

# Quantum algorithms 2021/2022: Exercices 2

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## 1 Implementation of Grover's diffuser operator

Our goal is to design a quantum circuit for Grover's diffuser operator  $U_\psi = 2|\psi\rangle\langle\psi| - 1$ , with  $|\psi\rangle = 1/\sqrt{N}\sum_x|x\rangle$ .

1. Write down a circuit  $U_1$  that prepares  $|\psi\rangle$  from  $|0\rangle^{\otimes n}$
2. Evaluate  $U_1^2$ .
3. We aim at implementing  $U_\psi$  as  $U_\psi = U_1U_2U_1$ . Write down the circuit corresponding to  $U_2$ .
4. Prove that  $U_2$  can be written as  $U_2 = -X^{\otimes n}U_3X^{\otimes n}$ , with  $U_3$  a  $n$ -qubit controlled  $Z$  gate
5. Write  $U_3$  in terms of the Toffoli gate.
6. Write and represent graphically the full circuit for  $-U_\psi$ . Comment on the role of the minus sign.