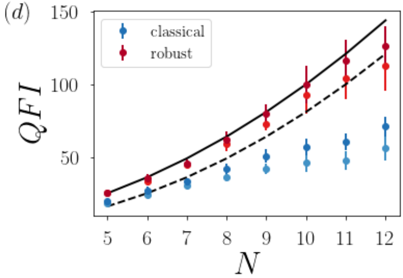
- We have introduced batch shadow estimators (thanks Richard), in Rath et al, PRXQ 2023. We build for each batch $b = 1, \ldots, n'$

$$\rho^{(b)} = \frac{n'}{M} \sum_{r=1}^{M/n'} \rho^{(b,r)} \tag{15}$$

- The asymptotic scalings of the variance are unchanged!
- Drawback: memory price $2^N \times 2^N$ (To be discussed: maybe machine-learning/tensor-network techniques can help there...)

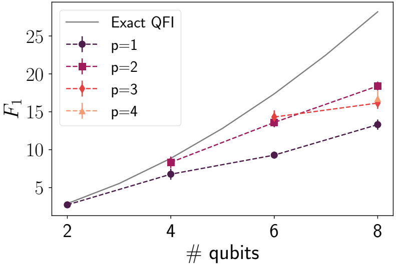
Observation of Heisebberg scaling of the QFI

- The GHZ state $|0\rangle^{\otimes N} + |1\rangle^{\otimes N}$ has Heisenberg scaling $F_Q = N^2$, offering optimal performance for quantum metrology, and showing genuine multipartite entanglement.
- Results for F_0 and F_1:



Multipartite entanglement at a quantum critical point

- We prepare the GS of the transverse Ising model via QAOA algorithm (open-loop)



- and observe the expected tradeoff as the depth p increases, wbere the QAOO circuit becomes at the same time more expressive and more subject to noise.

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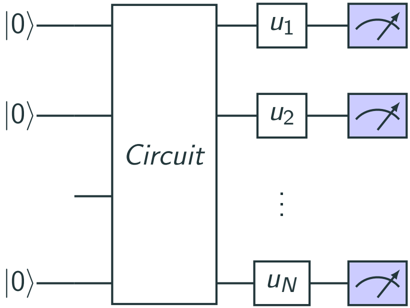
Common Randomized Measurements

- The data acquisition/postprocessing of RM is state-agnostic
- How to incorporate prior knowledge σ, density matrix from theory, in the framework, on order to reduce the measurement effort.
- With Common Randomized measurements (CRM), this information only enters during postprocessing by reducing statistical fluctuation of randomized measurements!

The two loops in randomized measurements



- Standard scenario for shadows: one measurement for each of the N_U random unitaries
- It is more convenient to repeat the protocol $N_M > 1$ times per random unitary
- Trapped ions, superconducting qubits $N_M = 100/1000$ is considered 'free', also some advantages in postprocessing.

Introducing CRM shadows

We define CRM shadows by adding a random component parametrized by the theory density matrix

$$\hat{\rho}_{\sigma}^{(r)} = \hat{\rho}^{(r)} - \sigma^{(r)} + \sigma, \tag{16}$$

where the term $\sigma^{(r)}$ is constructed from σ as

$$\sigma^{(r)} = \sum_{\mathbf{s}} P_{\sigma}(\mathbf{s}|U^{(r)})\mathcal{M}^{-1}\left(U^{(r)\dagger}\,|\mathbf{s}\rangle\,\langle\mathbf{s}|\,U^{(r)}\right), \tag{17}$$

- $P_{\sigma}(\mathbf{s}|U^{(r)})$: expected Born probability for unitary $U^{(r)}$.
- $\mathcal{M}^{-1}$: shadow channel inverse.

Insight: We reduce statistical fluctuations based on our prior knowledge and we still have an unbiased estimation of ρ.

Introducing CRM shadows

Theorem: For any multi-copy O_A acting on N_A qubits and n copies, the variance of the estimator from CRM shadows is bounded by

$$\mathbb{V}[\hat{O}] \leq \frac{n^2||O_A^{(1)}||_2^2}{N_U}\left(3^{N_A}||\rho_A - \sigma_A||_2^2 + \frac{2^{N_A}}{N_M}\right) + \mathcal{O}\left(\frac{1}{N_U^2}\right), \quad (18)$$

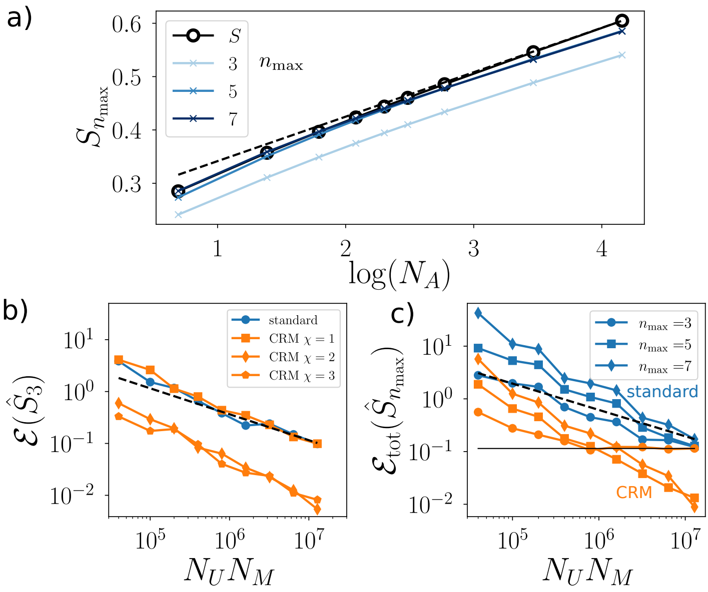
Proof: combines results on variances of multi-copy Os with approaches from 'muli-shot' shadow estimations.

- The variance is guaranteed to be reduced compared to standard shadows if $||\rho_A - \sigma_A||_2^2 \leq ||\rho_A||_2^2$, for any O_A.
- There is a way to build σ from prior experiments with shadows.

Application: Central charge measurement

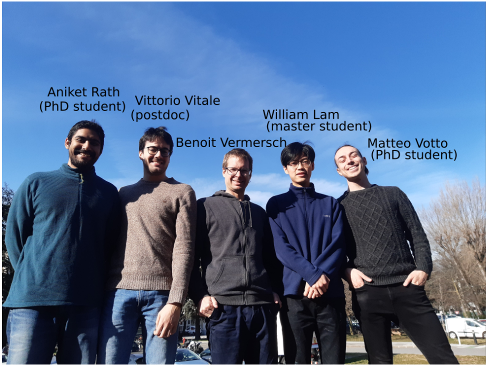
- For the ground state of $H = -\sum_{i=1}^{N} Z_i Z_{i+1} + X_i$, the von Neumann entropy $S(\rho_A) = -\mathrm{Tr}(\rho_A \log(\rho_A))$ at half-cut has a universal scaling $\sim c/12 \log(N_A)$ (Calabrese et al)
- Our idea: approximate S as polynomial function of the density matrix $S_n(\rho) = \sum_{n\geq 1} a_n \mathrm{Tr}(\rho^n)$
- Measure the multi-copy $S_n(\rho)$ via CRM shadows, using σ built from MPO

Application: Central charge measurement



Conclusion

Many open questions about quantum experiments and random datasets!



At IQOQI Innsbruck:

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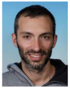

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