Quantum Algorithms 2021/2022: Exercices 1

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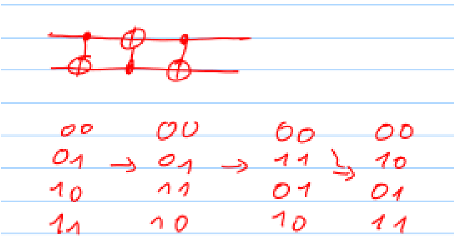
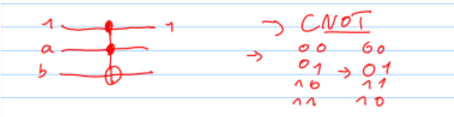
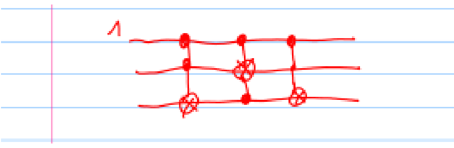
1 Universal reversible classical computing

Adapted from Jones/Jaksche, Oxford. The Toffoli gate is universal for resersible classical computing. We will illustrate this result by expressing common gates in terms of the Toffli gate.

1. Wikipedia
2. Wikipedia
3. Wikipedia
4. 

5. 

6. 

2 Universal quantum gates and measurement operations

The set of $(H, P, T, CNOT)$ form a set of universal quantum gates.

1. Wikipedia
2. Wikipedia
3. $Z = P^2$
4. $X = HZH = HP^2H$

$$\begin{aligned}
CZ &= |0\rangle\langle 0| \otimes 1 + |1\rangle\langle 1| \otimes Z \\
&= |0\rangle\langle 0| \otimes 1 + |1\rangle\langle 1| \otimes HXH \\
&= |0\rangle\langle 0| \otimes HH + |1\rangle\langle 1| \otimes HXH \\
&= (1 \otimes H)(|0\rangle\langle 0| \otimes 1 + |1\rangle\langle 1| \otimes X)(1 \otimes H) \\
&= (1 \otimes H)CNOT(1 \otimes H)
\end{aligned} \tag{1}$$

3 Measurements

1.

$$\begin{aligned}
\langle Z\rangle &= \langle\psi| Z |\psi\rangle \\
&= \langle\psi| (|0\rangle\langle 0| - |1\rangle\langle 1|) |\psi\rangle \\
&= |\langle\psi| |0\rangle|^2 - |\langle\psi| |1\rangle|^2
\end{aligned} \tag{2}$$

Therefore, we need to measure in the spin basis and substract the two measured probabilities

2.

$$\begin{aligned}\langle X\rangle &= \langle\psi| X |\psi\rangle = \langle\psi| HZH |\psi\rangle \\ &= \langle\psi| H(|0\rangle\langle 0| - |1\rangle\langle 1|)H |\psi\rangle \\ &= |\langle\psi H| |0\rangle|^2 - |\langle\psi H| |1\rangle|^2 \end{aligned} \tag{3}$$

Therefore, we need to apply H, then measure in the spin basis and substract the two measured probabilities.